Compartimental models to predict hospital bed utilisation

Title: Using Compartmental models to predict hospital bed occupancy

Abstract The use of acute care hospitals is a significant factor in the increasing cost of health care in many developed countries.

The modelling of hospital beds should lead to better decision-making in relation to this expensive resource. The average length of stay is inappropriate for such modelling. Millard and others have shown that compartmental models can be used for bed modelling. These models are plausible and easily interpreted.

Little work in relation to generalization and predictability has been undertaken. The purpose of this paper was to consider which methodology is likely to provide the best predictive decision-making in relation to hospital bed use for medical patients based upon the work of Millard and his colleagues. Our results showed that the annual average model performed best and offers a superior predictive capability over one-day census models. Model creation should be based upon the consideration of as many points as necessary to capture the variation within the data.

Improvement in model performance may be obtained by the creation of more complex models. Consideration about the method of optimisation used to create the models is also required to ensure that it coincides with the goals of the users. Keywords: hospital beds, occupancy, length of stay, modelling, prediction The Authors: Mark Mackay BSc(Hons) BEc BComm PhD Candidate, Department of Psychology University of Adelaide And Principal Project Officer Department of Human Services South Australia Ph: 61 8 8463 6130 Fax: 61 8 8226 8910 Email: mark. edu. au

Compartmental models of hospital bed occupancy and choice of data Introduction In recent years the Australian public health sector has seen increasing pressure to do more with the same or reduced levels of expenditure.

At the same time, the number of beds in public acute care hospitals has been declining, while the level of demand and patient expectation is increasing. Consequently, the provision of public health care has become a political risk, often with debate not progressing beyond the number of hospital beds provided by the different political parties. The percentage of the gross domestic product expended on the health sector in Australia has increased since 1972. While there was a slight reduction in the level of expenditure as a percentage of GDP during the early to mid 1980s, growth has continued during the 1990s, as shown in Table 1. This phenomenon, however, is not unique to Australia, but has occurred across countries with which Australia is often compared in relation to health care, such as the United States of America, the United Kingdom and New Zealand.

Table 1: Health Expenditure Trends in Australia, New Zealand, United Kingdom and the United States of America. Based upon OECD data 2001. During the same time, the percentage of expenditure that relates to provision of public health care services in Australia has grown. Similar growth has occurred in the USA, but not in New Zealand or the United Kingdom. It is widely acknowledged in journals and numerous reviews (e.

g. , Generational Health Review, 2003; Barclay, 2003) that, particularly in view of the ageing society, which is likely to result in increased demand for services, workforce issues and rising costs, the current level of health care provision is not sustainable. 1972 1977 1982 1987 1992 1998 Australia 5. 7 7. 5 7.

3 7. 4 8. 2 8. 5 New Zealand 5. 3 6. 6 6.

1 5. 9 7. 6 8. 1 United Kingdom 4.

7 5. 4 5. 8 6 6. 9 6.

7 United States of America 7. 3 8. 4 10 10. 9 13.

6 13. 6 Australia 62. 1 61. 6 60.

3 69. 6 66. 9 69. New Zealand 77.

3 76. 3 88 87. 2 79 77. 1 United Kingdom 87. 8 90 87. 6 84.

9 85. 3 83. 7 United States of America 37. 2 40 40.

7 40. 5 41. 3 44. 7 Year Total Health Expenditure as a Percentage of GDP Public Health Expenditure as a Percentage of Total Health Expenditure Expenditure Type Country Why Model? The ability to infer the underlying process that generated observed data is the goal of most behavioural research (Myung and Pitt, 1997) and the goal of modelling the use of acute care hospital beds is no different.

Developing a formal and quantitative model allows: Interpretation, understanding, and insight into how and why the distribution changes n The ability to generalise where data are not available (e. g. , other hospitals), and n The ability to make predictions where data cannot be available (e. g. , into the future). Ultimately, the use of modelling can be applied to the issue of resource allocation, including the determination of the number of hospital beds required for service provision for a given hospital or community.

Simple Models and the Average Length of Stay The modelling of hospital beds and patient length, which are intertwined, is not new. For example, work in this area has been undertaken Yates (1982), Pendergast and Vogel (1988), and Sorensen (1996). Some of this work, such as that by Sorensen, has the development of the models reliant upon the Average Length Of Stay (ALOS), which could be argued to be a slightly more advanced approach than what is often undertaken by health care managers and clinicians. It has been recognised that the use of the ALOS for modelling hospital bed issues is flawed (e. g. , Mackay and Millard, 1999; Costa, Ridley, Shahani, Harper, De Senna and Nielsen, 2003).

