

Summary and analysis of the compton effect



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$E_n = nhf$ (1) where E_n is the energy, n is a non-negative integer, h is Planck's constant, and f is the frequency of the photon. 2 In 1905, Albert Einstein extended Planck's inference to include not only black body radiation but all electromagnetic waves! Therefore, Einstein hypothesized that light is quantized with energy proportional to its frequency. 3 The obvious principle to be deduced from these discoveries is that light possessed attributes of waves and particles! In 1922, Arthur Holly Compton solidified Planck's assumption and therefore firmly established a new era of physics. Compton theorized and then experimentally demonstrated that electromagnetic waves had the properties of particles. Classically, x-rays would shake the electrons of a target material at the same frequency of the x-ray. Hence, the wavelength of radiation from the oscillating electrons would be identical to the wavelength of the incoming xrays. 1 However, it was observed that x-rays were more easily absorbed by materials than waves of longer wavelength. In other words, the scattered x-rays were of longer wavelength. 4 This was contrary to the predictions of classical physics. Compton realized though, that if the interaction was modeled as a collision between two particles (electron and photon), the scattered x-rays would-be of longer wavelength (compared to the incident-rays) because the recoiling electron would acquire some of the energy and momentum of the incoming x-ray. 4 Since wavelength is inversely proportional to frequency, the frequency of the scattered x-rays was less. From eq. (1), it is seen that the energy would also be decreased. When Compton carried out this experiment in 1922 using molybdenum as his target, he verified his theory and provided even more evidence that light also possessed a mass less particle nature

Detailed Description of Compton Effect

the elastic scattering of electromagnetic radiation by free electrons, accompanied by an increase in wavelength; it is observed during scattering of radiation of short wavelength-X rays and gamma rays. The corpuscular properties of radiation were fully revealed for the first time in the Compton Effect.

The Compton effect was discovered in 1922 by the American physicist A. Compton, who observed that X rays scattered in paraffin have a longer wavelength than the incident rays. Such a shift in wavelength could not be explained by classical theory. In fact, according to classical electrodynamics, under the influence of the periodic electric field of an electromagnetic (light) wave, an electron should oscillate with a frequency equal to that of the wave and consequently should radiate secondary (scattered) waves of the same frequency. Thus, in "classical" scattering (the theory of which was provided by the British physicist J. J. Thomson and is therefore called Thomson scattering) the wavelength of the light does not change.

An elementary theory of the Compton effect based on quantum concepts was given by Compton and independently by P. Debye. According to quantum theory a light wave is a stream of light quanta, or photons. Each photon has a definite energy $E = h\nu = hc/\lambda$ and a definite momentum $p = (h/\lambda)n$, where λ is the wavelength of the incident light (via its frequency), c is the speed of light, h is Planck's constant, and n is the unit vector in the direction of propagation of the wave (the subscript p denotes a photon). In quantum theory the Compton Effect appears as an elastic collision between

two particles, the incident photon and the stationary electron. In every such collision event the laws of conservation of energy and momentum are obeyed. A photon that has collided with an electron transfers part of its energy and momentum to the electron and changes its direction of motion (it is scattered); the decrease in the photon's energy signifies an increase in the wavelength of the scattered light. The electron, which previously had been stationary, receives energy and momentum from the photon and is set in motion (it experiences recoil). The direction of motion of the particles after the collision, as well as their energy, is determined by the laws of conservation of energy and momentum (Figure 1).

Elastic collision of a photon and an electron in the Compton effect. Before the collision the electron was stationary: p_i and p_f are the momentum of the incident and scattered photons, $p_e = mv$ is the momentum of the recoil electron (via its velocity), θ is the photon's scattering angle, and ϕ is the angle of escape of the recoil electron relative to the direction of the incident photon.

Simultaneous solution of the equations expressing the equality of the summed energies and momentums of the particles before and after the collision (assuming that the electron is stationary before the collision) gives Compton's formula for the shift in the wavelength of the light:

$$\lambda' - \lambda = \frac{h}{mc} (1 - \cos \theta)$$

Here λ' is the wavelength of the scattered light, θ is the photon's scattering angle, and $\lambda_0 = h/mc = 2.426 \times 10^{-10} \text{ cm} = 0.024 \text{ angstrom } (\text{\AA})$ is the "Compton wavelength" of the electron (via the mass of the electron). It

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follows from Compton's formula that the shift in the wavelength does not depend on the wavelength λ_0 of the incident light itself. It is solely determined by the scattering angle θ of the photon and is maximal when $\theta = 180^\circ$, that is, when scattering is straight back: $\Delta\lambda = 2\lambda_C$.

