Principles of finance

Principles of Finance Bang Dang Nguyen b. nguyen@jbs. cam. ac. uk http://www. jbs. cam. ac. uk/research/faculty/nguyenb. html Principles of Finance Page 1 Course Road Map I. Present Value and Stock Valuation II. Project Appraisal and Capital Budgeting III. Risk and Return and Portfolio Selection IV. CAPM and WACC V. Capital Structure and Dividend Policy VI. Options and Real Options Principles of Finance Present Value - Page 2 Present Value - Contents - Valuing Cash Flows — The Time Value of Money — Future Value — Present Value — Value Additivity - Project Evaluation — Net Present Value — The Net Present Value Rule - Shortcuts to Special Cash Flows — Perpetuities - Growing Perpetuities — Annuities - Growing Annuities - Compound Interest Rates — Compound Interest versus Simple Interest — Discrete Compounding — Continuous Compounding — Effective Annual Yield - Adjusting for Inflation Principles of Finance Present Value - Page 3 Valuing Cash Flows - Most investment decisions involve trade-offs over time. — Within a project - Trade-off between - payoff now, or - investing now and receive payoff later — Across projects - Trade-off between - investment 1 which involves a stream of payoffs, or - investment 2 with different stream of payoffs. Problem: How do we quantitatively compare cash flows that occur at different times? What is the time value of money? Principles of Finance Present Value - Page 4 The Time Value of Money Suppose you are asked if interested in: - Investing $1 today to - Receive $0. 50 each of the next 2 years. The answer is not ambiguous: You should certainly NOT do it The reason is that having one dollar today is worth more than having the same dollar two years in the future. Principles of Finance Present Value - Page 5 Time Value of Money If you have one dollar today, you can invest. If the rate of return is 5% per year, you would receive: - $1 Ã— 1. 05 = $1. 05 one year from now If after one year you invest the principal together with the interest for a second year, you then receive: $1. 05 Ã— 1. 05 = $1. 1025 two years from now [or $1 Ã— (1. 05)2] This certainly better than the proposed $0. 5+$0. 5. Principles of Finance Present Value - Page 6 Future Value If we continued one more year, we would receive: - $1. 1025 Ã— 1. 05 = $1. 157625 three years from now [or $1 Ã— (1. 05)3] More generally, the Future Value of a cash flow of C dollars in T years when invested at a rate-of-return r is: FV(C) = $C Ã— (1 + r)T Principles of Finance Present Value - Page 7 Present Value Let us now flip the story: - Question: How much is $1 to be received in 3 years, worth to us today We know it is less than $1 … Answer: It is worth today the amount we would have to invest today to receive $1 in 3 years. Principles of Finance Present Value - Page 8 Present Value - We have seen that, if we invest $C today at a rate-of-return r, it’s Future Value in 3 years is FV(C) = $C Ã— (1 + r)3 - Hence, to receive $1 in 3 years, we must deposit today an amount $C such that FV(C) = $1 That is: $C Ã— (1 + r)3 = $1 => $C ï€½ $1 (1 ï€« r)3 Principles of Finance Present Value - Page 9 Present Value - We say the Present Value of $1 to be received in 3 years is $C ï€½ $1 (1 ï€« r)3 - If the interest rate is 5%, then the present value of $1 to be received in 3 years from now is $1/(1. 05)3 = $0. 864. More generally, the Present Value of C dollars to be received in T years, when the interest rate is r, is PV (C) ï€½ where $C = $C ï‚´ [Discount factor at r , maturity T] (1 ï€« r)T 1 (1 ï€« r)T Present Value - Page 10 [Discount factor at r , maturity T] ï€½ Principles of Finance Present Value - Example - Receive either — A. $10M in 5 years, or — B. $15M in 15 years. Which is better if r = 5% ? Calculate the respective present values: PV A ï€½ PVB ï€½ $10 = 7. 84 (1 ï€« 0. 05)5 $15 = 7. 22 (1 ï€« 0. 05)15 we find that opportunity A is worth more than B. Principles of Finance Present Value - Page 11 Value Additivity - Real and financial assets typically have cash flows that span many periods. - Assessing the desirability of real projects or financial investments consists of determining whether a series of cash flows that occur at different times is worthwhile. Question: How to derive the Present Value of the total cash flows. Answer: To aggregate cash-flows occurring at different times, calculate what each of them is worth today (Present Value of each cash flow) then add-up these Present Value calculations Principles of Finance Present Value - Page 12 Project Evaluation - Consider a firm thinking of acquiring a new computer system that will enhance productivity for five years to come. - This computer system project essentially — requires an initial investment of $1 million today, — but yields in return the following sequence of cash inflows in the future, as a result from the enhanced productivity: Year 1: Years 2, 3, 4: Year 5: $100, 000 $300, 000 $100, 000 Principles of Finance Present Value - Page 13 Project Evaluation - To assess the computer system project, we first calculate the sum of the Present Values of future cash inflows. - To do this we should discount at the rate-of-return, r, we could obtain with an equivalent investment opportunity. Here again equivalence means cash flows match in terms of timing and risk. The discount rate, r, is the risk adjusted, expected return on an equivalent investment opportunity. It is the opportunity cost of capital. Take it to be 5%. Principles of Finance Present Value - Page 14 Project Evaluation - We obtain: PV ï€½ $100, 000 $300, 000 ï€« (1 ï€« r) (1 ï€« r)2 $300, 000 $300, 000 $100, 000 ï€« ï€« ï€« (1 ï€« r)3 (1 ï€« r)4 (1 ï€« r)5 ï€½ $951, 662 - Now, the present value of cash inflows is $951, 662, but to receive this cash requires an investment of $1, 000, 000. - The computer system project is therefore not worthwhile.

