# The hydrostatic thrust acting on a plane surface engineering essay

To determine the hydrostatic thrust acting on a plane surface immersed in water when the surface is partially submerged or fully submerged.

To determine the position of the line of action of the thrust and to compare the position determined by experiment with the theoretical position.

In this exercise a F1-10 Hydraulics Bench and F1-12 Hydrostatic Pressure Apparatus are used for this laboratory. The F1-10 and F1-12 are test devices created for use in physics and engineering laboratories by Armfield Limited, Ringwald, Hampshire England.

The hydrostatic force at any point on the curved surface is normal to the surface and therefore resolves through the pivot point because this is located at the origin of the radii. Hydrostatic forces on the upper and lower curved surfaces therefore have no net effect – no torque to affect the equilibrium of the assembly because all of these forces pass through the pivot.

The forces on the sides of the quadrant are horizontal and cancel out (equal and opposite).

Figure 1 Diagram of F1-12 Apparatus

The hydrostatic force on the vertical submerged face is counteracted by the balanced weight. The resultant hydrostatic force on the face can therefore be calculated from the value of the balance weight and the depth of the water as follows:-

When the system is in equilibrium, the moments about the pivot point are equal:

mgL = Fh

Where,

m is the mass on the weight hang,

g is the acceleration due to gravity,

L is the length of the balance arm,

F is the hydrostatic thrust, and

H is the distance between the pivot and the center of pressure.

Hence by calculating the hydrostatic thrust and center of pressure on the end face of the quadrant, we can compare theoretical and experimental results.

Partially Submerged Vertical Plane

For the case where the vertical face of the quadrant is partially submerged.

Figure 2 Partial submersion

Where:-

L is the horizontal distance between the pivot point and the weight hanger,

H is the vertical distance between the pivot and the base of the quadrant, D is the height of the quadrant face, B is the width of the quadrant face,

d is the depth of water from the base of the quadrant, and

h’ is the vertical distance between the surface and the center of pressure.

The forces shown are F, the hydrostatic thrust, and m. g, the weight.

Hydrostatic Thrust

The hydrostatic thrust can be defined as:

F = ÏgAh (Newtons)

Where,

A is the area = A = Bd

H is the mean depth of immersion = h = d/2

Therefore:

F = (1/2) ÏgBd$^2$ (1)

Experimental Depth of Pressure

The moment, M, can be defined as

M = Fh” (Newtons)

A balancing moment is produced by the weight, W, applied to the hanger at the end of the balance arm. The moment is proportional to the length of the balance arm, L.

For static equilibrium the two moments are equal, i. e.

Fh” = WL = mgL

By substitution of the derived hydrostatic thrust, F from (1), we have

h” = (mgl/F) = (2mL/ ÏBd²) (meters)

Theoretical Depth of Pressure

The theoretical result for depth of pressure P below the free surface is

h’ = (Ix/Ah) (2)

Where Ix is the 2nd moment of area of immersed section about an axis in the free surface. By use of the parallel axes theorem :

$Ix = Ic + Ah^2$

$= (Bd3/12) + Bd(d/2)^2 = (Bd3/3)$ (3)

The depth of the center of pressure below the pivot point is therefore given by:-

h” = h’ +H – d (m) (4)

Hence:

h” = H – (d/3)

The turning moment can be calculated.

Fully Submerged Vertical Plane

For the case where the vertical face of the quadrant is fully submerged.

Figure 3 Full submersion

Where :

d is the depth of submersion,

F is the hydrostatic thrust exerted on the quadrant,

h’ is the depth of the center of pressure,

h” is the distance of center of pressure below the pivot,

B is the width of the surface, and

D is the depth of the surface,

W is the weight on the hanger (= mg)

Hydrostatic Thrust

The hydrostatic thrust F can be defined as

F = ÏgBD(d – (D/2)) (5)

Experimental Depth of Pressure

The moment M can be defined as

M = Fh” (Nm)

The balancing moment is produced by the weight, W, applied to the hanger at the end of the balance arm. The moment is proportional to the length of the balance arm L.

For static equilibrium the two moments are equal, i. e.

Fh” = WL = mgL

By substitution of the derived hydrostatic thrust, F, from (5), we have

h” = (mL)/(ÏBD(d – (D/2))) (m)

Theoretical Depth of Pressure

The theoretical results for the depth of center of pressure below the free surface is

h’ = Ix/Ah

where Ix is the 2nd moment of area of immersed section about an axis in the free surface.

By use of the parallel axes theorem:

Ix = Ic + Ah$^2$

= BD (m4)

The depth of the center of pressure below the surface is

h” = h’ + H – d (m)

Substitution as before then gives the theoretical result of :

h" = H + – d

The turning moment can hence be calculated.

## Equipment

Figure 4 Hydraulic bench Figure 5 F1-12 Hydrostatic Pressure Apparatus

## Hydraulic Bench

– Gross Weight: 160kg

– Height: 1. 00m

– Width: 1. 13m

– Depth: 0. 73m

Centrifugal pump, max. Head 21m $H_2O$, max. Flow 1. 35 liters/sec.