Expressions for the energy E_e of the recoil, or "Compton," electron as a function of the angle θ of its escape may be obtained from the same equations. The dependence of the energy E_γ of the scattered photon on the scattering angle θ , as well as the dependence of E_e on θ , which is related to it, is shown in Figure 2. From the figure it is apparent that the recoil electrons always have a velocity component in the direction of motion of the incident photon (that is, θ does not exceed 90°).

Experiment has confirmed all the above theoretical predictions. The correctness of the corpuscular concepts of the mechanism of the Compton effect—and thus the correctness of the basic assumptions of quantum theory—has been experimentally proved.

In actual experiments on the scattering of photons by matter, the electrons are not free but are bound to atoms. If the energy of the photons is high in comparison with the binding energy of the electrons in the atom (X-ray and gamma-ray photons), then the electrons experience a recoil strong enough to expel them from the atom. In this case the photon scattering proceeds as if with free electrons. However, if the energy of the photon is not sufficient to tear the electron from the atom, then the photon exchanges energy and momentum with the entire atom. Since the mass of the atom is very great compared to the photon's equivalent mass (which, according to the theory of

relativity, equals $\frac{h\nu}{c^2}$), the recoil is virtually nonexistent; therefore, the photon

Dependence of the energy \hat{E}_γ of the scattered photon on the scattering angle $\hat{\theta}$ (for convenience, only the upper half of the symmetrical curve is depicted) and the dependence of the energy \hat{E}_e of the recoil electron on the angle of escape θ (lower half of the curve). Quantities related to the same collision event are labeled with identical numbers. The vectors drawn from point O, at which the collision between the photon with energy \hat{E}_γ^0 and the stationary electron occurred, to corresponding points on the curves depict the state of the particle after scattering: the magnitudes of the vectors give the energy of the particles, and the angles formed by the vectors with the direction of the incident photon define the scattering angle θ and the angle θ of the recoil electron's path. (The graph was plotted for the case of scattering of "hard" X rays with wavelength $\lambda = \frac{hc}{\hat{E}_\gamma^0} = \lambda_0 = 0.024 \text{ \AA}$.) is scattered without a change in its energy (that is, without a change in its wavelength, or "coherently"). In heavy atoms only the peripheral electrons are weakly bound (in contrast to the electrons filling the inner shells of the atom), and therefore the spectrum of the scattered radiation has both a shifted (Compton) line, from scattering by the peripheral electrons, and an un-shifted (coherent) line, from scattering by the entire atom. With increasing atomic number (nuclear charge) the electron binding energy increases, the relative intensity of the Compton line decreases, and that of the coherent line increases.

The motion of the electrons in atoms leads to a broadening of the Compton lines in the scattered radiation. This occurs because the wavelength of the

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incident light appears to be slightly changed for moving electrons; in addition, the amount of change depends on the magnitude and direction of the electron's velocity (the Doppler effect). Careful measurements of the intensity distribution in a Compton line, which reflects the velocity distribution of the electrons in the material, has confirmed the correctness of quantum theory, according to which electrons obey Fermi-Dirac statistics.

The simplified theory of the Compton Effect examined here does not permit the calculation of all characteristics of Compton scattering, particularly the intensity of photon scattering at various angles. A complete theory of the Compton Effect is provided by quantum electrodynamics. The intensity of Compton scattering depends on both the scattering angle and the wavelength of the incident radiation. Asymmetry is observed in the angular distribution of the scattered photons: more photons are scattered forward, and the asymmetry increases with increasing energy of the incident photons. The total intensity of Compton scattering decreases with an increase in the energy of the primary photons (Figure 3); this indicates that the probability of the Compton scattering of a photon passing through matter diminishes with decreasing energy. Such a dependence of intensity on energy determines the place of Compton scattering among the other effects of interaction between matter and radiation that are responsible for loss of energy by photons in their passage through matter. For example, in lead the Compton effect makes the main contribution to the energy loss of photons at energies of the order of 1-10 mega electron volts, or MeV (in a lighter element, aluminum, this range is 0.1-30.0 MeV); below this region it is surpassed by the photoelectric effect, and above it by pair production.

Compton scattering is used extensively in studying the gamma radiation of nuclei; it is also the basis of the principle of operation of some gamma spectrometers.

The Compton effect is possible not only for electrons but also for other charged particles, such as protons; however, because of the proton's large mass its recoil is noticeable only during the scattering of photons with very high energy.

The double Compton effect consists of the formation of two scattered photons in place of a single incident photon during scattering by a free electron. The existence of this process follows from quantum electrodynamics; it was first observed in 1952. Its probability is approximately a hundred times less than that of the ordinary Compton effect.

Graph showing the dependence of the total Compton scattering intensity
Inverse Compton effect.

