Tutorial solutions complex numbers and sequence essay sample

1. a) Z1 = 2-5i, z2 = 1 = 2i

Z1 = 22 + 52 = 4 +25 = 29

b) = (2-5i)(1+2i) = 2 + 4i-5i +10 = 12-i

c) (2 = {(2-5i) + [3(1 +2i)]}2

= {(2-5i + 3 +6i}2

= {(5 +i}2

= 25 +10i – 1

= 24 + 10i

d) {2 = +2(1+) + (1 +)2

= (2-5i)2 = 4- 10i +25 = 29-10i

2(1+) = 2(2-5i)(1 +1 +2i) =(4- 10i)(2 +2i) = 3 + 8i -2oi + 20

= 28- 12i

(1 +)2 = (1 + 1 = 2i)2 =(2 +2i)2 = 4 + 8i- 4 = 8i

{2 = 29-10i + 28- 12i + 8i = 57-14i

2. a) = =

= +

b) (2 +i)3 = 23 + 3(22i) + 3(2i2) +i3 = 8 +12i-6-I = 2 + 11i

c) 3 + = 3i + i

= + i

= 15 +6i

= 21 i

d) - = = =

3. a)

z = +i and w = 1 +

Z = r[cosθ + isinθ]

θ = =

r = = 2

z = 2[cos +isin]

w = r1 (cos α + isinα)

r = = 2

θ = =

w = 2[cos+ isin]

zw = rr1[cos(θ +α) + isin (θ +α)]

= 2*2 [cos + isin

= 4[cos + isin

= [[cos + isin

= cos - + isin

= cos - isin

= r[cosθ - isinθ]= 2[cos - isin]

b) z = +4i and w =-3 -

Z = r[cosθ + isinθ]

θ = =

r = = = 8

Z = 8[cos + isin]

w =-3 -

w = r1 (cos α + isinα

α = = = ; since it falls within the third quadrant

r = = = 3

w = 3 [cos+ isin]

zw = rr1[cos(θ +α) + isin (θ +α)]

= 8* 3 [cos + isin

= 24 [cos + isin

= [[cos + isin

= [[cos + isin]

=[cos + isin]

= r[cosθ - isinθ]= 8[cos - isin]

4. show that

a) = +

Let Z1 = a +bi and Z2 = x + yi

Z1 = a -bi and Z2 = x - yi

= . =

=

b) = .

Let Z1 = (r, θ) = r[cosθ +isinθ], = r[cosθ - isinθ],

And Z2 = (R, α) = R [cos α + isinα], = R [cos α - isinα]

= Rr [cos(θ +α) - isin (θ +α)

= R. r but R = and r =

=

c) =

z = r[cosθ + isinθ]

r. r[cos 2θ + isin2θ]

= r2 , but = r

= .

d) Z= r = r[cos θ + isinθ]

r[cos θ - isinθ]

But = cos θ - isinθ

r

5. a) (z +1)(2-i) = 3-4i

z +1 = = = = 2- i

b) = 1

let w = 3 +4i

= = 5

= = 1

= 1

= 1/5

= = = 1/5

6. Z5 = -32

Let z =(ρ, θ) and w = (r, α) = -32

ρ5 [cos 5θ + isin 5θ] = r [cos α + isinα]

ρ= = 2, α = π

θ =

for other solutions of z, θ = + k2π/5

k = 0

z = 2{cos }

k = 1 θ = + =

z = 2{cos }

k= 2, θ = + =

z = 2{cos }= -2

k= 3, θ = + =

Z= 2{cos}

K= 4, θ = + =

z = 2{cos }

b) Z4 +8i = 0

Z4 = -8i σ= falls within third quadrant

R = = 1. 682

4θ =

θ =

k = 0

z = 1. 682[cos + isin

k = 1

θ = + =

z= 1. 682[cos + isin

k= 2

θ = + =

z= 1. 682[cos + isin

k= 3

θ = + =

z= 1. 682[cos + isin

c) z3 = -1 +

R = = = 1. 587

3θ = π - = π-π/3 =

θ =

at k = 0

tz= 1. 587[cos + isin

k= 1

θ = + =

z= 1. 587[cos isin

k= 2

θ = + =

z= 1. 587[cos isin

7. Z2 –iz + (1 +3i) = 0

The sum of numbers = -i

Product = (1 +3i)

Nos. (1-2i) and (-1 + i)

Z = -(1-2i)

and z = -(-1+i)

=-1 + 2i

= 1-i

The roots of z are (-1 +2i) and (1-i)

Sequence and series tutorial solutions

1.

a) {2, 5, 8, 11} an = 2 + 3(n-1) , where n= 1, 2, 3,

b) {1/2, ¼, 1/6} aa = 1/(2n), where n= 1, 2, 3,

c) {-1, -1/3, 3/5, -5/6} an = (-1)n

d) {1, 0, 1, 0, 1} an =

e) {0, ½, 0, ½, 0} an =

2.

a) {n-} = ∞

Thus is the limit doess not exist and its divergent

b)

= =

the function converges to zero at infinity

c) = = = = ∞ ,

Since

The limit does not exist thus it is divergent.

d) Sin (+) = = 1

Since

+++++++++++++

Since

++++++

e) = = ∞

Since

++++++++++++++

Since

++++++++++++++*+++

f) = = . = 1 . 1 = 1

Since

+++++++++++++++

Since

++++++++++++++*+++++++

g) = =

= 0

Since

++++++++++++++++

Since

+++++++

h) =

= ln π = 0

= = 1

Since

+++++++++++++++++

Since

++++++++++++

i) + = ln π += 0

Since

++++++++++++++++++

Since

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3.

a) 5 + 3 + + + +

Since

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Since

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= 3/5 < 1 thus the series converges.

