Linear algebra

1. INTRODUCTION:

Linear algebra comprises of the theory and application of linear system of equation, linear transformation and Eigen value problem. In linear algebra, we make a systematic use of matrix and lesser extent determinants and their properties.

Determinants were first introduced for solving linear systems and have important engineering applications in system of differential equations, electrical networks, Eigen value problems and so on. Many complicated expression occurring in electrical and mechanical systems can be elegantly simplified by expressing them in the form of determinants.

Cayley discovered matrix in the year 1860. But it was not until the twentieth century was well advanced that engineer heard of them. These days, however,

Matrices have been found to be of great utility in many branches of applied mathematics such as algebraic and differential equation, mechanics, theory of electrical circuit,

Nuclear physics, aerodynamics and astronomy with the advent of computers, the usage of matrix method has been greatly facilitated.

2. MATRIX:-

A system of m n numbers arranged in a rectangular formation along m rows and n columns an bounded by a bracket [ ] is called an m by n matrix; which is written as m*n matrix. A matrix is also denoted by a single capital letter.

- (mathematics) a rectangular array of quantities or expressions set out by rows and columns; treated as a single element and manipulated according …
- (geology) amass of fine-grained rock in which fossils, crystals, or gems are embedded
- an enclosure within which something originates or develops (from the Latin for womb)
- the body substance in which tissue cells are embedded
- the formative tissue at the base of a nail

To locate any particular element of a matrix, the element respectively specify the rows and the columns. Thus aij is the

In this notation, the matrix is denoted by [aij].

3. HISTORY:-

The beginnings of matrices and determinants goes back to the second century BC although traces can be seen back to the fourth century BC. However it was not until near the end of the 17th Century that the ideas reappeared and development really got underway.

It is not surprising that the beginnings of matrices and determinants should arise through the study of systems of linear equations. The Babylonians studied problems which lead to simultaneous linear equations and some of these are preserved in clay tablets which survive. For example a tablet dating from around 300 BC contains the following problem:-

The Chinese, between 200 BC and 100 BC, came much closer to matrices than the Babylonians. Indeed it is fair to say that the textNine Chapters on the Mathematical Artwritten during the Han Dynasty gives the first known example of matrix methods. First a problem is set up which is similar to the Babylonian example given above:-

Now the author does something quite remarkable. He sets up the coefficients of the system of three linear equations in three unknowns as a table on a ‘ counting board’.

4. Other historical usages of the word “ matrix” in mathematics

The word has been used in unusual ways by at least two authors of historical importance.

Bertrand RussellandAlfred North Whiteheadin their Principia Mathematica (1910-1913) use the word matrix in the context of theirAxiom of reducibility. They proposed this axiom as a means to reduce any function to one of lower type, successively, so that at the “ bottom” (0 order) the function will be identical to itsextension:

“ Let us give the name of matrix to any function, of however many variables, which does not involve anyapparent variables. Then any possible function other than a matrix is derived from a matrix by means of generalization, i. e. by considering the proposition which asserts that the function in question is true with all possible values or with some value of one of the arguments, the other argument or arguments remaining undetermined”.

For example a function Φ(x, y) of two variables x and y can be reduced to a collection of functions of a single variable, e. g. y, by “ considering” the function for all possible values of “ individuals” ai substituted in place of variable x. And then the resulting collection of functions of the single variable y, i. e. ∀ai: Φ(ai, y), can be reduced to a “ matrix” of values by “ considering” the function for all possible values of “ individuals” bi substituted in place of variable y:

∀bj∀ai: Φ(ai, bj)

Alfred Tarskiin his 1946 Introduction to Logic used the word “ matrix” synonymously with the notion oftruth tableas used in mathematical logic.

(5): TYPES OF MATRIX:

(A): ROW MATRIX:

A matrix having a single row is called row matrix.

e. g, A = [1234]

(B): COLUMN MATRIX:

A matrix having a single column is called column matrix.

e. g

(C): SQUARE MATRIX:

A matrix having n rows and n columns is called square matrix.

e. g. 1 2

3 4

(D): DIAGONAL MATRIX:

A square matrix all of whose elements except those in the leading diagonal, are zero is called a diagonal matrix.

e. g. 1 0

0 2

(E): UNIT MATRIX:

A diagonal matrix of order n which for all its diagonal elements, is called a unit matrix or an identity matrix of order n.

e. g. 1 0

0 1

(F): NULL MATRIX:

I all the elements of a matrix are zero; it is called a null matrix.

e. g. 0 0

0 0

6. Eigen Values?

Many problems in mathematics and physics reduce " eigenvalue problems", and people often expend lots of effort trying to determine the eigenvalues of a general NxN system, and this is sometimes reduced to finding the roots of the Nth degree characteristic polynomial. The task of efficiently finding all the complex roots of a high-degree polynomial is non-trivial, and is complicated by the need to consider various special cases such as repeated roots.

