Golden rule for saving in solow growth model

Introduction:

This paper focuses on Golden Rule for saving in Solow growth model, and will solve three following problems: 1. using the Solow model with human capital, derive and demonstrate the golden rule for saving. 2. Describe the behaviour of the economy as it moves towards the corresponding steady state growth path. 3. What factors are important when it comes to assessing the desirability and feasibility of achieving the golden rule growth path?

Analysis:

Using the Solow growth model with human capital, derive and demonstrate the golden rule

for saving.:

Let k be the capital/labour ratio (i. e. capital per capita), y be the resulting per capita output (y = f(k)), and s be the savings rate. The steady state is defined as a situation in which per capita output is unchanging, which implies that k be constant. This requires that the amount of saved output be exactly what is needed to (1) equip any additional workers and (2) replace any worn out capital.

In a steady state, kt+1= kt= k, therefore: sf(k) = (n +)k, where n is the constant exogenous population growth rate, and d is the constant exogenous rate of depreciation of capital. Since n and d are constant and f(k) satisfies the Inada conditions, this expression may be read as an equation connecting s and k in steady state: any choice of s implies a unique value for k (thus also for y) in steady state. Since consumption is proportional to output (c = (1 âˆ’ s)f(k)), then a choice of value for s implies a unique level of steady state per capita consumption. Out of all possible choices for s, one will produce the highest possible steady state value for c and is called the golden rule savings rate.

To discover the optimal capital/labour ratio, and thus the golden rule savings rate, first note that consumption can be seen as the residual output that remains after providing for the investment that maintains steady state: c = f(k) âˆ’ (n +)k

Differential calculus methods can identify which steady state value for the capital/labour ratio maximises per capita consumption. The golden rule savings rate is then implied by the connection between s and k in steady state (see above).

The equation describing the evolution of the capital sock per unit of effective labour is given by

Substituting in for the intensive form of the Cobb-Douglas,, yields

On the balaced growth path, k is zero, investment per unit of efffective labor is equal to break-even investment per unit of offective labor and so k remains constant. Denoting the balanced-growth-path value of k as k*, we have , Rearranging to solve for k* yields

To get the balanced-growth-path value of output per unit of effective labor, substitute equation (2) into the intensive form of the production function, y = kÎ±;

Consumption per unit of effective labor on the balanced growth path is given by . Subsituting euation (3) into this expression yields

By definition, the golden rule level of the capital stock is that level at which consumption per unit f effective labor is maximized. To derive this level of k, take equation (2), which expresses the balanced-growth-path level of k, and rearrange it to solve for s:

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Now substitute equation (5) into equation (4):

After some straightforward algebraic manipulation, this simplifies to

Equation (6) can be easily interpreted. Consumption per unit of effective labour is equal to output per unit of effective labour, k*Î±, less actual investment per unit of effective labour, which on the balanced growth path is the same as break-even investment per unit of effective labour, (n+.

Now use equation (6) to maximize c* with respect to k*. The first-order condition is given by

Or simply

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Note that equation (7) is just a specific form of which is the general condition that implicitly defines the golden rule level of capital per unit of effective labour. Equation (7) has a graphical interpretation: it defines the level of k at which the slope of the intensive form of the production function is equal to the slope of the break-even investment line.

Solving equation (7) for the golden rule level of k yields

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To get the saving rate that will yield the golden rule level of k, substitute equation (8) into (5):

, which simplifies to

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With a Cobb-Douglas production function, the saving rate required to reach the golden rule is equal to the elasticity of output with respect to capital or capital’s share in output (if capital earns its marginal product).

Describe the behaviour of the economy as it moves towards the corresponding steady state growth path:

Normally, we assume that the policymaker can simply choose the economy’s steady state and jump there immediately. In this case, the policymaker would choose the steady state with highest consumption-the Golden Rule steady state. But if we suppose that the economy has reached a steady state other than the Golden Rule, then we must consider two cases: the economy might begin with more capital than in the Golden Rule steady state, or with less. It turns out that the two cases offer very different problems for policymakers.

Figure 1

Output, y

Consumption, c

Investment, i

t0 Time

(Reduce saving rate)

Considering the case in which the economy begins at a steady state with more capital than it would have in the Golden Rule steady state. In this case, the policymaker should pursue policies aimed at reducing the rate of saving in order to reduce the capital stock. Suppose that these policies succeed and that at some point-call it time t0-the saving rate falls to the level that will eventually lead to the Golden Rule steady state. Figure 1 shows the behaviour of the economy as it moves towards the golden rule growth path represented by output, consumption, and investment when the saving rate falls. The reduction in the saving rate causes an immediate increase in consumption and a decrease in investment. Because investment and depreciation were equal in the initial steady state, investment will now be less than depreciation, which means the economy is no longer in a steady state. Gradually, the capital stock falls, leading to reductions in output, consumption, and investment. These variables continue to fall until the economy reaches the new steady state. Because we are assuming that the new steady state is the Golden Rule steady state, consumption must be higher than it was before the change in the saving rate, even though output and investment are lower. Compared to the old steady state; consumption is higher not only in the new steady state but also along the entire path to it. When the capital stock exceeds the Golden Rule level, reducing saving is clearly a good policy, for it increases consumption at every point in time.

