History of mathematics teaching in the national curriculum

This research paper is to discuss about the nature and history of mathematics, how it has taken its place within the National Curriculum; the framework for teaching Mathematics in Secondary and finally investigation on a series of three lessons designed for Year 7 on Algebra.

INTRODUCTION

Education has made a difference in my life, the knowledge I have gained has given me the potential to explore, think and make decisions accordingly. In other words, Education is a powerful tool and plays a vital role to shape up a strong economy of a country.

As a Mathematics teacher, I clearly understand my key role in imparting knowledge and skills to the younger generation to make full use of their potential.

The perception of mathematics has been changed over the years. Hence, it is important to look back at the nature of mathematics, how it has taken its place within the national curriculum; how the teaching and learning of mathematics has been guided by the National Strategies Framework.

LITERATURE REVIEW

Nature of Mathematics

Even though mathematics is one of the many subjects in schools, there is a greater pressure on pupils to succeed in Mathematics other than subjects like History, Geography; why is that so?

As part of my investigation into the nature of Mathematics I referred to two sources that gave substantial evidence towards the nature of Mathematics.

The Enquiry Committee: A Major Enquiry Committee was set up in 1978 to consider the teaching of Mathematics in Primary and Secondary schools. After 4 years of study and research the committee came out with a report called The Cockcroft Report.

‘ It would be very difficult – perhaps impossible – to live a normal life in very many parts of the world in the twentieth century without making use of mathematics of some kind.’ (The Cockcroft Report (1982), Mathematics counts)

This fact itself for a thought is sufficient to reason out the purpose of importance given in teaching and learning mathematics in Schools.

The usefulness of Mathematics can be perceived in different ways; as arithmetic skills needed to use at Home and Office, as basis for development of Science and Technology and usage of Mathematical techniques as management tool in commerce and industry. Therefore, the Enquiry Committee in their report (The Cockcroft Report) concluded that all the perceptions on usefulness of mathematics arise from the fact that mathematics provides a mean of communication which is powerful, concise and unambiguous. Hence, providing a principal reason for teaching mathematics at all stages in the curriculum.

According to American Association for the Advancement of Science (AAAS), mathematics is closely related to Science, Technology and being greatly used in real life. The association has launched a program called Project 2061 where they relate mathematics into Science and Technology.

Project 2061 is an ongoing project that was launched in 1985 in America, where its main objective is to help all Americans to literate in Science, Mathematics and Technology. As part of the project, it has been clearly defined that mathematics does play an important role in developing Science and Technology in real life.

Besides communication, Mathematics can be used to present information by using charts, graphs and diagrams. As what AAAS has mentioned about the Mathematical representation, manipulation and derivation of information based on a mathematical relationship formed; the enquiry committee as well does mention in its report the usage of figures and symbols in mathematics for manipulation and to deduce further information from the situation the mathematics relate to. They gave 3 scenarios;

A car that has travelled for 3 hours at an average speed of 20 miles per hour; we can deduce that it has covered a distance of 60 miles.

To find the cost of 20 articles each costing 3p, the area of carpet required to cover a corridor 20 metres long and 3 metres wide

In the 3 scenarios, we made use of the fact that: 20 x 3 = 60; hence it provides an illustration of the fact that the same mathematical statement can arise from and represent many different situations. This fact has important consequences. Because the same mathematical statement can relate to more than one situation, results which have been obtained in solving a problem arising from one situation can often be seen to apply to a different situation.

Thus this characteristic of Mathematics does show its importance in the study of science and Technology as mentioned by both the Enquiry committee and the programme Project 2061 (AAAS).

History of Mathematics

By looking at the history of Mathematics; it has been further proven how the development of mathematics had impact on development of Science and Technology.