That is, it is not worthwhile because it is, in comparison, not worthwhile to forego the equivalent investment opportunity. Principles of Finance Present Value - Page 15 Project Evaluation - We therefore have a very simple intuitive decision making rule to follow to evaluate projects: Invest as long as — the present value of the cash inflows is greater than — the present value of the required investments. - Here, the present value of cash inflows is $951, 662 and it requires an investment of $1, 000, 000. - Hence, the computer system project is not worthwhile. Principles of Finance Present Value - Page 16 Net Present Value - The Net Present Value (NPV) of a project is defined as — the present value of the cash inflows minus — the present value of the cash outflows. - The NPV of the computer system project is NPV = $951, 662 - $1, 000, 000 = - $48, 338 It is a negative NPV project. Principles of Finance Present Value - Page 17 Net Present Value - More generally, consider a project involving a series of cash flows Cf0, Cf1, Cf2, … CfT, occurring in 0, 1, 2, … T years, respectively. - The Net Present Value of this project is NPV ï€½ Cf0 ï€« Cf1 Cf2 CfT ï€« ï€« ... ï€« 2 (1 ï€« r) (1 ï€« r) (1 ï€« r)T Principles of Finance Present Value - Page 18 The Net Present Value Rule - If the NPV is positive, the project is worthwhile, and if the NPV is negative it is not. Taking positive NPV projects increases the value of a firm by the amount of the NPV. Taking negative NPV projects decreases the value of a firm by the amount of the NPV. Principles of Finance Present Value - Page 19 The Net Present Value Rule The Net Present Value Rule is that one should - undertake all projects that have a positive NPV and - reject all projects that have a negative NPV. - This rule tells you the obvious: You have found a good project (and should undertake it) if — you can buy something for an amount (your investment) less than — the actual value of the resulting future cash flows (the PV of cash inflows). Principles of Finance Present Value - Page 20 The Net Present Value Rule More completely, the Net Present Value Rule, not only tells us if a single or several independent projects are worthwhile, but it permits to select among several mutually exclusive projects: - 1. For a single project: — undertake the project if it has a positive NPV and — reject the project if it has a negative NPV. - 2. For many independent projects: — undertake all projects that have a positive NPV and — reject all projects that have a negative NPV. - 3. For mutually exclusive projects: — undertake the one with positive and highest NPV Principles of Finance Present Value - Page 21 2. Present Value - Contents - Valuing Cash Flows — The Time Value of Money — Future Value — Present Value — Value Additivity - Project Evaluation — Net Present Value — The Net Present Value Rule - Shortcuts to Special Cash Flows — Perpetuities - Growing Perpetuities — Annuities - Growing Annuities - Compound Interest Rates — Compound Interest versus Simple Interest — Discrete Compounding — Continuous Compounding — Effective Annual Yield - Adjusting for Inflation Principles of Finance Present Value - Page 22 Shortcuts to Special Cash Flows Recall: PV(cash flows) ï€½ Cf0 ï€« Cf1 Cf2 CfT ï€« ï€« ... ï€« (1 ï€« r) (1 ï€« r)2 (1 ï€« r)T where T is the total number of periods. - For special cases such as perpetuities and annuities it is important to develop simple expressions. - These expressions will be used for interest rates and the valuation of bonds and stocks. Principles of Finance Present Value - Page 23 Perpetuities - A perpetuity is a constant stream of cash flows, C, that occur every unit period (say year) and continues forever: $ C C C C C C C C C C C C C C ... 1 2 3 4 5 6 7 20 21 22 23 24 25 26 ... 1000 Period - Examples: — Coupon Bonds — Preferred Stock — Some specific Projects (e. g. rental arrangements)

