

Gauss law and its
applications
philosophy essay



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The relationship between the net electric flux through a closed surface (often called as Gaussian surface) and the charge enclosed by the surface is known as Gauss's law. Consider a positive point charge q located at the center of a sphere of radius r . We know that the magnitude of the electric field everywhere on the surface of the sphere is $E = \frac{kq}{r^2}$. The field lines are directed radially outward and hence are perpendicular to the surface at every point on the surface. That is at each surface point, \vec{E} is parallel to the vector \hat{n} representing a local element of area dA surrounding the surface point. Therefore, $\vec{E} \cdot \hat{n} = E$ and the net flux through the Gaussian surface is $\Phi = \oint \vec{E} \cdot \hat{n} dA = \oint E dA$, where we have moved E outside of the integral because, by symmetry E is constant over the surface. The value of E is given by $E = \frac{kq}{r^2}$. Furthermore, because the surface is spherical, $\oint dA = 4\pi r^2$.

Hence, the net flux through the Gaussian surface is

This equation shows that the net flux through the spherical surface is proportional to the charge inside the surface. The flux is independent of the radius r because the area of the spherical surface is proportional to r^2 , whereas the electric field is proportional to $1/r^2$. Therefore, in the product of area and electric field, the dependence on r cancels.

Now, consider several closed surfaces surrounding a charge q . Surface is spherical, but surfaces and are not. The flux that passes through has value q/ϵ_0 . Flux is proportional to the number of lines through the nonspherical surfaces and. Therefore, the net flux through any closed surface surrounding a point charge q is given q/ϵ_0 and is independent of the shape of that surface.

Now consider a point charge located outside a closed surface of arbitrary shape. As can be seen from this construction, any electric field line entering the surface leaves the surface at another point. The number of electric field lines entering the surface equals the number leaving the surface. Therefore, the net electric flux through a closed surface that surrounds no charge is zero. The net flux through the cube is zero because there is no charge inside the cube.

Let's extend these arguments to two generalized cases: (1) that of many point charges and (2) that of a continuous distribution of charge. We use the superposition principle, which states that the electric field due to many charges is the vector sum of the electric fields produced by the individual charges. Therefore, the flux through any closed surface can be expressed as =

where is the total electric field at any point on the surface produced by the vector addition of the electric fields at that point due to the individual charges. Consider the system of charges, the surface S surrounds only one charge hence the net flux through S is. The flux through S due to charges outside it is zero because each electric field line from these charges that enters S at one point leaves it at another. The surface S' surrounds charges and hence the net flux through it is (+). Finally, the net flux through surface is zero because there is no charge inside this surface. That is, all the electric field lines that enter at one point leave at another. Charge does not contribute to the net flux through any of the surfaces because it is outside all the surfaces.

Gauss's law is a generalization of what we have just described and states that the net flux through any closed surfaces is

where E represents the electric field at any point on the surface and Q_{enc} represents the net charge inside the surface.

APPLICATIONS OF GAUSS'S LAW TO VARIOUS CHARGE DISTRIBUTIONS

Gauss's law is useful for determining electric fields when the charge distribution is highly symmetric. The following examples demonstrate ways of choosing the Gaussian surface over which the surface integral given by $\oint \mathbf{E} \cdot d\mathbf{A}$ can be simplified and the electric field is determined. In choosing the surface, always take advantage of the symmetry of the charge distribution so that E can be removed from the integral. The goal in this type of calculation is to determine a surface for which each portion of the surface satisfies one or more of the following conditions:-

The value of the electric field can be argued by symmetry to be constant over the portion of the surface.

The dot product in

can be expressed as a simple algebraic product $E dA$ because \mathbf{E} and $d\mathbf{A}$ are parallel.

The dot product in

is zero because \mathbf{E} and $d\mathbf{A}$ are perpendicular.

The electric field is zero over the portion of the surface.

Electric Field Due to a Line Charge – Cylindrical Symmetry

Let's find the electric field due to a line charge. Consider the field due to an infinitely long line of charge as opposed to the one of finite length. It's clear here that it's impossible to talk about a finite amount of charge stretched over an infinitely long distance. Instead, state that the line has a constant linear charge density.

Realistically, all line charges are finite. Consider the figure below which shows a view of the line charge and a point P a distance h away from it. We have to find the electric field at point P. To set up the integral, take infinitesimally small line segments of charge in pairs so that their horizontal components cancel and the vertical (i. e. radial) components add.

Figure: Calculation of the electric field at the midpoint of a line charge of length l .

q_{enclosed}

ϵ_0

$r_A t$

ϵ_0

r_t

$2\epsilon_0$

$(2 \cdot 10^{-6} \text{ C/m})(0.02 \text{ m})$

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$2(8.85 \cdot 10^{-12} \text{ C}^2/(\text{N} \cdot \text{m}^2))$

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