

# [Tutorial solutions complex numbers and sequence essay sample](https://assignbuster.com/tutorial-solutions-complex-numbers-and-sequence-essay-sample/)

1. a) Z1 = 2-5i, z2 = 1 = 2i

## Z1 = 22 + 52 = 4 +25 = 29

b) = (2-5i)(1+2i) = 2 + 4i-5i +10 = 12-i

c) (2 = {(2-5i) + [3(1 +2i)]}2

= {(2-5i + 3 +6i}2

= {(5 +i}2

= 25 +10i – 1

= 24 + 10i

d) {2 = +2(1+) + (1 + )2   
= (2-5i)2 = 4- 10i +25 = 29-10i

## 2(1+) = 2(2-5i)(1 +1 +2i) =(4- 10i)(2 +2i) = 3 + 8i -2oi + 20

= 28- 12i   
(1 + )2 = (1 + 1 = 2i)2 =(2 +2i)2 = 4 + 8i- 4 = 8i

{2 = 29-10i + 28- 12i + 8i = 57-14i

2. a) = =

= +   
b) (2 +i)3 = 23 + 3(22i) + 3(2i2) +i3 = 8 +12i-6-I = 2 + 11i

c) 3 + = 3i + i   
= + i   
= 15 +6i   
= 21 i   
d) - = = =   
3. a)

## z = +i and w = 1 +

Z = r[cosθ + isinθ]

θ = =

r = = 2   
z = 2[cos +isin]   
w = r1 (cos α + isinα)   
r = = 2   
θ = =   
w = 2[cos+ isin]   
zw = rr1[cos(θ +α) + isin (θ +α)]

= 2\*2 [cos + isin

= 4[cos + isin   
= [[cos + isin   
= cos - + isin   
= cos - isin   
= r[cosθ - isinθ]= 2[cos - isin]

b) z = +4i and w =-3 -

## Z = r[cosθ + isinθ]

θ = =

r = = = 8   
Z = 8[cos + isin]   
w =-3 -   
w = r1 (cos α + isinα   
α = = = ; since it falls within the third quadrant

r = = = 3   
w = 3 [cos+ isin]

zw = rr1[cos(θ +α) + isin (θ +α)]

= 8\* 3 [cos + isin

= 24 [cos + isin   
= [[cos + isin   
= [[cos + isin]   
=[cos + isin]   
= r[cosθ - isinθ]= 8[cos - isin]   
4. show that   
a) = +

## Let Z1 = a +bi and Z2 = x + yi

Z1 = a -bi and Z2 = x - yi

= . =   
=

b) = .

## Let Z1 = (r, θ) = r[cosθ +isinθ], = r[cosθ - isinθ],

And Z2 = (R, α) = R [cos α + isinα], = R [cos α - isinα]   
= Rr [cos(θ +α) - isin (θ +α)

= R. r but R = and r =   
=

c) =

## z = r[cosθ + isinθ]

r. r[cos 2θ + isin2θ]   
= r2 , but = r   
= .

d) Z= r = r[cos θ + isinθ]

## r[cos θ - isinθ]

But = cos θ - isinθ

r   
5. a) (z +1)(2-i) = 3-4i

## z +1 = = = = 2- i

b) = 1

## let w = 3 +4i

= = 5   
= = 1   
= 1   
= 1/5   
= = = 1/5

6. Z5 = -32

## Let z =(ρ, θ) and w = (r, α) = -32

ρ5 [cos 5θ + isin 5θ] = r [cos α + isinα]   
ρ= = 2, α = π   
θ =

## for other solutions of z, θ = + k2π/5

k = 0

## z = 2{cos }

k = 1 θ = + =   
z = 2{cos }   
k= 2, θ = + =   
z = 2{cos }= -2   
k= 3, θ = + =   
Z= 2{cos}   
K= 4, θ = + =   
z = 2{cos }

b) Z4 +8i = 0

## Z4 = -8i σ= falls within third quadrant

R = = 1. 682   
4θ =   
θ =   
k = 0

## z = 1. 682[cos + isin

k = 1   
θ = + =

## z= 1. 682[cos + isin

k= 2   
θ = + =

## z= 1. 682[cos + isin

k= 3   
θ = + =

## z= 1. 682[cos + isin

c) z3 = -1 +

## R = = = 1. 587

3θ = π - = π-π/3 =

θ =   
at k = 0   
tz= 1. 587[cos + isin   
k= 1

θ = + =

## z= 1. 587[cos isin

k= 2   
θ = + =

## z= 1. 587[cos isin

7. Z2 –iz + (1 +3i) = 0

## The sum of numbers = -i

Product = (1 +3i)   
Nos. (1-2i) and (-1 + i)   
Z = -(1-2i)   
and z = -(-1+i)

=-1 + 2i

= 1-i

## The roots of z are (-1 +2i) and (1-i)

Sequence and series tutorial solutions   
1.   
a) {2, 5, 8, 11} an = 2 + 3(n-1) , where n= 1, 2, 3,

b) {1/2, ¼, 1/6} aa = 1/(2n), where n= 1, 2, 3,

c) {-1, -1/3, 3/5, -5/6} an = (-1)n   
d) {1, 0, 1, 0, 1} an =   
e) {0, ½, 0, ½, 0} an =   
2.   
a) {n-} = ∞

## Thus is the limit doess not exist and its divergent

b)   
= =

## the function converges to zero at infinity

c) = = = = ∞ ,

## Since

The limit does not exist thus it is divergent.   
d) Sin (+ ) = = 1

## Since

+++++++++++++

## Since

++++++   
e) = = ∞

## Since

++++++++++++++

## Since

++++++++++++++\*+++   
f) = = . = 1 . 1 = 1

## Since

+++++++++++++++

## Since

+++++++++++++++\*+++++++   
g) = =   
= 0

## Since

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## Since

+++++++   
h) =   
= ln π = 0   
= = 1

## Since

+++++++++++++++++

## Since

++++++++++++   
i) + = ln π += 0

## Since

++++++++++++++++++

## Since

++++++++++++++   
3.   
a) 5 + 3 + + + +

## Since

+++++++++++++++++++

## Since

+++++++   
= 3/5 < 1 thus the series converges.