Principles of Finance Present Value - Page 24 Perpetuities The present value of a stream of cash flows, Ct, starting in one period (year) and lasting forever, is: PV(rent) ï€½ Cf1 Cf2 Cft ï€« ï€« ... ï€« ï€« ... 2 (1 ï€« r) (1 ï€« r) (1 ï€« r)t - When each cash flow, Cft, is equal to a constant, C, we can use the Perpetuity formula (proof in Appendix A): PV(perpetuity) ï€½ ï€½ Principles of Finance C (1 ï€« r) ï€« C (1 ï€« r) 2 ï€« ... ï€« C (1 ï€« r)t ï€« ... C r Present Value - Page 25 Example of Perpetuity - Suppose you own a plot of land on which the people of your local community have for centuries performed their annual fertility celebrations. This rite is expected to continue forever. - You have an agreement to rent out the property (which is otherwise worthless) each year for $1, 000. - The city offers to buy the land from you for $17, 000. - If the interest rate remains at 5% per annum forever. Question: Should you accept the offer? Principles of Finance Present Value - Page 26 Example of Perpetuity - The value of continuing the rental agreement is obtained applying the perpetuity formula: PV(rental) ï€½ $1, 000 0. 05 ï€½ $20, 000 - The city is only offering $17, 000. Hence you should not accept. - Of course from the town's point of view, if you accepted this would be worth +$3, 000. Principles of Finance Present Value - Page 27 Example of Perpetuity - What about if the interest rate goes up to 7% per annum in two years from now, and stay there forever: - The value of renting the property out to the town becomes: $1, 000 $1, 000 $1, 000 PV(rental) ï€½ ï€« ï€« 0. 072 ï€½ $14, 817 1. 05 (1. 05)2 (1. 05) - Here, given that the city is offering $17, 000, you should accept. Principles of Finance Present Value - Page 28 Remark - Observe (as you will in many instances) that — the value of a fixed stream of cash flows goes down — when the interest rate goes up. - Essentially, when you are not selling, — today you don't have the

$17, 000 the town offers you, — instead you receive cash in the future. Principles of Finance Present Value - Page 29 Remark - You therefore — forego the opportunity to invest $20, 000 at the interest rate, r, today — to receive cash in the future. - The higher the interest rate, r, — the more attractive the opportunity you forego, hence — the less valuable cash in the future becomes. Principles of Finance Present Value - Page 30 Growing Perpetuities - A growing perpetuity is stream of cash flows , Ct, which occur every unit period (say year) and continues forever, where — the cash flow equals, C, at the end of the first year, and — grows at a constant rate, g, every unit period (year) after. $ C ... 1 2 3 4 5 6 7 20 21 22 23 24 25 26 ... 1000 Period Principles of Finance Present Value - Page 31 Growing Perpetuities - The Growing Perpetuity formula gives the present value of this stream of cash flows, when the unit period interest rate is a constant, r (proof in Appendix B): PV(grow. perp.) ï€½ C (1 ï€« g) C (1 ï€« g)2 ï€« (1 ï€« r) (1 ï€« r)2 (1 ï€« r)3 C (1 ï€« g)t ï€1 ï€« ... ï€« ï€« ... (1 ï€« r)t C ï€½ r ï€g C ï€« - Notice that if g ï‚³ r, then PV(growing perpetuity) = ï‚¥ï€ . This is not a problem, as g ï‚³ r is not economically meaningful. Principles of Finance Present Value - Page 32 Example of Growing Perpetuity - Suppose your contract with the town specifies that the rent you can charge for the land is allowed to grow by 2% p. a. each year in order to cover rising costs - If the interest rate is again r = 5% p. a. forever, $1, 000 $1, 000 ï‚´ (1. 02) ï€« 1. 05 (1. 05)2 $1, 000 ï‚´ (1. 02)2 ï€« ï€« ... (1. 05)3 - Applying the growing perpetuity formula, we obtain that the plot of land is now worth PV(grow. perp.) ï€½ PV(grow. perp.) ï€½ Principles of Finance $1, 000 ï€½ $33, 333 0. 05 ï€ 0. 02 Present Value - Page 33 A Remark on the Timing of the First Cash flow - The formulae for a perpetuity assume that the first cash flow occurs one period from now (same with growing perpetuity and annuities as seen below). - If a first cash flow occurs today, then consider that you have — an immediate first cash flow which must be added to — a standard perpetuity where cash flows start in one period. - The present value of a perpetuity with first cash flow today is PV(perp.) ï€½ C ï€« - Similarly, Principles of Finance C r PV(grow. perp.) ï€½ C ï€« C ï‚´ (1 ï€« g) r ï€g Present Value - Page 34 Annuities - An annuity is constant stream of cash flows, C, that occur every year for a fixed number of unit periods (typically years). - The annuity has a maturity of T periods (years) when T is the period (year) of the last cash flow. $ C C C C C C C C C C ... 