If the electrons on which electromagnetic radiation is scattered are relativistic (that is, if they are moving with speeds close to the speed of light), then in an elastic collision the wavelength of the radiation will decrease: the energy and momentum of the photons will increase at the expense of the energy and momentum of the electrons. This phenomenon is called the inverse Compton effect and is often used to explain the radiation mechanism of cosmic X-ray sources, the production of the X-ray component of the background galactic radiation, and the transformation of plasma waves into high-frequency electromagnetic waves.

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Description of the phenomenon

By the early 20th century, research into the interaction of X-rays with matter was well underway. It was known that when a beam of X-rays is directed at an atom, an electron is ejected and is scattered through an angle θ . Classical electromagnetism predicts that the wavelength of scattered rays should be equal to the initial wavelength; however, multiple experiments found that the wavelength of the scattered rays was greater than the initial wavelength.

In 1923, Compton published a paper in the *Physical Review* explaining the phenomenon. Using the notion of quantized radiation and the dynamics of special relativity, Compton derived the relationship between the shift in wavelength and the scattering angle:

Where

λ_0 is the initial wavelength,

λ is the wavelength after scattering,

h is the Planck constant,

m is the mass of the electron,

c is the speed of light, and

θ is the scattering angle.

The quantity $\frac{h}{mc}$, which is known as the Compton wavelength of the electron; it is equal to 2.43×10^{-12} m. The wavelength shift $\lambda - \lambda_0$ is at least zero

(for $\theta = 0^\circ$) and at most twice the Compton wavelength of the electron (for $\theta = 180^\circ$).

Compton found that some X-rays experienced no wavelength shift despite being scattered through large angles; in each of these cases the photon failed to eject an electron. Thus the magnitude of the shift is related not to the Compton wavelength of the electron, but to the Compton wavelength of the entire atom, which can be upwards of 10,000 times smaller.

Compton Scattering

the scattering of X-rays from electrons in a carbon target and found scattered X-rays with a longer wavelength than those incident upon the target. The shift of the wavelength increased with scattering angle according to the Compton formula:

Compton explained and modeled the data by assuming a particle (photon) nature for light and applying conservation of energy and conservation of momentum to the collision between the photon and the electron. The scattered photon has lower energy and therefore a longer wavelength according to the Planck relationship.

At a time (early 1920's) when the particle (photon) nature of light suggested by the photoelectric effect was still being debated, the Compton experiment gave clear and independent evidence of particle-like behavior. Compton was awarded the Nobel Prize in 1927 for the "discovery of the effect named after him".

Compton Scattering Data

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Compton's original experiment made use of molybdenum K-alpha x-rays, which have a wavelength of 0.0709 nm. These were scattered from a block of carbon and observed at different angles with a Bragg spectrometer. The spectrometer consists of a rotating framework with a calcite crystal to diffract the x-rays and an ionization chamber for detection of the x-rays. Since the spacing of the crystal planes in calcite is known, the angle of diffraction gives an accurate measure of the wavelength.

Examination of the Compton scattering formula shows that the scattered wavelength depends upon the angle of scattering and also the mass of the scatterer. For scattering from stationary electrons, the formula gives a wavelength of 0.0733 nm for scattering at 90 degrees. That is consistent with the right-hand peak in the illustration above. The peak which is near the original x-ray wavelength is considered to be scattering off inner electrons in the carbon atoms which are more tightly bound to the carbon nucleus. This causes the entire atom to recoil from the x-ray photon, and the larger effective scattering mass proportionally reduces the wavelength shift of the scattered photons. Putting the entire carbon nuclear mass into the scattering equation yields a wavelength shift almost 22,000 times smaller than that for an unbound electron, so those scattered photons are not seen to be shifted.

The scattering of photons from charged particles is called Compton scattering after Arthur Compton who was the first to measure photon-electron scattering in 1922. When the incoming photon gives part of its energy to the electron, then the scattered photon has lower energy and according to the Planck relationship has lower frequency and longer wavelength. The wavelength change in such scattering depends only

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upon the angle of scattering for a given target particle. The constant in the Compton formula above can be written

and is called the Compton wavelength for the electron. The formula presumes that the scattering occurs in the rest frame of the electron

Compton scattering occurs when the incident x-ray photon is deflected from its original path by an interaction with an electron. The electron is ejected from its orbital position and the x-ray photon loses energy because of the interaction but continues to travel through the material along an altered path. Energy and momentum are conserved in this process. The energy shift depends on the angle of scattering and not on the nature of the scattering medium. Since the scattered x-ray photon has less energy, it has a longer wavelength and less penetrating than the incident photon.

Compton Effect was first observed by Arthur Compton in 1923 and this discovery led to his award of the 1927 Nobel Prize in Physics. The discovery is important because it demonstrates that light cannot be explained purely as a wave phenomenon. Compton's work convinced the scientific community that light can behave as a stream of particles (photons) whose energy is proportional to the frequency.