The 17th century saw an unprecedented explosion of mathematical and scientific ideas across Europe. Galileo, an Italian, observed the moons of Jupiter in orbit about that planet, using a telescope based on a toy imported from Holland. Tycho Brahe, a Dane, had gathered an enormous quantity of mathematical data describing the positions of the planets in the sky. His student, Johannes Kepler, a German, began to work with this data. In part because he wanted to help Kepler in his calculations, John Napier, in Scotland, was the first to investigate natural logarithms. Kepler succeeded in formulating mathematical laws of planetary motion. This explains the relationship between mathematics and science or another word, how knowledge of mathematics has been used to develop science over the years.

The 19th century saw the beginning of a great deal of abstract algebra. Hermann Grassmann in Germany gave a first version of vector spaces, the British mathematician George Boole devised an algebra that soon evolved into what is now called Boolean algebra, in which the only numbers were 0 and 1 and in which, famously, 1 + 1 = 1. Boolean algebra is the starting point of mathematical logic and has important applications in computer science.

Abel and Galois's investigations into the solutions of various polynomial equations laid the groundwork for further developments of group theory, and the associated fields of abstract algebra. In the 20th century physicists and other scientists have seen group theory as the ideal way to study symmetry.

The 20th century saw mathematics become a major profession. Every year, thousands of new Ph. D. s in mathematics was awarded, and jobs are available in both teaching and industry.

Therefore, from the 20th Century is where importance has been given to teaching of mathematics.

National Curriculum of Mathematics

This further explains how the national curriculum for Mathematics has been formed in Britain. Let's look at the various views of Mathematics usage in Industry before the Enquiry Committee was set up;

From 1973 to 1976 there were a large volume of complaints which seemed to be coming from employers about lack of mathematical competence on the part of some school leavers;

In his speech made at Ruskin College, Oxford in October 1976, Mr James Callaghan, at that time Prime Minister, said:

' I am concerned on my journeys to find complaints from industry that new recruits from the schools sometimes do not have the basic tools to do the job that is required. … There is concern about the standards of numeracy of school leavers. Is there not a case for a professional review of the mathematics needed by industry at different levels? To what extent are these deficiencies the result of insufficient coordination between schools and industry? Indeed how much of the criticism about basic skills and attitudes is due to industry's own shortcomings rather than to the educational system?' (The Cockcroft Report (1982)

In written evidence to the Parliamentary Expenditure Committee, the Confederation of British Industry (CBI) stated:

' Employers are becoming increasingly concerned that many school leavers, particularly those leaving at the statutory age have not acquired a minimum acceptable standard in the fundamental skills involved in reading, writing, arithmetic and communication. This shows up in the results of nearly every educational enquiry made amongst the CBI membership, and is backed up by continuing evidence from training officers in industry and further education lecturers that young people at 16+ cannot pass simple tests in mathematics and require remedial tuition before training and further education courses can be started.' (The Cockcroft Report (1982)

In oral evidence to the Expenditure Committee a CBI representative stated:

' Mathematics, I think – or arithmetic, which is really the primary concern rather than mathematics themselves – is the one area which is really brought up every time as a problem. It seems that industry's needs are greater in this respect than almost any other. This is the way, certainly, in which shortfall in the education of children makes itself most manifest immediately to an employer.' (The Cockcroft Report (1982)

Written evidence to the Expenditure Committee from the Engineering Industry Training Board (EITB) stated:

' The Engineering Industry Training Board, over the last two years, received from its industry increasing criticism, with supporting evidence, of the level of attainment, particularly in arithmetical skills, of school leavers offering themselves for craft and technician training … In the view of the Engineering Industry Training Board the industry needs a higher level of attainment in basic mathematics among recruits than it is now getting and believes that, with closer cooperation between school and industry, children can while still at school be motivated to achieve this … Mathematics is, however, not simply a question of basic manipulative skills. An understanding of the concepts is also needed and these are better taught by ' innovative methods', which also appear to enhance the ability to acquire planning and diagnostic skills, of great importance to craft and technician employees.' The Cockcroft Report (1982)

These are the examples of complaints received and the main reason for the enquiry committee to set up in 1978 to investigate complaints about low levels of numeracy among young entrants to employment and the need for improved liaison between schools and industry. Hence we could deduce that due the mathematical knowledge demand in the work force has brought mathematics an important place in the national curriculum to promote numeracy skills among the young people.