1 2 3 4 5 6 7 T-2 T-1 T T+1 T+2 T+3 T+4 1000 Period Principles of Finance Present Value - Page 35 Annuities - The cash flows of an annuity with a maturity of T can be replicated with two perpetuities. - The formula for the value of an annuity with maturity T years is (proof in Appendix C): PV(annuity) ï€½ 1 C C ï€ ï‚´ T r r (1 ï€« r) ï€½ C ï‚´ [Annuity factor at r , maturity T] 1 ïƒ¦ 1 ïƒ¶ where [Annuity factor at r , maturity T] ï€½ r ï‚´ ïƒ§ 1 ï€ (1 ï€« r)T ïƒ· ïƒ¨ ïƒ¸ Principles of Finance Present Value - Page 36 Example of Annuity - Suppose your daughter will enter college next year, and you would like to put enough money into the bank to afford her $10, 000 for each of the next 4 years. - You therefore essentially would like to buy a security that pays her $10, 000 every year, for 4 year. - The fair price you will have to pay the bank today for this annuity is: PV(annuity) ï€½ $10, 000 $10, 000 ï€« 1 (1 ï€« 0. 05) (1 ï€« 0. 05)2 $10, 000 $10, 000 ï€« ï€« 3 (1 ï€« 0. 05) (1 ï€« 0. 05)4 Present Value - Page 37 Principles of Finance Example of Annuity - For long annuities, i. e. more than 4 years, this sort of calculations can be painful without a computer. - However we can derive the present value of the annuity, directly using the formula: PV(annuity) ï€½ $10, 000 ïƒ¦ 1 ïƒ¶ ï‚´ ïƒ§1 ï€ ïƒ· 0. 05 (1. 05)4 ïƒ¸ ïƒ¨ ï€½ $35, 460 Principles of Finance Present Value - Page 38 Growing Annuities - A growing annuity is stream of cash flows , Ct, which occur every unit period (say year) for a fixed number of periods. — the cash flow equals, C, at the end of the first period, and — grows at a constant rate, g, every period after, — until the period T, when the last cash flow occurs. The annuity is then said to have a maturity of T periods. $ C ... 1 2 3 4 5 6 7 T-2 T-1 T 23 24 25 26 1000 Period Principles of Finance Present Value - Page 39 Growing Annuities - The cash flows of an growing annuity with a maturity of T can be replicated with two growing perpetuities. - The Growing Annuity formula gives the present value of this stream of cash flows, when the interest rate is a constant, r (proof in Appendix D). PV(Gro. Ann.) ï€½ C r ï€g C ï€ 1 C (1 ï€« g)T ï‚´ r ï€g (1 ï€« r)T ïƒ¦ (1 ï€« g)T ïƒ¶ ï€½ ï‚´ ïƒ§1 ï€ ïƒ· r ï€g ïƒ¨ (1 ï€« r)T ïƒ¸ Principles of Finance Present Value - Page 40 Shortcuts: Summary - The present value of a Perpetuity is: PV(perpetuity) ï€½ C r C r ï€- g ïƒ¶ ïƒ·] ïƒ¸ - The present value of a Growing Perpetuity is: PV(grow. perp.) ï€½ - The present value of an Annuity is: 1 ïƒ¦ 1 PV(annuity) ï€½ C ï‚´ [ï‚´ ïƒ§1 ï€ r ïƒ¨ (1 ï€« r)T - The present value of a Growing Annuity is: PV(Gro. Ann.) ï€½ Principles of Finance ïƒ¦ (1 ï€« g)T ïƒ¶ ï‚´ ïƒ§1 ï€ ïƒ· r ï€g ïƒ¨ (1 ï€« r)T ïƒ¸ C Present Value - Page 41 Combining Perpetuities and/or Annuities - Real world projects often involve combinations of different types of perpetuities and/or annuities. - It is often convenient to keep track of cash flow patterns graphically. - This enables a decomposition in perpetuities and/or annuities, hence simple derivations of the Net Present Value of projects. - This best illustrated with an example: Principles of Finance Present Value - Page 42 Example: Retirement Example 1: Your favorite aunt has asked you for some advice about investing for retirement. - She wishes to work for the next 15 years. - She wishes to save enough to provide an income of $100, 000 per year starting 16 years from now and ending 25 years from now. - She currently has $100, 000 saved for retirement in the bank. - She plans to save an amount growing at 5% in each of the next 15 years ($A in one year, $A(1. 05) in two years...). - Assume she can earn a rate of return equal to 6%. What investment, $A, will be required in one year, in order for her to meet her goal? Principles of Finance Present Value - Page 43 Example: Retirement Calculate annual investment $A to meet her goal: - Work for the next 15 years - Save $A per year growing at g = 5% per year. - Currently has $100, 000 in bank - Earn $100, 000 per year from years 16 to 25 - r = 6% per year $100, 000 in bank at t= 0 t= 1 n= 15, r= 6% p. a., g= 5% p. a., $A? t= 15 n= 10, r= 6% p. a., $0. 1m t= 25 t= 16 Your problem: Solve for $A Principles of Finance Present Value - Page 44 Example: Retirement $A PV (savings) ï€½ $100, 000 ï€« r ï€g ïƒ© (1 ï€« g)15 ïƒ¹ ïƒª1 ï€ ïƒº (1 ï€« r)15 ïƒ» ïƒ« ï€½ $100, 000 ï€« 13. 