There are several numerical and practical reasons that using the ALOS is inappropriate for use in the development of models. First, the length of stay profile typically has a highly skewed distribution and that is not well summarised by its mean value. Second, the length of stay distribution is complex, often consisting of mixtures of patient types (i. e.

, medical and surgical, planned and unplanned admissions, young and elderly) and mixtures of outcomes (i. e. , some patients die, some are discharged home, some to alternative care services such as nursing homes). While it might be argued that the introduction of casemix categories could reduce some of the complexity, recent work indicates that the problems associated with the average length of stay are still not overcome (Wang, Yau and Lee, 2002). Furthermore, the ALOS does not take into account the time of day when a hospital is most busy.

That is not to say, however, that some of the work previously undertaken has not yielded interesting findings, such as the focus on discharge destination by Sorensen (1996). Compartmental ModellingCompartmental models were introduced as a means of looking resource implications concerning hospital beds by Harrison, McClean and Millard (Harrison and Millard, 1991; Harrison, 1994; McClean and Millard, 1994; 1995; 1998). These models recognise that patients do not flow through the hospital in a uniform manner and describe the flow of patients through notional time-related compartments. The model work was developed using data relating to the hospitalization of geriatric patients in the South of England.

Mackay and Millard (1999; see also Mackay, 2001) extended the model to the acute care sector using data relating to the hospitalization of atients in acute care settings in Australia. Compartmental models describe the flow of something, such as patients, through a system, where the system is comprised of a finite number of homogeneous subsystems known as compartments (Godfrey, 1983). Figure 2 describes the flow of patients. According to Godfrey (1983), compartmental models have been widely applied as modelling solutions in the areas of biomedicine, pharmacokinetics and ecology. Harrison and Millard (1991) drew an analogy to the decay of drugs and the decline of patient stay over time when initially developing this form of bed modelling. Although the compartment models may consist of many compartments, work to date has focused on two or three compartment models to describe the patient stay profile within Compartment 1 – e.

g. short stay patients Compartment 2 – e. g. long stay patients Compartment n Patients enter the system Patients leave the system Patients leave the system Patients leave the system Flow to next compartment Flow to next compartment Compartments Figure 1: A diagrammatic representation of the flow of patients through the hospital system.

The compartments are notional, in the sense that the patients may not actually change location within the physical hospital. the hospital (Harrison, 1994, and Mackay, 2001), with additional compartments being added to incorporate the community (Taylor, McClean and Millard, 1996). The modelling work of Harrison was incorporated into software known as the Bed Occupancy Management and Planning System (BOMPS). This software is available as freeware at http://www2.

wmin. ac. uk/hscmg/. The software provided two mechanisms for creating the bed occupancy profiles: a daily census or an average census. Most of the work undertaken has focussed on the use of the daily census. The flow model results are plausible and easily interpreted.

However, relatively little work has focussed on the ability of these models to generalise and make predictions. For example, if the census method of model creation is employed based upon sampling data from a Monday, it is reasonable to question whether the obtained LOS distribution will generalise to data from other days. It is also not clear whether generalisability is adequately addressed in the average day census approach. The purpose of this paper is to consider which methodology is likely to provide the best results for the predictive modelling of hospital bed use under a flow model.

Methodology De-identified data from a large teaching hospital within South Australia were used as the basis for this research. The data related to patients treated within a medical division or service and therefore excluded the majority of patients who had been admitted for surgical procedures. Same-day elective or planned admission patient data were excluded from the analysis, because the beds for these patients were managed differently. Often the “ beds” were chairs, or the area was only staffed Monday to Friday during normal business days, or both, and thus these “ beds” were not generally available to all patients.

The data included the date and time of patient admission and discharge. A subset of the data was used to create a profile of the busiest time of day at the hospital based upon bed occupancy at various times of the day using the admission and discharge data. As a consequence of this analysis, instead of using midnight census data for the remainder of the analysis, a midday day bed census profile was created for each day of 1998 and 1999 calendar years. The profiles provided a count of how many patients were in bed at midday for a given date and how many days patients had been in bed (i.

e. , days since admission). The “ days since admission” profiles did not indicate the total number of beds occupied on a given day for all patients admitted on that day or before, which is how the data are represented in BOMPS software. To put the data in the format of that used in the BOMPS software, a reverse cumulative distribution was created.