However, depending on what we're really trying to accomplish, we may not need to find the eigenvalues at all. Of course, the description of an action is usually simplest when its components are resolved along the directions of the eigenvectors, e. g., the eigenvectors of a rotation matrix define its axis of rotation. As a result, it's easy to write equations for the positions of points on a rotating sphere if we work in coordinates that are aligned with the eigenvectors of the rotation, whereas if we work in some other arbitrary coordinates the equations describing the motion will be more complicated.

However, there are many situations in which the characteristic axes of the phenomenon can be exploited without ever explicitly determining the eigenvalues. For example, suppose we have the system of three continuous variables x(t), y(t), and z(t) that satisfy the equations

$ax + by + cz = x'$

$dx + ey + fz = y'$

$gx + hy + iz = z'$

6. Matrix eigenvalues

Eigenvalues are a special set of scalars associated with alinear system of equations(i. e., amatrix equation) that are sometimes also known as characteristic roots, characteristic values (Hoffman and Kunze 1971), proper values, or latent roots (Marcus and Minc 1988, p. 144).

The determination of the eigenvalues and eigenvectors of a system is extremely important in physics and engineering, where it is equivalent tomatrix diagonalizationand arises in such common applications as stability analysis, the physics of rotating bodies, and small oscillations of vibrating systems, to name only a few. Each eigenvalue is paired with a corresponding so-calledeigenvector(or, in general, a correspondingright eigenvectorand a correspondingleft eigenvector; there is no analogous distinction between left and right for eigenvalues).

Let be alinear transformationrepresented by amatrix. If there is avectorsuch that

for somescalar, then is called the eigenvalue of with corresponding (right)eigenvector

.

Letting be asquare matrix

with eigenvalue, then the correspondingeigenvectorssatisfy

which is equivalent to the homogeneous system

Equation (4) can be written compactly as

where is theidentity matrix. As shown inCramer's rule, alinear system of equationshas nontrivial solutionsiffthedeterminantvanishes, so the solutions of equation (5) are given by

This equation is known as thecharacteristic equationof , and the left-hand side is known as thecharacteristic polynomial.

7. Existence and multiplicity of eigenvalues

For transformations on real vector spaces, the coefficients of the characteristic polynomial are all real. However, the roots are not necessarily real; they may include complex numbers with a non-zero imaginary component. For example, a matrix representing a planar rotation of 45 degrees will not leave any non-zero vector pointing in the same direction. Over a complex vector space, thefundamental theorem of algebraguarantees that the characteristic polynomial has at least one root, and thus the linear transformation has at least one eigenvalue.

As well as distinct roots, the characteristic equation may also have repeated roots. However, having repeated roots does not imply there are multiple distinct (i. e., linearly independent) eigenvectors with that eigenvalue. The algebraicmultiplicityof an eigenvalue is defined as themultiplicity of the corresponding rootof the characteristic polynomial. The geometric multiplicity of an eigenvalue is defined as the dimension of the associated eigenspace, i. e. number of linearly independent eigenvectors with that eigenvalue.

Over a complex space, the sum of the algebraic multiplicities will equal the dimension of the vector space, but the sum of the geometric multiplicities may be smaller. In a sense, then it is possible that there may not be sufficient eigenvectors to span the entire space. This is intimately related to the question of whether a given matrix may be diagonalized by a suitable choice of coordinates.

Shear

Horizontal shear. The shear angle φ is given by k = cot φ.

Shear in the plane is a transformation in which all points along a given line remain fixed which while other points are shifted parallel to that line by a distance proportional to their perpendicular distance from the line. Shearing a plane figure does not change its area. Shear can be horizontal − along the

X axis, or vertical − along the Y axis. In horizontal shear (see figure), a point P of the plane moves parallel to the X axis to the place P’ so that its coordinate y does not change while the x coordinate increments to become x’ = x + k y, where k is called the shear factor.

The matrix of a horizontal shear transformation is . The characteristic equation is λ2 − 2 λ + 1 = (1 − λ)2 = 0 which has a single, repeated root λ = 1. Therefore, the eigenvalue λ = 1 has algebraic multiplicity 2. The eigenvector(s) are found as solutions of

The last equation is equivalent to y = 0, which is a straight line along the x axis. This line represents the one-dimensional eigenspace. In the case of shear the algebraic multiplicity of the eigenvalue (2) is greater than its geometric multiplicity (1, the dimension of the eigenspace). The eigenvector is a vector along the x axis. The case of vertical shear with transformation matrix is dealt with in a similar way; the eigenvector in vertical shear is along the y axis. Applying repeatedly the shear transformation changes the direction of any vector in the plane closer and closer to the direction of the eigenvecto

Eigenvalues of 3×3 matrices

If then thecharacteristic polynomialof A is

Alternatively the characteristic polynomial of a 3×3 matrix can be written in terms of thetracetr(A)anddeterminantdet(A)as

whereI3is the 3×3identity matrix.