25 ï‚´ $A PV (retirement income) ï€½ 1 $100, 000 ïƒ© 1 ïƒ¹ ï‚´ 15 ïƒª1 ï€ (1 ï€« r)10 ïƒº r (1 ï€« r) ïƒ« ïƒ» ï€½ $307, 111 - Since PV(retirement income) must equal PV(savings), $307, 111 ï€½ $100, 000 ï€« 13. 25 ï‚´ $A - which gives $A ï€½ 15, 631 Principles of Finance Present Value - Page 45 2. Present Value - Contents - Valuing Cash Flows — The Time Value of Money — Future Value — Present Value — Value Additivity - Project Evaluation — Net Present Value — The Net Present Value Rule - Shortcuts to Special Cash Flows — Perpetuities - Growing Perpetuities — Annuities - Growing Annuities - Compound Interest Rates — Compound Interest versus Simple Interest — Discrete Compounding — Continuous Compounding — Effective Annual Yield - Adjusting for Inflation Principles of Finance Present

Value - Page 46 Compound Interest Rates Preliminary Remark: Rates of Return versus Rates of Interest - The rate of return is what you actually care about: Rate of Return ï€½ Rate of Return ï€½ Valuetomorrow - Valuetoday Valuetoday Vt ï€« 1 ï€ Vt Vt - The rate of interest is what is quoted in the market — Always quoted on a per year (per annum) basis — Always quoted with a compounding frequency — Always quoted for a particular maturity date. Principles of Finance Present Value - Page 47 Compound Interest versus Simple Interest - Simple interest - earn no interest on interest - compound interest - earn interest on interest Simple Interest Starting Ending Balance Interest Balance 100 10 110 110 10 120 120 10 130 … … … 190 10 200 … … … 290 10 300 … … … Compound Interest Starting Ending Balance Interest Balance 100 10 110 110 11 121 121 12. 1 133. 1 … … … 135. 79 23. 58 259. 37 … … … 611. 59 61. 16 672. 75 … … … Present Value - Page 48 Year 1 2 3 10 20 Principles of Finance Discrete Compounding - Interest rates are quoted on a per year (per annum) basis. However, this does not mean that interest is paid only once a year. - Many securities pay interest — semi-annually (e. g. Corporate Bonds), — monthly (most savings accounts), — daily, or even — continually. Principles of Finance Present Value - Page 49 Discrete Compounding - If you invest $1 in a security for T years, and the compounding frequency is not annual, then — the actual rate of return you will earn is not equal to — the stated rate of interest. - Actually, future values increase with the compounding frequency. - Consequently present values decrease with the compounding frequency. - We see this with an example: Principles of Finance Present Value - Page 50 Discrete Compounding The future value of $1 in 5 years if r is 6% per year under: - annual compounding (6% every year, for 5 years) FV ï€½ $1 ï€¨1. 06 ï€© ï€½ 1. 338 5 - Semiannual compounding (6%/2 = 3% every six months, for 10 half-years) FV ï€½ $1 ï€¨1. 03ï€© 10 ï€½ 1. 344 - Monthly compounding (6%/12 = 0. 5% per month, for 60 months) FV ï€½ $1 ï€¨1. 005ï€© Principles of Finance 60 ï€½ 1. 349 Present Value - Page 51 Discrete Compounding - More generally, the future value of 1 dollar in T years from now at a discrete compounding frequency of m times per year is: m ï‚´T r ïƒ¶ ïƒ¦ FV ï€½ $1 ïƒ§1 ï€« ïƒ· mïƒ¸ ïƒ¨ - Similarly, the present value of $1 to be received in T years is PV ï€½ $1 r ïƒ¶ ïƒ¦ ïƒ§1 ï€« m ïƒ· ïƒ¨ ïƒ¸ m ï‚´T Principles of Finance Present Value - Page 52 Continuous Compounding - Continuous compounding corresponds to the limiting case where interests are paid every instant. - That is, the discrete compounding frequency (of m times per year) becomes very large, and tends to infinity. - The future value of 1 dollar in T years with continuous compounding is: m ï‚´T ïƒ©ïƒ¦ r ïƒ¶ ïƒª 1ï€« ïƒ· m ï‚® ï‚¥ ïƒªïƒ§ mïƒ¸ ïƒ« ïƒ¨ lim ïƒ¹ ïƒºï€½ ïƒº ïƒ» erï‚´T where e = 2. 