Pythagorean triples

PYTHAGOREAN TRIPLES The Pythagorean Theorem is probably one of the first principles learned in a Trigonometry It is about a property of all right triangles showing a relationship between their sides given by the equation c2 = a2 + b2; that is, the square of the longest side is equal to the sum of the squares of the remaining two sides (Knott 2009). Hence, given any two sides of a right triangle (triangle with a 90-degree angle) the unknown side can always be solved by algebraically manipulating the equation presented above.

A special case of such triangles is when all the three sides, a, b and c, are integers, thus generating a Pythagorean Triple (Bogomolny 2009). The most popular example of which is the 3-4-5 triangle, the triple which, according to Knott (2009), was known to the Babylonians since way back 5, 000 years and was possibly used as a basis in making true right angles in ancient building construction.

Then again, the 3-4-5 triangle is just one of the infinitely many Pythagorean Triples, and mind you, there are various ways of generating such triples. One is, given two integers n and m, where n > m, then sides a, b and c are define as n2 - m2, 2nm and n2 + m2, respectively, following a simple proof (Bogomolny 2009):

Taking for instance the triple 8-15-17, which is generated by taking n = 4 and m = 1, then a = n2 - m2 = 42 - 12 = 16 - 1 = 15; b = 2mn = 2(1)(4) = 8, and; c = n2 + m2 = 42 + 12 = 16 + 1 = 17. Another example is 7-24-25, which can be verified using n = 4 and m = 3. Such triples are examples of Primitive Pythagorean Triples, or those triples that are not multiples of another and are found using the n-m formula (Knott 2009). Other Pythagorean Triples can be found using a variety of methods as presented by Bogomolny (2009) and Knott (2009), some of which are:

a) by generating multiples of Primitive Pythagorean Triples, ka, kb, kc: 2 * (3, 4, 5) = 6, 8, 10;

b) by taking a series of multiples 1, 11, 111, …: 3-4-5, 33-44-55, 333-444-555, …;

c) by Two-fractions method—choose any two fractions whose product is 2, add 2 to each fraction, then cross multiply, getting the two shorter sides of the triple: 4/2, 2/2 → 8/2, 6/2 → 16, 12 → 162 + 122 = 202, and;

d) by taking (m+1) for n, with m as powers of 10, hence simplifying the triples into 2m+1, 2m(m+1), 2m2+2m+1: 2(10) + 1 = 21, 2(10)(10+1) = 220, 2(10)2 + 2(10) + 1 = 221.

To sum it up, there are infinitely many Pythagorean Triples existing. But one thing is for sure, a variety of techniques are available that will serve useful in generating patterns among such triples. Hence, if you cannot list them all, be familiar of their patterns at least.

References:

Bogomolny, A. (2009). Pythagorean Triples. Retrieved November 20, 2009, from Interactive Mathematics Miscellany and Puzzles Web site: http://www. cut-the-knot. org/pythagoras/pythTriple. shtml

Knott, R. (2009). Pythagorean Triangles and Triples. Retrieved November 20, 2009, from The University of Surrey, Mathematics Web site: http://www. maths. surrey. ac. uk/hosted-sites/R. Knott/Pythag/pythag. html