## Since

++++++++++++++++++++\*+++   
b) 2 + 0. 4 + 0. 08 + 0. 016 + +

## Since

+++++++++++++++++++++

## Since

the series converge ; since = 0. 2 < 1the sum = = = = c) 1/9 -1/3 + 1- 3 + (-1)(n+1)a(3)n-1

## Since

++++++++++++++++++++++

## Since

+++++++   
d) + + + +

## Since

+++++++++++++++++++++++\*+   
e)

## Since

++++++++++++++++++++++++

## Since

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The sum = 0

f) = 1/5 + 1/50 + 1/500 + 1/5000 + + a(1/10)n-1

## Since

+++++++++++++++++++++++++

## Since

++++++++   
The series converges since = 1/10 < 1The sum = = = g)

## Since

++++++++++++++++++++++   
Sk= - }+{ } + + + + + +   
Sk = ½ + ¼ = ¾   
= sk = ¾

4)   
a)

## Since

++++++++++++++++   
And = thus the series is divergent.   
b)

## Since

++++++++++++++++++++++++++++

## Since

++++   
Therefore; = ∞   
Thus the series is divergent   
c)

## Since

++++++++++   
= 0 an converges, so that = 0

## Since

++++++++++++++++++++++++++++++\*++++   
d)

## Since

+++++++++++++++++++++++++++++   
e)

## Since

+++++++++++++++++++++++++++++   
5. 1 +2r + r2 +2r3 +r4 + 2r5+ r6 +   
= {1 + r2+r4+ r6 + + r2n} +{2r + +2r3 + 2r5 ++ 2r2n+1}

## Since

+++

if the series converge

-1

## Since

+++++++++++++++++

-1

## Since

++++++++++++++++++++++++++   
6. determine whether the series is absolutely convergent, conditionally convergent or divergent   
a)

= the series is absolutely convergent.   
b) -

= - = 1-1 = 0

## Since

+++++++++++++++++++++++++++++++++   
c)   
= . = 0

## Since

+++++++++++++++++++++++++++++++++   
d)

= = 1 test inclusive

## Since

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= {ln (x +5)} = the series is divergent

7. Find the radius of convergence and interval of convergence of the power aeries   
a)   
= = . .=

## Since

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2   
I (-8, 2)

b)   
=. = 0 < 1

## Since

++++++++++++++++++++++++++++++++++++++++

## Since

++   
c)

= . =

## Since

+++++++++++++++++++++++   
For so that I(-4, 3)   
d)   
= 3x. = 3x

## Since

+++++++++++++++++++++++   
For

## Since

+++++++++   
e)   
= . =

## Since

++++++++++++++++++   
For

## Since

++++++++++++   
f)   
= . =

## Since

+++++++++++++++++++   
At

## Since

++++++++++++++++++++++++   
g)   
=

## Since

++++++++++++++++++++++   
At

## Since

++++++++++++++++++

8. the Taylor series for the functions   
a) f(x) = 3x2 + 2x + 1 at a = 3

## Since

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f1(3) = 6x+ 2 = 20

f11(3) = 6   
f(x) = 34 + 20(x-3) + (x-3)2   
b)   
f(x) = sinx a = π/2

## Since

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g(x) =

## Since

++++   
h1(

## Since

++++

h111(   
h(x) = 1- +   
f(x) = g(x)h(x)   
f(x) = {[1- ]. [   
f(x)=   
f(x)=   
c) f(x) = 3x a = 1

## Since

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f1(x) = 3x ln3 f(1) = 3 ln3

f11(x) = 3x ln3. ln3; f11(1) = 3 (ln3)2

f111(x) = 3x ln3. ln3 . ln3 ; f111(1) = 3(ln3)3   
f(x)= 3 + 3 ln3(x-1) + + +   
d) f(x) = a = 2

## Since

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f1(x) = , f1(2) = 1

f11(x) = f11(2) = 2

f111(x) = , f111(2) = 6   
f(x) = 1 + + + +   
9. Maclaurin series for the following functions   
a) f(x) =

## Since

+++

f1(x) = f1(0) = 1

f11(x) = f11(0) = 2

f111(x) = f111(0) = 6   
f(x) = 1 + x + + 6 +   
f(x) = 1+ x+ x2 + x3 + + xn   
b)

## Since

+++++++

f(0) = 1

f1(x) = -3sinx = 0 at x = 0

f11(x) = -9cos x = -9 at x= 0

f111(x) = 27sin x = 0

f111(x) = 81cos x = 81

f1v(x) = -243sin x = 0

fv(x) = -729cosx = -729   
f(x) = 1 -3 + 81 -729 + +   
c) f(x) = x

## Since

+++   
f1(x) = = 1 at x = 0   
f11(x) = = 2   
f111(x) = = 3   
f1v(x) = = 4   
fv(x) = = 5   
f(x) = 0 + x + 2 + 3 + 4 +   
f(x) = x + + + + +