This profile showed how many patients had been in bed for at least x days. Thus, at 0 days, all patients currently admitted at the midday census would have been in bed for at least 0 days. A common statistical assumption in modeling count data is that the counts follow a Poisson distribution (e. g.

, Kohler, 1985). Previous research involving the counts that define length of stay distributions in hospitals has used this assumption successfully (e. g. , Wang, Yau and Lee 2002; Xiao, Lee and Vemuri 1999).

To test of whether the current data were consistent with a Poisson distribution, the means and variances of the counts for each day of the week across the training year were calculated. The rationale for this analysis is that, to a first level of approximation, it is reasonable to assume that Mondays throughout the year are roughly consistent in their length of stay distribution, and so can be treated as repeated measures. The correlation between the means and variances over all possible length of stay, and over all days of the week, was greater than 0. 98. Since the defining characteristic of the Poisson distribution is that the mean and variance are correlated, this analysis provides a useful heuristic justification for assuming each count value is Poisson distributed, with a mean and variance given by the count itself.

The BOMPS software uses the least squares method do determine the equation that bestfits the cumulative pattern of bed occupancy. This method of analysis is easier to program but less statistically tractable. To overcome this problem Matlab software was used to create compartmental models. The

Matlab script was based upon the Poisson distribution. The models were optimised by maximizing the log likelihood values.

The choice of optimisation strategies affects the results. While the script enabled any number of compartments to be modelled, initial piloting of the software indicated that a double compartment model was appropriate to describe the data. Double compartment models provided the best balance between the fit of the model to the data, but minimizing model complexity. The analysis for this paper is therefore based upon the creation of double compartment models. A compartment model for each day of the week was created using data from 1998.

The days were chosen at random. A model for the day of minimum and maximum occupancy during the same year was also created. An annual average model was also created for the entire 1998 year. A minimum of 10 optimisations were run in an effort to ensure that the model fitting achieved levels close to the best possible fit.

The goodness-of-fit achieved by optimisation was measured for each census day model against the one day in 1998 from which it was generated, and for the annual average model against the average day for 1998. While a number of measures of goodness-of-fit are possible (Hastie, Tibshirani and Friedman, 2001), the absolute error was used as this provided a simpler statistic that could be more meaningfully interpreted. The prediction capabilities of the census day models were tested against the data for the remaining non-census days of 1998 (with 364 test days) and all of the 1999 data (with 365 test days). The prediction ability of the annual average model was also tested against the 1999 data, thus providing a means to compare performance against the census models. Occupancy, correlation and absolute error statistics were then derived for each model.

While both the absolute error and squared error can be used to measure the performance of such models ResultsThe occupancy profile based upon time since admission is shown in Figure 2. It is clear that the distribution is not Normal, but highly skewed. The absolute error statistics for each model are reported in Table 2. The absolute error statistic value was obtained from the annual average model when the performance of the models were analysed against both the training and test year data. 0 20 40 60 80 100 120 140 160 0 4 8 12 16 20 24 28 32 36 40 44 48 52 56 60 64 68 72 76 80 84 88 92 Time since admission (days) No.

of beds occupied or patients lower 95% CI average profile upper 95% CIFigure 2: The average midday census occupancy profile for 1998 with a 95 per cent confidence interval. On an average day, the number of beds occupied was 149+/-2. 5 and approximately 13 per cent of patients were admitted for more than two weeks. 1 09-Feb-99 Tuesday random 102. 0 0. 9964 920 791 2 19-Sep-98 Saturday random 84.

1 0. 9975 442 421 3 27-May-99 Thursday random 65. 3 0. 9987 663 544 4 07-Dec-98 Monday random 62.

6 0. 9988 921 796 5 28-Oct-98 Wednesday random 72. 6 0. 9969 649 536 6 10-Jan-99 Sunday random 71. 5 0.

9973 872 794 7 16-Apr-99 Friday random 100. 9 0. 9963 1, 754 1, 652 27-Jul-98 Monday max occupancy 95. 6 0.

9986 494 461 9 25-Dec-98 Friday min occupancy 48. 9 0. 9981 1, 134 1, 027 10 Annual 54. 8 0. 9992 226 244 Ave.

Absolute Error – 1999 Year Model Date Day Selection Correlation between census/annual average data and model Absolute Error – model census day/average year: 1998 year Ave. Absolute Error – 1998 Year (Excl Census Day) Figure 3 presents the results in a more visually appealing manner. It can be seen that all of the models fit the individual days on which they were trained well. This is supported by the correlations between the model and training data. However, their predictions for the remainder of 1998 are much worse than the average annual model. This of course may be attributed to the fact that the error relates to the training period for this model, whereas this is a test period for the census models.