Programme of Study (POS)

The national curriculum through the Mathematics Programme of Study (POS) aims to develop;

- Successful learners – where pupils should be numerate, creative and able to tackle problems with more than one approach and to solve open-ended problems.
- Confident Individuals – Pupils are given the opportunity to express their ideas using strategies that they are familiar and secure with.
- Responsible citizens – the emphasis on analyzing and justifying conclusions in mathematical situations helps prepare pupils for taking critical and analytical approaches to real-life situations.
- The framework has set out a number of key concepts that pupils need to know in order to deepen and broaden their knowledge, skills and understanding of Mathematics;
- Competence – should be able to apply a range of mathematical techniques to assess risk, problem solving and decision making
- Creativity – Able to combine understanding, experiences, imagination and reasoning to construct new knowledge and usage of existing mathematical knowledge to create solutions
- Application and Implication of Mathematics – Able to understand that mathematics is used as a tool in a wide range of contexts, such as for Financial issues, Engineering, computer security and so on
- Critical Understanding – Recognizing the limitations and scope of a model or representation. For example, mathematical skills are required to compare different methods of borrowing and paying back of money

but the final decision may rely on other factors like comparing the merits of using a credit card that might offer the lowest overall costs.

The framework has a set of key processes for both Key Stage 3 and 4 that are essential skills that pupils need to learn to make progress within the Subject.

Representing

Identify the mathematical aspects of a situation or problem, able to choose between representations to simplify a situation or problem in order to represent it mathematically, using appropriate variables, symbols, diagrams and models to select mathematical information, methods and tools to use.

Analysing

Use mathematical reasoning, pupils should be able to: make connections within mathematics use knowledge of related problems visualise and work with dynamic images identify and classify patterns; make and begin to justify conjectures and generalisations, considering special cases and counter-examples; explore the effects of varying values and look for invariance and covariance; take account of feedback and learn from mistakes; work logically towards results and solutions, recognising the impact of constraints and assumptions; appreciate that there are a number of different techniques that can be used to analyse a situation; reason inductively and deduce.

Use appropriate mathematical procedures

Pupils should be able to: make accurate mathematical diagrams, graphs and constructions on paper and on screen; calculate accurately, selecting mental methods or calculating devices as appropriate ; manipulate numbers, algebraic expressions and equations and apply routine algorithms; use accurate notation, including correct syntax when using ICT; record methods, solutions and conclusions; estimate, approximate and check working.

Interpreting and evaluating

Pupils should be able to: form convincing arguments based on findings and make general statements; consider the assumptions made and the appropriateness and accuracy of results and conclusions; be aware of the strength of empirical evidence and appreciate the difference between evidence and proof ; look at data to find patterns and exceptions; relate findings to the original context, identifying whether they support or refute conjectures; engage with someone else's mathematical reasoning in the context of a problem or particular situation; consider the effectiveness of alternative strategies.

Communicating and reflecting

Pupils should be able to: communicate findings effectively; engage in mathematical discussion of results; consider the elegance and efficiency of alternative solutions; look for equivalence in relation to both the different approaches to the problem and different problems with similar structures; make connections between the current situation and outcomes, and situations and outcomes they have already encountered.

The framework sets out an outline for teachers to follow in teaching the key concepts and key processes. The range and content for both Key stages are as follow:

Key Stage 3:

Number and algebra

rational numbers, their properties and their different representations

rules of arithmetic applied to calculations and manipulations with rational numbers

applications of ratio and proportion

accuracy and rounding

algebra as generalised arithmetic

linear equations, formulae, expressions and identities

analytical, graphical and numerical methods for solving equations

polynomial graphs, sequences and functions

Geometry and measures

properties of 2D and 3D shapes

constructions, loci and bearings

Pythagoras' theorem

transformations

similarity, including the use of scale

points, lines and shapes in 2D coordinate systems units, compound measures and conversions

perimeters, areas, surface areas and volumes

Statistics

the handling data cycle

presentation and analysis of grouped and ungrouped data, including time series and lines of best fit

measures of central tendency and spread

experimental and theoretical probabilities, including those based on equally likely outcomes. Rules of arithmetic: This includes knowledge of operations and inverse operations and how calculators use precedence. Pupils should understand that not all calculators use algebraic logic and may give different answers for calculations such as 1 + 2 X 3.