The change in wavelength of the scattered photon is given by:

Where:

L

=

wavelength of incident x-ray photon

λ'

=

wavelength of scattered x-ray photon

h

=

Planck's Constant: The fundamental constant equal to the ratio of the energy E of a quantum of energy to its frequency ν : $E = h\nu$.

m_e

=

the mass of an electron at rest

c

=

the speed of light

Q

=

The scattering angle of the scattered photon

The applet below demonstrates Compton scattering as calculated with the Klein-Nishina formula, which provides an accurate prediction of the angular distribution of x-rays and gamma-rays that are incident upon a single

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electron. Before this formula was derived, the electron cross section had been classically derived by the British physicist and discoverer of the electron, J. J. Thomson. However, scattering experiments showed significant deviations from the results predicted by Thomson's model. The Klein-Nishina formula incorporates the Breit-Dirac recoil factor, R , also known as radiation pressure. The formula also corrects for relativistic quantum mechanics and takes into account the interaction of the spin and magnetic moment of the electron with electromagnetic radiation. Quantum mechanics is a system of mechanics based on quantum theory to provide a consistent explanation of both electromagnetic wave and atomic structure.

The applet shows that when a photon of a given energy hits an atom, it is sometimes reflected in a different direction. At the same time, it loses energy to an electron that is ejected from the atom. θ is the angle between the scattered photon direction and the path of the incident photon. ϕ is the angle between the scattered electron direction and the path of the incident photon.

Derivation of the scattering formula

A photon with wavelength λ is directed at an electron in an atom, which is at rest. The collision causes the electron to recoil, and a new photon with wavelength λ' emerges at angle θ . Let ϕ denote the electron after the collision.

From the conservation of energy,

Compton postulated that photons carry momentum; thus from the conservation of momentum, the momenta of the particles should be related by

Assuming the initial momentum of the electron is zero.

The photon energies are related to the frequencies by

Where h is the Planck constant. From the relativistic energy-momentum relation, the electron energies are

Along with the conservation of energy, these relations imply that

Then

From the conservation of momentum,

Then by making use of the scalar product,

Thus

The relation between the frequency and the momentum of a photon is $p = hf/c$, so

Now equating 1 and 2,

Then dividing both sides by $2hf/c$,

Since $c = \lambda f$, $\lambda = c/f$,

Detector characteristics

Even large Compton-scatter telescopes have relatively small effective areas. This is because only a small number of the incident gamma-rays actually Compton scatter in the top level. So even if an instrument like COMPTEL has a geometric area of several thousand cm², the effective area (weighted for the probability of an interaction) is a few tens of cm².

Energy resolution is fairly good for these detectors, typically 5-10%. This is limited by uncertainties in the measurements of the energy deposited in each layer. Compton scatter telescopes have wide fields-of-view and can form images even though the so-called point spread function (the probability that an event came from a certain area on the sky) is a ring.

Applications

Compton scattering is of prime importance to radiobiology, as it is the most probable interaction of gamma rays and high energy X rays with atoms in living beings and is applied in radiation therapy. [4]

In material physics, Compton scattering can be used to probe the wave function of the electrons in matter in the momentum representation.

Compton scattering is an important effect in gamma spectroscopy which gives rise to the Compton edge, as it is possible for the gamma rays to scatter out of the detectors used. Compton suppression is used to detect stray scatter gamma rays to counteract this effect.

Inverse Compton scattering

Inverse Compton scattering is important in astrophysics. In X-ray astronomy, the accretion disks surrounding a black hole is believed to produce a thermal

spectrum. The lower energy photons produced from this spectrum are scattered to higher energies by relativistic electrons in the surrounding corona. This is believed to cause the power law component in the X-ray spectra (0.2-10 keV) of accreting black holes.

The effect is also observed when photons from the cosmic microwave background move through the hot gas surrounding a galaxy cluster. The CMB photons are scattered to higher energies by the electrons in this gas, resulting in the Sunyaev-Zel'dovich effect. http://en.wikipedia.org/wiki/Sunyaev-Zel'dovich_effect HYPERLINK "http://en.wikipedia.org/wiki/Sunyaev-Zel'dovich_effect" dovlch effect. Observations of the Sunyaev-Zel'dovich effect provide a nearly redshift-independent means of detecting galaxy clusters.

Some synchrotron radiation facilities scatter laser light off the stored electron beam. This Compton backscattering produces high energy photons in the MeV to GeV range subsequently used for nuclear physics experiments.

Future developments

Current research on Compton telescopes is emphasizing ways of tracking the scattered electron. By measuring the direction of the scattered electron in the top level, a complete solution for the incoming trajectory of the cosmic gamma-ray can be found. This would allow Compton telescopes to have more conventional data analysis approaches since the "event circle" would no longer exist.