250 liter Sump tank capacity.

60 liter low flow volumetric tank.

Height of working surface is 1 meter above the floor

Volumetric flow measurement via remote sight gauge. Two ranges for flows from 1 to 6 liters per minute and 5 to 40 liters per minute.

## F1-12 Hydrostatic Pressure Apparatus

– Tank capacity: 5. 5 liters

– Cross-sectional area of quadrant (torroid): 7. 5 x 10-3 m2

– Total depth of completely immersed quadrant: 160 mm

– Height of fulcrum above quadrant: 100 mm

– Distance between suspended mass and fulcrum: 275 mm

Flotation tank with adjustable feet

Lever arm with weight hanger and counterbalance

Quick release connection to hydraulics bench

Plastic quadrant

Knife edges for measuring

Acrylic tank

Scale on the side of the quadrant to indicate water level

## Graduated Cylinder

## Procedure

The equipment was checked to make sure it was set up correctly. The built in spirit level was checked to verify that the tank was level. The counter balance weight was then adjusted until the balance arm was horizontal, which was indicated by a central index mark on the beam level indicator. The dimensions of the quadrant end-face and the distance H and L were obtained from the lab manual, verified, and recorded.

First, a 50g mass was added to the hanger. Then the floatation tank was filled with water, avoiding wetting the quadrant or level, until the balance arm was horizontal. Once it was level, the depth of immersion was recorded from the scale on the face of the quadrant. The above procedure was repeated by incrementing 50g weights until the water level reached the top of the tank. Finally, the procedure was again repeated in the reverse order. After all the data was recorded, the lab area was cleaned up of spilled water and tools were put away.

# Experimental Results

# Table 1 Constants

Height of End Face

D

=

0. 1 m

Width of End Face

B

=

0. 075 m

Length of Arm

L

=

0. 275 m

Height of Pivot

H

=

0. 2 m

## Table 2 Experimental and calculated values

Temperature and Density

Sample Calculation

Calculated Values for Partial Submersion

Hydrostatic Thrust

Experimental Distance to Center of Pressure

Measured Turning Moment

Theoretical Distance to Center of Pressure

Theoretical Turning Moment

Calculated Values for Full Submersion (d > 0. 1 m)

Hydrostatic Thrust

Experimental Distance to Center of Pressure

Measured Turning Moment

Theoretical Distance to Center of Pressure

Theoretical Turning Moment

## Figure 6 Turning moment vs depth

## Figure 7 Distance to center of pressure vs depth

## Discussion of Results

As the depth of the water increased, the hydrostatic thrust increased and the distance to the center of pressure decreased. The center of pressure moved closer to the center of the vertical face as the depth increased. The experimental values for the distance to the center of pressure were smaller than the theoretical distances at nearly all submersion depths. Since the same hydrostatic force is used to calculate the turning moments, the experimental turning moment was also smaller than the theoretical turning moment at nearly all submersion depths.

Both the experimental and theoretical values are calculated using the depth and end face dimensions. Although the experimental and theoretical values are calculated using different relations between the distance and depth, some of the error in those measurements is probably present in both sets of calculations. The experimental distance to the center of pressure is also calculated using the mass and water density. One possible cause of the lower experimental distances is that the mass is slightly larger than labeled. If the weights have dirt, oils, or other debris attached to them, they would weigh more than labeled, and this would result in an underestimation of the experimental distance to the center of pressure. Variation in the density of the water also introduces error into the experimental calculation. Temperature variations during the experiment result in variations of the water density. If the temperature increases, the water density decreases, which results in a higher distance to the center of pressure. The distance to the center of pressure would be underestimated using the higher density from the beginning of the experiment. Error is also introduced when water splashes onto the quadrant or balance arm. Water that splashes onto the balance arm or quadrant weighs the quadrant down, requiring more water to increase the hydrostatic force to balance the weight of the stray water. This overestimation of the depth results in an underestimation of the experimental distance to the center of pressure.

Other sources of error include inaccuracy in the measurement of the depth and inaccuracy in the determination of when the balance arm is balanced on the pivot.

## Conclusion

In this lab the turning moment and the distance to center of pressure in relation to depth were determined. The objectives of this lab were to determine the hydrostatic thrust acting on a plane surface immersed in water when the surface is partially submerged or fully submerged, to determine the position of the line of action of the thrust and to compare the position determined by experiment with the theoretical position. The objectives of the lab were accomplished using a F1-10 Hydraulics Bench and F1-12 Hydrostatic Pressure Apparatus. As the depth of the water increased, the hydrostatic thrust increased and the center of pressure moved closer to the center of the vertical face. The experimental distances to the center of pressure were lower than the theoretical distances to the center of pressure. Many factors may have contributed to this discrepancy. Water splashing onto the balance arm or quadrant would cause overestimation of the water depth for equilibrium, changes in water temperature would cause variations in the water density, and excess weight on the masses would cause underestimation of the experimental distance to the center of pressure.