718 Principles of Finance Present Value - Page 53 Continuous Compounding - The future value of 1 dollar in T years from now with continuous compounding is: FV ï€½ e r ï‚´ T - Similarly, the present value of $1 to be received in T years with continuous compounding is: PV ï€½ e - r ï‚´ T - Continuous compounding is a popular reference case because it is extremely simple to use. - It is very suitable in financial asset pricing, where uncertainty is often best captured using a continuous time framework. Principles of Finance Present Value - Page 54 Effective Annual Yield - We have seen that when the compounding frequency is not annual, — the actual rate of return you are earning is not equal to — the stated rate of interest (r% per annum). - The actual rate of return is known as the effective annual yield. - We have seen that the Future Value of 1$, increases with the compounding frequency. - This clearly means that the effective annual yield increases with the compounding frequency. Principles of Finance Present Value - Page 55 Calculating Effective Annual Yield - To calculate the effective annual yield, we relate it to the above calculation of the Future Value of 1$: — the future value of 1 dollar in T years from now at a compounding frequency of m times per year is: r ïƒ¶ ïƒ¦ FV ï€½ $1 ïƒ§1 ï€« ïƒ· mïƒ¸ ïƒ¨ m ï‚´T — in the meantime, the effective annual yield, y, is the actual return earned per year. Hence it solves: FV ï€½ $1 ï€¨1 ï€« y ï€© T Principles of Finance Present Value - Page 56 Calculating Effective Annual Yield - Therefore, with compounding frequency of m times per year, the effective annual yield, y, is: r ïƒ¶ ïƒ¦ y ï€½ ïƒ§1 ï€« ïƒ· ï€ 1 mïƒ¸ ïƒ¨ - With continuous compounding, the effective annual yield, y, is y ï€½ er ï€1 m Principles of Finance Present Value - Page 57 Annual Percentage Rate - Typically compound interest is quoted using an annual percentage rate (A. P. R.) with an associated compounding interval. Example: A bank offers a one-year Certificate of Deposit (CD) compounded semi-annually, paying 6% A. P. R. If you invest $10, 000, how much money would you have at the end of one year? In this case the six-month return is (6%)(1/2) = 3%. In the second six-month period you earn interest on your interest (compound interest). At the end of one year you have 10, 000 (1+0. 03)(1+0. 03) = 10, 000 (1. 0609) = 10, 609. You earn an effective annual rate of 6. 09% Principles of Finance Present Value - Page 58 Example 1: Mortgage Example 1: Suppose you wish to buy a house worth $500, 000 with a mortgage from a bank. The terms that seem to suit you best are as follows: - You make an immediate $100, 000 down payment - You borrow the difference, and will repay it — making constant monthly payments, C, — over 30 years. - For such mortgage terms (monthly compounding), the bank offers to lend at a fixed rate of rm = 8. 5% per annum. How is the required monthly payment, C, computed? Principles of Finance Present Value - Page 59 Example 1: Mortgage The required (fair) monthly payment, C, must cover the $400, 000 borrowed, with 30 years of payments (30x12 = 360 payments). - It therefore solves $400, 000 ï€½ ïƒ¥ t ï€½1 360 C 0. 085 ïƒ¶ ïƒ¦ ïƒ§1 ï€« 12 ïƒ· ïƒ¨ ïƒ¸ t - This is an annuity with — maturity in T = 360 periods, which here are months, and — the interest rate is 0. 085/12 per unit period. Principles of Finance Present Value - Page 60 Example 1: Mortgage - We can therefore use the annuity formula. Here, ïƒ¦ ïƒ§ C $400, 000 ï€½ ï‚´ ïƒ§1 ï€ ïƒ¦ 0. 085 ïƒ¶ ïƒ§ ïƒ§ ïƒ§ 12 ïƒ· ïƒ§ ïƒ¨ ïƒ¸ ïƒ¨ ï€½ 1 0. 085 ïƒ¶ ïƒ¦ ïƒ§1 ï€« 12 ïƒ· ïƒ¨ ïƒ¸ 360 ïƒ¶ ïƒ· ïƒ· ïƒ· ïƒ· ïƒ· ïƒ¸ C ïƒ¦ 0. 085 ïƒ¶ ïƒ§ 12 ïƒ· ïƒ¨ ïƒ¸ ï‚´ (0. 9212) Thus, C = $3, 075. Principles of Finance Present Value - Page 61 Example 1: Mortgage - Notice that the lender’s (bank) effective annual yield, y, is: y = (1 + (0. 085/12))12 - 1 = 8. 839% which is a larger figure than the quoted rm = 8. 5% per annum. - That is, the quoted rate depends on the frequency of compounding (here monthly). Principles of Finance Present Value - Page 62 Example 1: Mortgage - The monthly payment, C, is then each month broken down in (a) interest payment and (b) capital Amount $ 400, 000. 00 r(annual) 8. 50% 8 50% r(monthly) 0. 708% Month 0 1 2 3 99 100 101 359 360 Payment $ $ $ $ $ $ $ $ 3, 075. 65 3, 075. 65 3, 075. 65 3 075 65 3, 075. 65 3, 075. 65 3, 075. 65 3, 075. 65 3, 075. 65 Interest $ $ $ $ $ $ $ $ 2, 833. 33 2, 831. 62 2, 829. 89 2 829 89 2, 591. 70 2, 588. 28 2, 584. 82 43. 11 21. 63 T(years) T(month) Payment Capital $ $ $ $ $ $ $ $ 242. 32 244. 04 245. 77 245 77 483. 95 487. 38 490. 83 3, 032. 54 3, 054. 02 $ 30 360 3, 075. 65 Balance $ 400, 000. 00 $ 399, 757. 68 $ 399, 513. 64 $ 399 267 88 399, 267. 88 $ 365, 403. 68 $ 364, 916. 30 $ 364, 425. 47 $ 3, 054. 02 $ 0. 00 Present Value - Page 63 Principles of