Calculations and manipulations with rational numbers: This includes using mental and written methods to make sense of everyday situations such as temperature, altitude, financial statements and transactions.

Ratio and proportion: This includes percentages and applying concepts of ratio and proportion to contexts such as value for money, scales, plans and maps, cooking and statistical information (eg 9 out of 10 people prefer).

Accuracy and rounding: This is particularly important when using calculators and computers.

Linear equations: This includes setting up equations, including inequalities and simultaneous equations. Pupils should be able to recognise equations with no solutions or an infinite number of solutions.

Polynomial graphs: This includes gradient properties of parallel and perpendicular lines.

Sequences and functions: This includes a range of sequences and functions based on simple rules and relationships.

2D and 3D shapes: These include circles and shapes made from cuboids.

Constructions, loci and bearings: This includes constructing mathematical figures using both straight edge and compasses, and ICT.

Scale: This includes making sense of plans, diagrams and construction kits.

Compound measures: This includes making sense of information involving compound measures, for example fuel consumption, speed and acceleration.

Surface areas and volumes: This includes 3D shapes based on prisms.

The handling data cycle: This is closely linked to the mathematical key processes and consists of:

- specifying the problem and planning (representing)
- collecting data (representing and analysing)
- processing and presenting the data (analysing)
- interpreting and discussing the results (interpreting and evaluating).

Presentation and analysis: This includes the use of ICT.

Spread: For example, the range and inter-quartile range.

Probabilities: This includes applying ideas of probability and risk to gambling, safety issues, and simulations using ICT to represent a probability experiment, such as rolling two dice and adding the scores.

Key Stage 4

Number and algebra

real numbers, their properties and their different representations

rules of arithmetic applied to calculations and manipulations with real numbers, including standard index form and surds

proportional reasoning, direct and inverse proportion, proportional change and exponential growth

upper and lower bounds

linear, quadratic and other expressions and equations

graphs of exponential and trigonometric functions

transformation of functions

graphs of simple loci

Geometry and measures

properties and mensuration of 2D and 3D shapes

circle theorems

trigonometrical relationships

properties and combinations of transformations

3D coordinate systems

vectors in two dimensions

conversions between measures and compound measures

Statistics

the handling data cycle

presentation and analysis of large sets of grouped and ungrouped data, including box plots and histograms, lines of best fit and their interpretation

measures of central tendency and spread

Experimental and theoretical probabilities of single and combined events.

Functional Skills in Mathematics

The revised mathematics programme of study has given importance in embedding Functional Maths into teaching. Functional Mathematics requires learners to be able to use mathematics in ways where it make them effective and involve as citizens, able to operate confidently in life and to work in a wider range of contexts. The framework has divided the functional skill into two levels, where level 1 is linked to key stage 3 and level 2 to key stage 4. (Please refer to Appendix 1)

The key concept of competence emphasises the need for students to be able to adapt and apply their understanding in a widening range of contexts within the classroom and beyond. This is also at the heart of functional skills. In this way functional skills are much more than a set of technical competencies in mathematics; students have to use mathematics to tackle tasks and problems. All teaching needs to be designed in a way that contributes to the development of functional skills.

When planning opportunities for students to develop and understand functional skills you should consider whether you have:

provided opportunities for different skills you are focusing on in representing, analysing and interpreting to be developed in combination

ensured that students understand that they are learning skills that they will use and apply in a variety of contexts

given students the chance to select the skills and tools (including ICT) they need for a particular task

provided opportunities for students to apply these skills for real purposes and contexts beyond the classroom.