Since

++++++++++++++++++++*+++

b) 2 + 0. 4 + 0. 08 + 0. 016 + +

Since

++++++++++++++++++++

Since

the series converge ; since = 0. 2 < 1the sum = = = = c) 1/9 -1/3 + 1- 3 + (-

1)(n+1)a(3)n-1

Since

+++++++++++++++++++++

Since

+++++++

d) + + + +

Since

+++++++++++++++++++++++*+

e)

Since

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Since

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The sum = 0

f) = 1/5 + 1/50 + 1/500 + 1/5000 + + a(1/10)n-1

Since

+++++++++++++++++++++++++

Since

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The series converges since = 1/10 < 1The sum = = = g)

Since

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Sk= - }+{ } + + + + + +

Sk = ½ + ¼ = ¾

= sk = ¾

4)

a)

Since

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And = thus the series is divergent.

b)

Since

++++++++++++++++++++++++++++

Since

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Therefore; = ∞

Thus the series is divergent

c)

Since

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= 0 an converges, so that = 0

Since

+++++++++++++++++++++++++++++++*++++

d)

Since

+++++++++++++++++++++++++++++

e)

Since

+++++++++++++++++++++++++++++

5. 1 +2r + r2 +2r3 +r4 + 2r5+ r6 +

= {1 + r2+r4+ r6 + + r2n} +{2r + +2r3 + 2r5 ++ 2r2n+1}

Since

+++

if the series converge

-1

Since

++++++++++++++++

-1

Since

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6. determine whether the series is absolutely convergent, conditionally convergent or divergent

a)

= the series is absolutely convergent.

b) -

= - = 1-1 = 0

Since

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c)

= . = 0

Since

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d)

= = 1 test inclusive

Since

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= {ln (x +5)} = the series is divergent

7. Find the radius of convergence and interval of convergence of the power

aeries

a)

= = . .=

Since

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2

I (-8, 2)

b)

=. = 0 < 1

Since

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Since

++

c)

= . =

Since

++++++++++++++++++++++

For so that I(-4, 3)

d)

= 3x. = 3x

Since

+++++++++++++++++++++++

For

Since

+++++++++

e)

= . =

Since

++++++++++++++++++

For

Since

++++++++++++

f)

= . =

Since

+++++++++++++++++++

At

Since

++++++++++++++++++++++++

g)

=

Since

++++++++++++++++++++++

At

Since

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8. the Taylor series for the functions

a) f(x) = 3x2 + 2x + 1 at a = 3

Since

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f1(3) = 6x+ 2 = 20

f11(3) = 6

f(x) = 34 + 20(x-3) + (x-3)2

b)

f(x) = sinx a = π/2

Since

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g(x) =

Since

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h1(

Since

++++

h111(

h(x) = 1- +

f(x) = g(x)h(x)

f(x) = {[1-]. [

f(x)=

f(x)=

c) f(x) = 3x a = 1

Since

+++

f1(x) = 3x ln3 f(1) = 3 ln3

f11(x) = 3x ln3. ln3; f11(1) = 3 (ln3)2

f111(x) = 3x ln3. ln3 . ln3 ; f111(1) = 3(ln3)3

f(x)= 3 + 3 ln3(x-1) + + +

d) f(x) = a = 2

Since

+++

f1(x) = , f1(2) = 1

f11(x) = f11(2) = 2

f111(x) = , f111(2) = 6

f(x) = 1 + + + +

9. Maclaurin series for the following functions

a) f(x) =

Since

+++

f1(x) = f1(0) = 1

f11(x) = f11(0) = 2

f111(x) = f111(0) = 6

f(x) = 1 + x + + 6 +

f(x) = 1+ x+ x2 + x3 + + xn

b)

Since

+++++++

f(0) = 1

f1(x) = -3sinx = 0 at x = 0

f11(x) = -9cos x = -9 at x= 0

f111(x) = 27sin x = 0

f111(x) = 81cos x = 81

f1v(x) = -243sin x = 0

fv(x) = -729cosx = -729

f(x) = 1 -3 + 81 -729 + +

c) f(x) = x

Since

+++

f1(x) = = 1 at x = 0

f11(x) = = 2

f111(x) = = 3

f1v(x) = = 4

fv(x) = = 5

f(x) = 0 + x + 2 + 3 + 4 +

f(x) = x + + + + +