The absolute error values that measure the performance of the models compared to the 1999 data, however, represent a period of true comparison of prediction capability for all models. It can be seen that the annual average model continues to perform best and offers a superior predictive capability over all one-day census models. 200 400 600 800 1000 1200 1400 1600 1800 2000 Absolute Error – census day/average year: training year Ave. Absolute Error – 1998 Year (Excl Census Day) Ave. Absolute Error – 1999 Year Absolute Error Type Absolute Error Model 1 (worst census model) Model 2 Model 5 Model 7 Model 8 Model 9 (best census model) Model 10 (annual average) Table 2: Absolute error statistics and correlations for each model.

The annual average model performed best, as measured by the absolute error and also correlation compared to the other models against both the 1998 and 1999 year data. The absolute error statistics derived over an entire year were averaged to enable comparison to the same statistics derived from the census day. Figure 3: Plot of Absolute error values for training and prediction situations. The annual average model performs best. Figure 4 illustrates the plot of the total occupancy with the model and lower and upper confidence intervals.

While the model captures much of the occupancy profile it does not capture the entire profile. Table 3 details the performance of the model in terms of total bed occupancy, which is the key outcome in terms of the decision-maker. 0 50 100 150 00 250 1/07/1998 1/08/1998 1/09/1998 1/10/1998 1/11/1998 1/12/1998 1/01/1999 1/02/1999 1/03/1999 1/04/1999 1/05/1999 1/06/1999 1/07/1999 1/08/1999 1/09/1999 1/10/1999 1/11/1999 1/12/1999 1/01/2000 1/02/2000 1/03/2000 1/04/2000 1/05/2000 1/06/2000 Date Total Occupancy data Model 1 Lower 95% CI Model 1 average Model 1 Upper 95% CI Figure 4: The annual average model and the training and test data. It can be seen that despite the inclusion of a confidence interval, the model does not capture 95 per cent of the data points.

The models were optimised based on log-likelihood values, which has statistical meaning. It can be seen that the average annual model (model 10) has approximately equal days of under, appropriate and over occupancy, which would be expected with such model optimisation. Discussion Hamel and Prahalad (1994) have argued that implementation failures are really past foresight problems. The creation of credible models that can be used to describe and forecast hospital bed needs can help improve decision-making relating to hospital beds and reduce foresight problems that result in poor outcomes.

Such modelling is important for three reasons. Firstly, the models can be used to assist in decision making around current and future resource utilisation, such as bed management and workforce planning, ideally leading to better outcomes. These outcomes may not solely relate to the number of beds opened or closed, but may be more diverse, such as influencing the number of training places made available for health professionals to meet future forecast demands. Secondly, the creation of a bed model also provides decision makers with the ability to pretest decisions relating to how current configurations might be changed in the future.

McClean and Millard (1995) have previously emphasized this as an important reason that such work should be undertaken. It enables decision makers to experiment with various Table 3: The adequacy of bed supply based upon the model output. The average under of over supply of beds is inadequate to fully evaluate the performance of each model. The number of days of over and under supply and the extent of under or over supply must be taken into consideration. average over or under supply of beds stdev maximum under supply of beds maximum over supply of beds no. of days insufficient beds forecast no.

f days sufficient beds forecast (occupancy 85-100%) no. of days too many beds forecast 1 8% 18% -27% 88% 110 137 118 2 26% 21% -15% 119% 48 52 265 3 7% 17% -28% 86% 114 142 109 4 5% 17% -30% 82% 132 139 94 5

15% 19% -23% 99% 81 96 188 6 8% 18% -28% 87% 110 141 114 7 0% 16% -33% 73% 177 127 61 8 59% 26% 6% 175% 0 8 357 9 -41% 10% -60% 2% 364 1 0 10 – lower 95% CI -7% 15% -38% 61% 256 89 20 10 7% 17% -29% 85% 121 137 107 10 Upper 95% CI 21% 20% -19% 110% 64 62 239 Supply statistics Model options prior to implementation, thus reducing the likelihood of mistakes occurring when the decision is implemented. Thirdly, the models can provide the foundation for more complex modelling work, assuming that they have a sound theoretical basis and capture the events being modelled in a useful manner (e. g. , El-Darzi, Vasilakis, Chaussalet and Millard, 1998).