For example, a year 10 project asked students to recommend to school managers a method for electing representatives for the school council. Students explored methods used in politics, including ‘ first past the post’ and different methods of proportional representation. They collected data about different voting methods and carried out simulations, which enabled them to produce a clear recommendation with justification.

This project has the potential to be developed in conjunction with ICT, English and citizenship colleagues as it addresses wider curricular issues and also offers opportunities to develop functional skills in ICT and English as well as mathematics.

The following are case studies on Functional skills taken from the National Curriculum website (http://curriculum. qcda. gov. uk);

Wellacre Technology and Vocational College

Objective:

To help learners understand the relevance of mathematics in real life

Year 9 science project and a Year 7 design and technology project. Both required pupils to solve real-world product design problems;

In the year 9 science project, skiing was used as a context for developing learners' understanding of pressure, mass, surface area and speed. Pupils had to work out how wide skis would need to be for individual pupils to ensure that their skis did not sink into the snow. This required pupils to rearrange formulae and calculate the surface area of their feet and pressure.

For the year 7 design and technology project, pupils were given a budget and challenged to raise as much money as they could for their partner school in Newcastle, South Africa. Pupils considered a range of products before settling on key fobs. Maximising the amount of profit was the main design criterion and pupils were encouraged to use tessellation to ensure their designs minimised waste. As part of the project they also use formulae to calculate break-even points, profit and loss.

In both projects, working with real figures proved both an incentive and a challenge – pupils were not able to fall back on a set of answers in a textbook. This generated discussion as pupils collaborated to check their calculations. The nature of the tasks also encouraged learners to think independently and creatively to solve problems.

Opened ended mathematical Enquiries- Lancaster Girls' Grammar School

Objective:

to develop pupils' functional mathematics and problem-solving skills

Introducing open-ended projects that required pupils to use mathematics to solve real-life problems

Mobiles and Mathematics' in year 8 and ' Music and Mathematics' in year 10.

Both projects were based around open-ended problems without a ' right' answer. Pupils were given the broad topic areas and told to devise their own projects. Pupils were given two months to prepare, which encouraged them to make their own choices about how they would work and what they would explore.

The range of investigations devised by pupils was broad. Year 8 pupils explored different tariffs, compared costs between pay as you go contracts and investigated different usage patterns of people over and under 30.

In year 10 pupils were encouraged to make links between mathematics and music. Some considered what kinds of functions might be used to model sound waves. Others explored the connections between the Fibonacci sequence and the layout of a keyboard.

In both projects, pupils defined their own problem, decided on the data to collect and how to collect it, gathered information from a number of sources, including their parents or other pupils, considered how to analyse their data, used and applied mathematics skills and drew conclusions. At the end of the projects, they presented their findings and evaluated how successful they had been.

Staff and pupils embraced the new way of working. The head of department acknowledged that ‘ it was a considerable risk to introduce this way of teaching but it paid off. Initially, staffs were concerned about setting problems when they didn’t know the answers but once the work was underway they enjoyed a different way of teaching. The projects offered opportunities to stretch pupils and encourage them to make connections between different parts of their learning.’

Many of the pupils were nervous about working on a project when they ‘ didn’t have an indication of what type of project to make’. However they soon began to enjoy the freedom of the approach. At the end of the project, a year 8 pupil reflected: ‘ This was a break from everyday work and we can use our imagination as we aren’t being spoon fed the information. We could decide what we wanted to do – I have learnt to make decisions. There were different ways to present information on this project and this made it even more exciting. I could be creative with my choices as I didn’t have to do exactly what the teacher said.’

ASSESSING PUPILS' PROGRESS IN MATHEMATICS (APP)

Finally, in my literature review, I am going to look into embedding APP guidance into teaching and learning of mathematics.