Figure 2

Output, y

Consumption, c

Investment, i

t0 Time

(Increase saving rate)

When the economy begins with less capital than in the Golden Rule steady state, the policymaker must raise the saving rate to reach the Golden Rule. Figure 2 shows what happens. The increase in the saving rate at time t0 causes an immediate fall in consumption and a rise in investment. Over time, higher investment causes the capital stock to rise. As capital accumulates, output, consumption, and investment gradually increase, eventually approaching the new steady-state levels. Because the initial steady state was below the Golden Rule, the increase in saving eventually leads to a higher level of consumption than that which prevailed initially.

The increase in saving that leads to the Golden Rule steady state eventually raises economic welfare, because the steady-state level of consumption is higher. But achieving that new steady state requires an initial period of reduced consumption. Note the contrast to the case in which the economy begins above the Golden Rule. When the economy begins above the Golden Rule, reaching the Golden Rule produces higher consumption at all points in time. When the economy begins below the Golden Rule, reaching the Golden

Rule requires initially reducing consumption to increase consumption in the future.

What factors are important when it comes to assessing the desirability and feasibility of achieving the golden rule growth path?

For this question, we firstly discuss the desirability of achieving the golden rule growth path. When deciding whether to try to reach the Golden Rule steady state, policymakers have to take into account that current consumers and future consumers are not always the same people. Reaching the Golden Rule achieves the highest steady-state level of consumption and thus beneï¬ts future generations. But when the economy is initially below the Golden Rule, reaching the Golden Rule requires raising investment and thus lowering the consumption of current generations. Thus, when choosing whether to increase capital accumulation, the policymaker faces a trade-off among the welfare of different generations. A policymaker who cares more about current generations than about future generations may decide not to pursue policies to reach the Golden Rule steady state. For instance, some poor country like Ethiopia have to care more about current generations, since most of their consumption only for basic maintenance, then they can't increase saving rate. By contrast, a policymaker who cares about all generations equally will choose to reach the Golden Rule. Even though current generations will consume less, an inï¬nite number of future generations will beneï¬t by moving to the Golden Rule.

Thus, optimal capital accumulation depends crucially on how we weigh the interests of current and future generations. The biblical Golden Rule tells us,

“ do unto others as you would have them do unto you. ” If we heed this advice, we give all generation’s equal weight. In this case, it is optimal to reach the Golden Rule level of capital-which is why it is called the “ Golden Rule. ” (Mankiw, 2010)

Second, we discuss the feasibility of achieving the golden rule growth path. The key of this problem is whether policymaker can change the saving rate effectively by various policies. Thus, we will discuss some policy in different national conditions in this part.

Various economic policies can have an effect on the savings rate and, given data about whether an economy is saving too much or too little, can in turn be used to approach the Golden Rule level of savings. Consumption taxes, for example, may reduce the level of consumption and increase the savings rate, whereas capital gains taxes may reduce the savings rate. These policies are often known as savings incentives in the west, where it is felt that the prevailing savings rate is “ too low” (below the Golden Rule rate), and consumption incentives in countries like Japan where demand is widely considered to be too weak because the savings rate is “ too high” (above the Golden Rule).

Japan’s high rate of private saving is offset by its high public debt. A simple approximation of this is that the government has borrowed 100% of GDP from its own citizens backed only with the promise to pay from future taxation. This does not necessarily lead to capital formation through investment (if the revenue from bond sales is spent on present government consumption rather than infrastructure development). Compared to China’s high rate of private saving, the behaviour of Japan may derive from traditional view or culture, while Chinese people have to keep high rate of private saving since the pressure from deficient social security system. Therefore, instead of fiscal policy, building a perfect social security system will bestially release potential consumption. US provide a typical example in this respect.

If consumption tax rates are expected to be permanent then it is hard to reconcile the common hypothesis that rising rates discourage consumption with rational expectations (since the ultimate purpose of saving is consumption (Frankel, 1998)). However, consumption taxes tend to vary (e. g. with changes in government or movement between countries), and so currently high consumption taxes may be expected to go away at some point in the future, creating an increased incentive for saving. Actually, some countries like UK and New Zealand even have increased their consumption taxes. The efficient level of capital income tax in the steady state has been studied in the context of a general equilibrium model and Judd (1985) has shown that the optimal tax rate is zero. However, Chamley (1986) says that in reaching the steady state (in the short run) a high capital income tax is an efficient revenue source.

Conclusion:

In economics, the Golden Rule savings rate is the rate of savings which maximizes steady state level or growth of consumption (Phelps, 1966), as for example in the Solow growth model. Although the concept can be found earlier in John von Neumann and Maurice Allais's works, the term is generally attributed to Edmund Phelps who wrote in 1961 that the Golden Rule " do unto others as you would have them do unto you" could be applied inter-generationally inside the model to arrive at some form of " optimum".

In the Solow growth model, a steady state savings rate of 100% implies that all income is going to investment capital for future production, implying a steady state consumption level of zero. A savings rate of 0% implies that no new investment capital is being created, so that the capital stock depreciates without replacement. This makes a steady state unsustainable except at zero output, which again implies a consumption level of zero. Somewhere in between is the " Golden Rule" level of savings, where the savings propensity is such that per-capita consumption is at its maximum possible constant value.

When assessing the desirability and feasibility of achieving the golden rule growth path, initial economic situation (whether below or over Golden rule), national culture, and some relational economic model must be considered.