Finance Example 2: Purchasing a New Car Example 2: You are considering the purchase of a new car, and you have been offered two different deals from two different dealerships. - Dealership A offers to sell you the car for $20, 000, but allows you to put down $2, 000 and pay back $18, 000 over 36 months (fixed payment each month) at a rate of 8% compounded monthly. - Dealership B offers to sell you the car for $19, 500 but requires a down payment of $4, 000 with repayment of the remaining $15, 500 over 36 months at 10% compounded monthly. What does your opportunity cost have to be for you to choose the offer of Dealership B? Principles of Finance Present Value - Page 64 Example 2: Purchasing a New Car First use the annuity formula to determine the monthly payments Ca and Cb for dealerships A and B, respectively, ignoring the initial down payments: - Dealership A: PVa ï€½ $18, 000 ra ï€½ 0. 08 / 12 and t ï€½ 36 months ïƒž Ca ï€½ $564. 05 - Dealership B: PVb ï€½ $15, 500 rb ï€½ 0. 10 /12 and t ï€½ 36 months ïƒž Cb ï€½ $500. 14 Principles of Finance Present Value - Page 65 Example 2: Purchasing a New Car - If the monthly discount rate is currently r, then the net present values of the two packages are NPVa ï€½ $2, 000 ï€« Ca ïƒª ï€ 36 ïƒº ïƒ« r r (1 ï€« r) ïƒ» NPVb ï€½ $4, 000 ï€« Cb ïƒª ï€ 36 ïƒº ïƒ« r r (1 ï€« r) ïƒ» It is clearly more advantageous to accept dealership A's offer if and only if NPVa < NPVb. - Substituting the expressions from above and simplifying, ïƒ©1 1 ïƒ¹ ïƒ©1 1 ïƒ¹ NPVa ï€¼ NPVb iif ïƒª ï€ ï€¼ 31. 29 36 ïƒº ïƒ« r r (1 ï€« r) ïƒ» Principles of Finance Present Value - Page 66 ïƒ©1 1 ïƒ¹ Example 2: Purchasing a New Car - By trial and error, the cross-over point is a monthly discount rate of r = 0. 00778. That is a rate of 0. 00778 ï‚´12 = 9. 34 % per annum with monthly compounding. - The conclusion is that: — If the current annual interest rate for a 36-month period (compounded monthly) is below 9. 34%, NPVb < NPVa and you should choose dealership B. — Conversely, if the current annual interest rate for a 36month period (compounded monthly) is above 9. 34%, you should choose dealership A. Principles of Finance Present Value - Page 67 2. Present Value - Contents - Valuing Cash Flows — The Time Value of Money — Future Value — Present Value — Value Additivity - Project Evaluation — Net Present Value — The Net Present Value Rule - Shortcuts to Special Cash Flows — Perpetuities - Growing Perpetuities — Annuities - Growing Annuities - Compound Interest Rates — Compound Interest versus Simple Interest — Discrete Compounding — Continuous Compounding — Effective Annual Yield - Adjusting for Inflation Principles of Finance Present Value - Page 68 Adjusting for Inflation - If you invest $1 in a bank deposit offering an interest rate of r = 10%, you are promised $1 Ã— (1+r) = $1. 10 in one year. - However, what — you will be able to purchase in one year with $1. 10 is less than what — you are able to purchase today with $1. 10 because of inflation. - Suppose the inflation over the next year is i = 6%. Then, in one year, $1. 10 will only buy the same goods as $1. 10 / 1. 06 = $1. 038 can buy today. Principles of Finance Present Value - Page 69 Adjusting for Inflation Introducing some terminology: - If you earn a 10% nominal rate, rnomi, on investment of $1 - and 6% of that return is “ eaten up" by an inflation rate, i, then your real return is: (1 ï€« rnomi) 1. 10 ï€½ $1 ï‚´ ï€½ $1. 038 $1 ï‚´ (1 ï€« i) 1. 06 - The real rate of interest, rreal , is then the rate such that $1. 038 ï€½ $1 ï‚´ (1 ï€« rreal) 1 ï€« rreal ï€½ 1 ï€« rnomi , approx. to 1ï€« i rreal ï‚» rnomi ï€ i . Principles of Finance Present Value - Page 70 Adjusting for Inflation - We have considered all cash flows as nominal cash flows, but these are hardly comparable across time. - Real cash flows are nominal cash flows adjusted for inflation. They are comparable across time in that they have the same purchasing power. A nominal cash flow, Cnomi , at a future period t is converted to a real cash flow, Creal , at a future period t as follows: C real ï€½ Cnomi . (1 ï€« i)t Principles of Finance Present Value - Page 71 Adjusting for Inflation - To calculate present values: either (a) Discount nominal cash flows, Cnomi , with nominal rate, rnomi or (b) Discount real cash flows, Creal , with real rate, rreal - Let us illustrate the validity: Principles of Finance Present Value - Page 72 Adjusting for Inflation When a bank offers a nominal interest rate of rnomi = 10%, it promises a nominal cash flow of $1. 