The modelling of hospital (or institutional e. g. nursing home) beds is important, because this is where much of the service provision activity occurs in relation to admitted patients. However, linking such modelling to other activities that are involved in the total service provision may also be informative. For example, linking pharmacy, theatre, pathology, allied health and hotel services may provide a more complete picture of the relationship between different components of the acute care system.

Changes implemented in one part of the system, be it an increase in the number of beds opened or a reduction in available theatre time, are likely to influence other components of the systems. Sometimes such outcomes may be counter intuitive. The one-day bed census methodology developed by Harrison and Millard (1991) and validated and extended by Harrison (1994) and McClean (McClean and Millard 1994, 1995 and 1998) fitted the data well. The models we tested all performed well in terms absolute error and correlation statistics when measured against the training data.

The results of our work suggest, however, that a well-fitted model based upon a single day census derived model may not lead to the best model when compared to one based upon the annual average census model. The results presented in Figure 3 indicate that in terms of absolute error, the annual average model (model 10) was consistently better than the other census models when measured against both 1998 and 1999 data. This finding is consistent with the common modelling findings (e. . , Hastie, Tibshirani and Friedman, 2001; Pitt, Myung and Zhang, 2002) that the training error of a model does not predict the test error well.

The reason for the superior prediction performance of the annual average model over the census models is likely to stem from the fact that the model is based upon the consideration of far more data: approximately 23, 600 for the year compared to a maximum of 94 for any of the census days. The effect of increased number of data is to smooth the seasonal and daily patterns of fluctuation in demand and supply. The provision of medical acute care services, as with many other services, does not occur at a uniform rate. Rather, work patterns, weather or seasonal patterns, disease patterns, the ability to access other forms of care and other factors influence the number of patients admitted on any given day. Thus, an annual average model of acute hospital services will be insufficient to enable bed planning or work force planning (St George, 1988), unless length of stay distributions are very stable across the year.

However, the use of confidence intervals may overcome this shortfall to some extent. In terms of implementation, the models are not intended to suggest that a hospital manager should decide that a specific number of beds be opened all year round to avoid patient turnaway. Such a decision would result in periods of either substantial under utilisation and would be a poor use of expensive resources. Rather the model output is designed to alert managers to the range of beds that will be required throughout the year.

This can provide the manager or decision-maker with ability to plan for the staffing of extra beds or to implement alternative strategies should this not be possible prior to bed crises occurring. The actual data fell out of the 95 per cent confidence interval more than five per cent of occasions as shown in Figure 4. This can be explained by at least three factors. First, the graph is only looking at one component of the data and that is the total number of patients in bed on a given date at the midday. The model, however, fits the data to the entire time distribution of bed occupancy and not just at least zero days (or the total number of beds occupied).

Secondly, the difficulty in trying to look at occupancy performance measures is that no account of the method of model optimisation occurs. As previously mentioned, the models were not optimised for achieving a specific level of occupancy, but for maximizing log-likelihood. The method of optimisation used to create the models may be statistically useful, but may not necessarily lead to the best prediction of total beds. Thirdly, models that are more complicated may be required to adequately model the number of hospital beds required. The creation of meaningful hospital bed models requires the input of a range of expertise, including those with the appropriate modelling skills and those who understand the acute care sector. The output of such models, however, is targeted at the decision-makers.

The decision-makers are unlikely to care about the intricacies of the model as long as it provides useful output (see Table 3). It can be seen that the average annual model (model 10) has approximately equal days of under, appropriate and over occupancy, which would be expected with the method of model optimisation chosen. Log-likelihood values, however, have little meaning to those typically charged with making decisions relating to the use of hospital beds. Thus, in their eyes, it is likely that the model would be viewed as perhaps being informative, but not necessarily leading to ideal outcomes in terms of resource deployment. This highlights the need to not only consider statistical concepts when optimizing models, but also the needs of the end user. The issue of what should be optimized is not, however, a simple one.

It depends upon the goals and utilities of the decision-maker. In a hospital, different players may have different goals, such as minimizing costs compared to minimizing waiting times for admission from the Emergency Department into a bed. Future Work Given the results of our work, it is likely that models that are more complex may be required. However, increasing model complexity to achieve increased fit to the data does not come without a price. A tradeoff between fit and generalization exists and there is a need to establish performance in relation to the key capabilities of model prediction. Consideration of how statistical indicators of model performance, such as absolute error, and operational indicators, such as the level of occupancy, interact is also required.

