

A study on hooke's law mechanics essay



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HOOKE'S LAW: Hooke's law of elasticity is an approximation that states that the extension of a spring is in direct proportion with the load added to it as long as this load does not exceed the elastic limit. Materials for which Hooke's law is a useful approximation are known as linear elastic or "Hookean" materials.

If a metal is lightly stressed, a temporary deformation permitted by an elastic displacement of atoms in space takes place. Removal of stress results in a gradual return of metal to its original shape.

Mathematically, Hooke's law states that

Where,

x is the displacement of the end of the spring from its equilibrium position;

F is the restoring force exerted by the material; and

k is the force constant (or spring constant).

DIAGRAMMATICALLY:-

When no weight is applied to the spring, the strain is zero,

And, we can measure its length,

and when we apply a force F to the spring it stretches

And it extends length, x , that is, the strain, caused by the stress is

$$F = mg.$$

Also,

In terms of mechanics hooke's law states that:-

“ For an elastic material stress applied on a body is directly proportional to the strain produced”

That is, $\sigma \propto e$

Or $\sigma = E e$

Where,

σ is the stress applied

e is the strain developed

E is the YOUNG'S MODULUS OF ELASTICITY

Now

STRESS it is the force causing the deformation.

It is measured in units of force per unit area of cross-section (N. m^{-2}) denoted by σ (“ sigma”).

That is $\sigma = F/A$

Units of stress are Pascal...

Strain is the deformation that takes place in the body.

It is the ratio of the increase in length, ΔL to the original length (L),

Represented by symbol ϵ (“ epsilon”) or e .

That is

$$e = \Delta L / L$$

It is dimensionless.

And according to hook's law: $\sigma = E e$

Or, $E = \sigma / e$

Putting values of stress and strain in above equation we get:-

$$E = \frac{F \cdot L}{A \cdot \Delta L}$$

Young's modulus of elasticity (E) is defined as the ratio of unit stress to unit strain .

GENERALIZED HOOKE'S LAW:

The generalized Hooke's Law can be used to predict the deformations caused in a given material by an arbitrary combination of stresses.

The linear relationship between stress and strain applies for

The generalized Hooke's Law also reveals that strain can exist without stress. For example, if the member is experiencing a load in the y-direction (which in turn causes a stress in the y-direction), the Hooke's Law shows that strain in the x-direction does not equal to zero. This is because as material is being pulled outward by the y-plane, the material in the x-plane moves inward to fill in the space once occupied, just like an elastic band becomes thinner as you try to pull it apart. In this situation, the x-plane does not have

any external force acting on them but they experience a change in length. Therefore, it is valid to say that strain exist without stress in the x-plane.

STRESS-STRAIN CURVE:-

The stress-strain curve is a graphical representation of the relationship between stress, derived from measuring the load applied on the sample, and strain, derived from measuring the deformation of the sample, i. e. elongation, compression, or distortion. The nature of the curve varies from material to material.

ELASTIC LIMIT:

The elastic limit is where the graph departs from a straight line. If we go past it, the spring won't go back to its original length. When we remove the force, we're left with a permanent extension.

Below the elastic limit, we say that the spring is showing " elastic behaviour": the extension is proportional to the force, and it'll go back to it's original length when we remove the force.

Beyond the elastic limit, we say that it shows " plastic behaviour". This means that when a force is applied to deform the shape, it stays deformed when the force is removed.

YIELD POINT:

The yield strength or yield point of a material is defined in engineering and material science as the stress at which a material begins to deform plastically . Prior to the yield point the material will deform elastically and will return to its original shape when the applied stress is removed. Once the

yield point is passed some fraction of the deformation will be permanent and non-reversible.

True elastic limit:

The lowest stress at which dislocations move. This definition is rarely used, since dislocations move at very low stresses, and detecting such movement is very difficult.

Proportionality limit:

Up to this amount of stress, stress is proportional to strain hooke's law so the stress-strain graph is a straight line, and the gradient will be equal to the elastic modulus of the material.