10 next year. Still assume that the inflation rate is i = 6%. - Calculating a present value consists of either (a) discounting a nominal cash flow , Cnomi = $1. 10, received next year, with the nominal rate, rnomi = 10%, gives the correct: Cnomi $1. 10 ï€½ ï€½ $1 PV ï€½ 1 ï€« rnomi 1. 10 Principles of Finance Present Value - Page 73 Adjusting for Inflation Or, calculating a present value alternatively consists of - (b) Discounting a real cash flow , Creal, received next year, with the real rate, rreal: — The real cash flow , Creal, is the nominal cash flow adjusted for inflation: C real ï€½ Cnomi $1. 10 ï€½ . 1ï€« i 1. 06 — and the real rate, rreal , also adjusts for inflation: 1 ï€« rreal ï€½ Principles of Finance 1 ï€« rnomi 1. 10 . ï€½ 1ï€« i 1. 06 Present Value - Page 74 Adjusting for Inflation - With way (b) we also obtain the correct present value: $1. 10 C real PV ï€½ ï€½ 1. 06 ï€½ $1 1. 10 1 ï€« rreal 1. 06 - Therefore both ways (a) and (b) are equal and correct. - The most commonly employed is clearly the easier way (a), - but importantly, remember that doing so does not mean that inflation is ignored. Inflation simply cancels out when consistently using nominal cash flows and nominal interest rates. Principles of Finance Present Value - Page 75 Homework Readings: Chapters

3, 4 of Brealey, Myers, Allen. Principles of Finance Present Value - Page 76 Appendix A: Perpetuity Formula - A simple proof of the perpetuity formula is as follows (do not memorize it): V ï€½ ï€¨1 ï€« r ï€© ï€¨1 ï€« r ï€© C ï€« C ï€« C 2 ï€« C ï€¨1 ï€« r ï€© ï€« ... 3 ï€« ... (1 ï€« r) V ï€½ C ï€« ï€¨1 ï€« r ï€© ï€¨1 ï€« r ï€©2 C r C subtract the first equation from the second one r V = C (or) V = Principles of Finance Present Value - Page 77 Appendix B: Growing Perpetuity Formula - A simple proof of the growing perpetuity formula is as follows (do not memorize it): V ï€½ ï€¨1 ï€« r ï€© C ï€« C (1 ï€« g) ï€¨1 ï€« r ï€© 2 ï€« C (1 ï€« g)2 ï€¨1 ï€« r ï€© 3 ï€« ... C (1 ï€« g) C (1 ï€« g)2 C (1 ï€« g)3 (1 ï€« r) V ï€½ C ï€« ï€« ï€« ï€« ... 3 ï€¨1 ï€« r ï€© ï€¨1 ï€« r ï€©2 ï€¨1 ï€« r ï€© C (1 ï€« g) C (1 ï€« g)2 C (1 ï€« g)3 ï€« ï€« ï€« ... (1 ï€« g) V ï€½ 3 ï€¨1 ï€« r ï€© ï€¨1 ï€« r ï€©2 ï€¨1 ï€« r ï€© subtract the third equation from the second one r V - g V = C (or) V = Principles of Finance C r-g Present Value - Page 78 Appendix C: Annuity Formula Given that: annuity 1 2 3 4 5 6 7 T-2 T-1 T T+1 T+2 T+3 T+4 1000 $ C C C C C C C C C C ... Period equals $ perpetuity I C C C C C C C C C C C C C C C ... 1 2 3 4 5 6 7 T-2 T-1 T T+1 T+2 T+3 T+4 ... 1000 Period minus $ C C C C C perpetuity II (displaced) 1 2 3 4 5 6 7 T-2 T-1 T T+1 T+2 T+3 T+4 ... 1000 Period Principles of Finance Present Value - Page 79 Appendix C: Annuity Formula - The present value of perpetuity I is readily available: PV(perpetuity I) ï€½ C r - For perpetuity II it is more complicated: — The perpetuity formula gives us the value of perpetuity II, but only at date T. That is, its future value at date T. FVT (perpetuity) ï€½ C r - This FVT needs to be discounted back to date 0. The present value of perpetuity II is therefore 1 C PV(perpetuity II) ï€½ ï‚´ T r (1 ï€« r) Principles of Finance Present Value - Page 80 Appendix C: Annuity Formula - Therefore, the formula for the value of an annuity with maturity T years is: PV(annuity) = PV(Perpetuity I) - PV(perpetuity II) PV(annuity) ï€½ C 1 C ï€ ï‚´ T r r (1 ï€« r) Principles of Finance Present Value - Page 81 Appendix D: Growing Annuity Formula - We can similarly synthesize a growing annuity with cash flow C, maturity T, and growth rate, g per unit period, with — a growing perpetuity I - with starting cash flow, C, in one period - then growing at a rate g minus — a growing perpetuity II - with starting cash flows, C Ã—(1 + g)T, - then growing at a rate g, - where the series of cash flows starts in period T+1. Principles of Finance Present Value - Page 82 Appendix D: Growing Annuity Formula - The present value of the growing annuity can then be obtained as PV(Gro. Ann.) ï€½ PV(Gro. Perp. I) ï€ FVT (Gro. Perp. II) (1 ï€« r)T 1 C C (1 ï€« g)T ï€½ ï€ ï‚´ r ï€g r ï€g (1 ï€« r)T ï€½ ïƒ¦ (1 ï€« g)T ïƒ¶ ï‚´ ïƒ§1 ï€ ïƒ· r ï€g ïƒ¨ (1 ï€« r)T ïƒ¸ C Principles of Finance Present Value - Page 83