ConclusionMillard, Harrison and McClean have previously demonstrated that the ability to create compartmental models that fit hospital bed occupancy profiles well exists. This work confirms that such models can be created for a medical acute care service. Model creation should be based upon the consideration of as many points as necessary to capture the variation within the data. Our results suggest that a single day census model is unlikely to do this. Rather, an annual average model appears to provide better performance in terms of capturing the variation within the training data and predicting future events. Improvement in model performance may be obtained by the creation of more complex models.

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Medscape Medical News 2003 http://www. medscape. com/viewarticle/449476 accessed on 20 February 2003. Costa AX, Ridley SA, Shahani AK, Harper PR, De Senna V, Nielsen MS.

(2003) Mathematical modelling and simulation for planning critical care capacity. Anaesthesia Apr; 58(4): 320-327 El-Darzi, E, Vasilakis, C, Chaussalet, T and Millard, PH (1998). A simulation modelling approach to evaluating length of stay, occupancy, emptiness and bed blocking in a hospital geriatric department. Health Care Management Science, vol 1, pp 143-149.

Generational Health Review (2003) Generational Health Review – Progress Report January 2003. Generational Health Review, Adelaide (Australia), 2003.

Godfrey, K (1983). Compartmental models and their application. Academic Press Inc. London, 1983.

Hamel, G. and Prahald, C. K. (1994). Competing for the future.

Harvard University Press, Boston. Harrison, G. W. and Millard, P.

H. (1991) Balancing acute and long term care: the mathematics of throughput in departments of geriatric medicine. Meth. Inform. Med. 30, 221-228.

Harrison, G. W. (1994) Compartmental models of hospital patient occupancy patterns. In: Millard, P.

H. and McClean, S. I. , (Eds.) Modelling hospital resource use: a different approach to the planning and control of health care systems, London: Royal Society of Medicine] Hastie, T.

, Tibshirani, R. and Friedman, J (2001). The Elements of Statistical Learnin; Data Mining, Inference and Prediction. Springer. Canada, 2001.

Kohler, H (1985). Statistics for Business and Economics. Scott, Foresman and Company. United States of America , 1985. Mackay, M. , Practical experience with bed occupancy management and planning systems: an Australian view, Health Care Management Science, 4 (2001) 47-56.

Mackay, M. and Millard, P. H. (1999) Application and comparison of two modelling techniques for hospital bed management.

Aust Health Rev 22, 118-143. McClean, S. I. and Millard, P. H. (1994) Go with the flow: Modelling bed occupancy and patient flow through a geriatric department.

OR Insight 7 (3), 2-4. McClean, S. I. and Millard, P.

H. (1995) A decision support system for bed-occupancy management and planning hospitals. IMA J. Math. App.

Med. Biol. 12, 225-234. McClean, S.

I. and Millard, P. H. (1998) A three compartmental model of the patient flows in a geriatric department: a decision support approach.

Health Care Manag Sci 1, 159-163. Myung, I. J. and Pitt, M. A. (1997).

Applying Occam's razor in modelling cognition: A Bayesian Approach. Psychonomic Bulletin and Review, v(1), 79-95. Sorensen, J (1996). Multi-phased bed modelling. Health Services Management Research, vol 9, pp 61-67.

OECD (2001). Health at a glance. OECD Publications France 2001. Pendergast, J F and Vogel, W B (1988).

A multistage model of hospital bed requirements. Health Services Research, vol 23, pp 381-399 Pitt, M. A. , Myung, I. , & Zhang, S. (2002).

Toward a method of selecting among computational models of cognition. Psychological Review, 109, 472-491. St George D. (1988).

How many beds? Helping consultants to estimate their requirements. BMJ: Sep 17; 297(6650): 729-31 Taylor, G. , McClean, S. and Millard, P.

(1996) Geriatric-patient flow-rate modelling. IMA Journal of Mathematics Applied in Medicine & Biology 13, 297-307. Wang, K, Yau, K. K.

W, and Lee, A. H. (2002). A hierarchical Poisson mixture regression model to analyse maternity length of hospital stay. Statistics in Medicine 21: 3639-3654.

Xiao, J, Lee, A. H. and Vemuri, S. R.

(1999). Mixture distribution analysis of length of hospital stay for efficient funding. Socioe-Economic Planning Sciences 33: 39-59. Xiao-Ming, H (1995).

A planning model for requirement of emergency beds. IMA Journal of Mathematics Applied in Medicine & Biology, vol 12, pp 345-353. Yates J. Hospital beds: a problem for diagnosis and management. 1 ed.

London: William Heinemann; 1982.