Assessing Pupils' Progress (APP) is a structured approach to periodic assessment, enabling teachers to:

- track pupils' progress over a key stage or longer;
- use diagnostic information about pupils' strengths and weaknesses to improve teaching and learning

Using APP materials, teachers can make more consistent level-related judgements in National Curriculum

The APP focuses on how as mathematics teacher can use AFL (Assessment for learning) strategy in lessons in order to generate evidence pupils' learning. The diagram shown below tells how the APP cycle works.

Review a range of evidence for periodic assessment (APP)

Collect and feedback to pupils' evidence of their progress during day to day teaching and learning

Plan for progression from learning objectives (Secondary Framework and Planning toolkit)

Make level related assessment using APP Criteria

Adjust Planning, Teaching and learning by referring to Secondary Framework

The focused assessment materials are on the APP assessment criteria and organised in National Curriculum levels. There is a set for each level from 4 to 8. The materials include examples of what pupils should know and able to do and some probing questions for teachers to initiate dialogue as to assist in their assessment judgement. The following is an example from the level 6 focused assessment materials.

Add and subtract fractions by writing them with a common denominator, calculate fractions of quantities (fraction answers); multiply and divide an integer by a fraction

Examples of what pupils should know and be able to do

Probing questions

Add and subtract more complex fractions such as

11)„ 18 + 7)„ 24, including mixed fractions.

Solve problems involving fractions,

e. g.:

In a survey of 24 pupils, 1)„ 3 liked football best, 1)„ 4 liked basketball, 3)„ 8 liked athletics and the rest liked swimming. How many liked swimming?

Why are equivalent fractions important when adding or subtracting fractions?

What strategies do you use to find a common denominator when adding or subtracting fractions?

Is there only one possible common denominator?

What happens if you use a different common denominator?

Give pupils some examples of adding and subtracting of fractions with common mistakes in them. Ask them to talk you through the mistakes and how they would correct them.

How would you justify that 4 ÷ 1)„ 5 = 20?

How would you use this to work out 4 ÷ 2)„ 5?

Do you expect the answer to be greater or less than 20? Why?

Probing questions are an important tool in a lesson as it could be used to confirm pupils' understanding in a particular topic or their misconceptions.

' Before we talked about it I always thought if the shape had three numbers you just times them. But now I know that you split the shape into rectangles and I can find the area of a rectangle. It's so easy. I understand it fully now.' (Source: APP: Secondary Mathematics Guidance)

That was a comment from a pupil after dialogue about understanding and using the formula for the area of a rectangle using the probing questions.

KANGAROO MATHS – http://www. kangaroomaths. com/index. html

Kangaroo Maths is the home page of ' Bring on the Maths' where interactive activities for teachers can be purchased from Key stage 2 to ' A' level. It has an APP page that provides supporting materials for teachers from Key stage

1 to Key stage 3. The assessment policy from the website (Appendix 5) has been rewritten to reflect the APP and to help with the on going development of APP, it has an evaluation tool (Appendix 6) where it allows teachers to self evaluate themselves in focusing, developing and establishing APP criteria with regards to pupils' engagement, lesson planning and evidence gathering. Further more, to understand the assessment criteria on the ' A3 grid', Kangaroo maths has developed the levelopaedias that provide exemplifications and probing questions for each of the assessment criteria.

DISCUSSION/FINDINGS:

To add on to my findings, I am going to look into the topic Algebra and analyse how it has developed across the levels using the APP criteria (Appendix 7a) and Kangaroo maths Level Ladders(Appendix 7b). Then, based on level 5 work on Algebra, I am going to design 3 series of lesson plans with the guidance of the level ladders.

The word ' ALGEBRA' seems to be a put off to most students when unknown numbers or using formulas to real life context. It is a topic that requires accumulative understanding building on from level 2 onwards as shown below (taken from APP guidelines);

Algebra

Level 5

Construct, express in symbolic form and use simple formulae involving one or two operations.

Level 4

Begin to use simple formulae expressed in words

Level 3

Recognise a wider range of sequences

Begin to understand