Elastic limit (yield strength):

Beyond the elastic limit, permanent deformation will occur. The lowest stress at which permanent deformation can be measured. This requires a manual load-unload procedure, and the accuracy is critically dependent on equipment and operator skill. For elastomers such as rubber the elastic limit is much larger than the proportionality limit. Also, precise strain measurements have shown that plastic strain begins at low stresses.

Offset yield point (proof stress)

This is the most widely used strength measure of metals, and is found from the stress-strain curve. A plastic strain of 0.2% is usually used to define the offset yield stress, although other values may be used depending on the material and the application. The offset value is given as a subscript, e. g., $R_{p0.2} = 310$ MPa. In some materials there is essentially no linear region and

so a certain value of strain is defined instead. Although somewhat arbitrary, this method does allow for a consistent comparison of materials.

Upper yield point and lower yield point

Some metals, such as mild steel reach an upper yield point before dropping rapidly to a lower yield point. The material response is linear up until the upper yield point, but the lower yield point is used in structural engineering as a conservative value. If a metal is only stressed to the upper yield point, and beyond rubber band can develop.

NUMERICALS:-

Q1) When a 13.2-kg mass is placed on top of a vertical spring, the spring compresses 5.93 cm. Find the force constant of the spring.

Solution: Mass = 13.2 kg

$$\text{Weight} = 13.2 \times 9.8$$

$$= 129$$

$$\text{Compression (x)} = 5.93 = 0.0593 \text{ m}$$

From Hooke's Law: $F = kx$

The force on the spring is the weight of the object, i. e. $(F) = 129 \text{ N}$

Putting values of force and compression in above equation;

$$129 = (0.0593) \times k$$

Or,

$$k = 2181 \text{ N/m Answer}$$

Q2) A 3340 N ball is supported vertically by a 2m diameter steel cable assuming cable has a length of 10m, determine stress and strain in the cable. Young's modulus for steel is 200N/sq. m.

Solution:

$$\text{Force (F)} = 3340\text{N}$$

$$\text{Diameter} = 2\text{m}$$

$$\text{Radius (r)} = 1\text{m}$$

$$\text{Length of cable} = 10\text{m}$$

$$\text{Young's modulus (E)} = 200\text{N/sq. m}$$

Now we know,

$$\text{Stress } (\sigma) = F/A$$

$$\text{Area} =$$

$$= 3.14 \times 1^2 = 3.14$$

$$\text{So, } \sigma = 3340/3.14$$

$$\sigma = 1063.69\text{N/m. sq}$$

$$\text{Also, strain (e)} = \sigma/E$$

Putting values...

$$e = 1063.69/200$$

$$e = 5.3184$$

Answer

Q3) If a spring has a spring constant of 400 N/m, how much work is required to compress the spring 25.0 m from its undisturbed position?

Solution: spring constant (K) = 400 N/m

compression (x) = 25m

we know, force required for compression:-

$$F = kx$$

$$\text{i. e. } F = 400 \times 25 = 10000\text{N}$$

and work done = force x compression

$$w = F \times X$$

$$w = 10000 \times 25$$

$$w = 25,000 \text{ Joules Answer}$$

Q4) On a of steel rod of length 15m and diameter 6m a force of 60N is applied. Calculate the extension and new length of the rod. Young's modulus of steel is 250N/m. sq.

Solution: :

$$\text{Force (F) = 60 N}$$

$$\text{Diameter = 6m}$$

So, Radius (r) = 3m

Length (L) = 15 m

Young's modulus (E) = 250N/m. sq.

Now,

Area (A)

$$A = 3.14 \times 3 \times 3$$

$$A = 28.26 \text{ sq. m}$$

$$\text{Also, } E = \frac{F \cdot L}{A \cdot \Delta L}$$

$$\text{Or, } \Delta L = \frac{F \cdot L}{A \cdot E}$$

$$\Delta L = \frac{60 \cdot 15}{28.26 \cdot 250}$$

$$\Delta L = 0.127 \text{ m}$$

$$\text{SO, new length} = 15 + 0.127$$

$$L' = 15.127 \text{ m ANSWER}$$

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