8. Identifying eigenvectors

With the eigenvalues in hand, we can solve sets of simultaneous linear equations to determine the corresponding eigenvectors. Since we are solving for the system, if$\lambda = 2$then,

Now, reducing torow echelon form:

allows us to solve easily for the eigenspaceE2:

We can confirm that a simple example vector chosen from eigenspaceE2is a valid eigenvector with eigenvalue$\lambda = 2$:

Note that we can determine the degrees of freedom of the solution by the number of pivots.

IfAis arealmatrix, the characteristic polynomial will have real coefficients, but its roots will not necessarily all be real. Thecomplexeigenvalues come in pairs which areconjugates. For a real matrix, the eigenvectors of a non-real eigenvaluez, which are the solutions of$(A-zI)v = 0$, cannot be real.

Thespectral theoremfor symmetric matrices states that ifAis a real symmetricn-by-nmatrix, then all its eigenvalues are real, and there existnlinearly independent eigenvectors forAwhich are mutuallyorthogonal. Symmetric matrices are commonly encountered in engineering.

9: PROPERTIES OF EIGEN VALUE

(i) 1. Any square matrix A and its transpose A’ has the same eigen value

(ii) 2. The eigen value of a triangular matrix are just the diagonal element of the matrix

(iii) 3. The eigen value of an idempotent matrix are either zero or unity

(iv) 4. The sum of the eigen value of matrix is the sum of the of the element of the principal diagonal

(v) 5. The product of the eigen value of a tmatrix A is equal to its determinant

(vi) 6. If λ is the eigen value of a matrix A then 1/λ is the eigen value of A-1.

(vii) 7. If λ is the eigen value of an orthogonal matrix then 1/λ is its eigen value.

(viii) 8. If λ1, λ2,…………, λn are the eigen value of a matrix A, then Am has the eigen value λ1m λ2m…………….. λnm

(ix)

(x) A is real Eigen value are real or complex conjugates in pairs.

2 -1 -1

A = 0 3 -1

0 0 2

The characteristic equation

A-λI = 0

2-λ -1 -1

0 3-λ -1 = 0

0 0 2-λ

(2-λ)(3-λ)(2-λ)= 0

λ = 2, 2, 3

Eigen value= 2, 2, 3

We can write Eigen value is the form of complex

First complex Eigen value= 2+0i

Second complex Eigen value= 2-0i

Hence it is prove that A is real Eigen value are real or complex conjugates in pairs.

(xi) A-1 exist if and only if 0 is not an Eigen

Value of A it has Eigen values

1/λ1, 1/λ2,……………….. 1/λn

If λ is an Eigen value of matrix A, then 1/λ is the Eigen value of A-1

If x be the Eigen vector corresponding to λ,

Then AX= λX

Multiply by A both side,

A-1 A= A-1 λX

IX= λ (A-1 X)

X= λ (A-1X)

A-1 X= 1/λ X

Shows that 1/λ is an Eigen value of the inverse matrix A-1.

If Eigen value of matrix A is 0 then,

Eigen value of inverse matrix A-1 will not exist because

Eigen value of matrix A is λ and Eigen value of inverse matrix A-1 is 1/λ.

PROFF OF THE GIVEN STATEMENT:

A=

So its eigen value are 1, 3, -2

Now,

A-1=

R2→R2+R3

R2/3

R3/-3 :

R1→R1-2R2+R3 :

So eigen vector of A-1 are 1, 3/2 -1/2.

λ 1 = 1

λ 2= 3

λ 3=-2

λ 1'= 1/ λ 1= 1/1

λ 2'= 1/ λ 2= 1/3

λ 3'= 1/ λ3= 1/2

Its eigen value are real and also A is real and satisfied the first condition of the topic

When A-1 exist it has the eigen values 1, 1/3, -1/2 i. e. 1/λ1, 1/ λ2, 1/ λ3 satisfied the second condition of the topic.

CONLUSION:

🗣 This is to conclude that while doing this topic " EIGN VALUE & VECTOR" I came to know many more things about EIGEN VALUE & VECTOR and their applications. which I not even heard of. It was great experience doing this topic. It has also increased my knowledge .