The strengths and limitations of duration analysis

As stated by the US Federal Reserve, interest rate risk impacts on a various range of stakeholders, and hence financial actors are interested in quantifying its impact. The most important practical tool to manage interest rate risk and to satisfy this main function for banks is duration analysis. In general duration Analysis is an econometric tool and in terms of Financial Economics it is defined as the mean length of time that passes until the present value is returned by a stream of fixed payments according to Macaulay (1938). Hence, Duration is a measure of the sensitivity of asset´s prices to interest movements. My following essay defines duration according to Macaulay and presents special terms from the practice. Moreover, it considers immunization, hedging and Duration Gap Analysis as practical applications. The next part will discuss strengths and weaknesses of duration analysis. It concludes with today’s importance of Duration analysis.

There are two main reasons to study Duration according to Kopprasch (2006). Firstly, firms and especially financial intermediaries have tied up huge amounts of capital in fixed income instruments. These include bonds partly with optional characteristics or recent financial innovations like swaps, interest rate options or floaters. Hence, proper hedging of these instruments becomes important. Secondly, the key figure duration provides an intuitive approach to educate potential customers. This leads to a better understanding of financial instruments in general and how they behave when interest rates change.

Bodie, Kane, Marcus (2006)

Empirical studies and Figure 16. 1 show six bond-pricing relationships:

Firstly, there’s an inverse relation between bond price and yield to maturity. Secondly, an increase in a bond’s yield to maturity results in a smaller price change than a decrease of equal magnitude. Thirdly, long-term bonds are more sensitive than short-term bonds. Fourthly, interest rate risk, which is measured by the sensitivity of bond prices to changes in yields, is less than proportional to bond maturity. Fifthly, there is an inverse relationship between interest rate risk and bond’s coupon rate, because a bond with a higher coupon rate pays a greater percentage of its present value prior to maturity. Sixthly, the yield to maturity at which the bond is currently sold is inversely related to the sensitivity of the bond’s price to a change in its yield. This five observations were described Mankiel and are known as Malkiel bond-pricing relationships. The sixth property was demonstrated by Homer and Liebowitz (1972). Ingersoll, Skelton, Weil, (1978) stated that the key figure Duration can be interpreted as an attempt to quantify this qualitative observations through a single and numerical measure.

The duration concepts has its origins in the work of Macaulay(1938), Samuelson (1945), Hicks (1939) and Redington (1952). Macaulay(1938) defined duration as the mean length of time that pass until the present value is returned by a stream of fixed payments. The proof that duration is an elasticity was provided by Hicks in 1939. This means that the price elasticity of a bond in response to an infinitesimal change in its yield to maturity is proportional to duration. But Fisher (2006) casts doubt on Hicks’s derivation. Nevertheless, his proof is generally acknowledged. Redington (1952) derived the duration independently and used it for portfolio immunization. The standard definition according to Macaulay is:

subject to

The weight is calculated by .

In the special case of a zero bond, the duration equals the maturity, because no payments occur before maturity.

Kopprasch (2006) mentions several different practical methods which are based on Macauley's duration and are used in practice. " Effective duration" is determined by the price movement to an incremental movement while holding the " option adjusted spread" constant. " Option adjusted Spread" (OAS) is a flat spread which is added to the yield curve in a pricing model and considers options like prepayments opportunities for mortgage backed securities. Hence, OAS is model dependent and incorporates volatility like variable interest rates or prepayment rates. " Portfolio duration" quantifies the Duration of a portfolio of different assets. It is based on the additivity of single durations. Additivity means that the duration of a portfolio is the weighted-average of the durations of the individual securities. The weights are the current market value of each security. The term " Modified duration" is calculated by the formula:

Furthermore, the term " partial durations" or " key rate durations" is a vector of durations, where each duration is only valid for a limited maturity range. " Spread duration" recognizes that a change in the spread can affect the bond. This key figure was designed especially to value floaters which trade near par by definition. It often turns out that the market doesn't seem to trade the instruments with the predicted duration. Hence, " empirical duration" was developed to deal with these times. It is calculated by regressing price movements of the asset versus some market benchmark.

The next paragraph considers two applications of duration in risk management: Hedging and immunization for a portfolio and Duration Gap Analysis.

The change in an asset price due to change in interest rates can be calculated by:

Fooladi (2000) describes that the realized rate of return encompass interest accumulated from reinvestment of coupon income and the capital gain or loss at the end of the planning period when the portfolio is sold. The two components impact the realized rate of return in opposite directions. Hence, in one point the two opposite effects of coupon reinvestment and capital gain or loss offset one another. When the portfolio duration equals the length of the planning period, the portfolio is immunized and the realized return will not fall below the promised rate of return.

The second described application is Duration Gap Analysis which is an extension to the immunization approach, because it includes liabilities. A main function of banks is to provide maturity transformation. Hence, banks usually have short-term liabilities and long-term assets. As a consequence of this duration mismatch and shown by the third following equation, changes in interest rates have a direct effect on the banks' equity value. The gap between the durations of the assets and liabilities (is a measure of the interest rate risk of banks' equity.

Fooladi (2000) describes that banks may take modest bets by setting a duration gap or set the duration gap close to zero. The second equation shows how banks can adjust their duration gap by shifting weights on assets or liabilities. Bierwag and Fooladi (2006) specify that banks use off-balance-sheet securities like interest rate futures, options and swaps to reduce adjustment time and to save costs.

Despite the shown strengths, there are weaknesses in duration analysis. As one can see in Figure 16. 3, Duration is only valid for small changes, because the relationship between duration and price changes is derived by a first-order Taylor series approximation.

Furthermore, Mishkin/Eakins (2006) criticise that interest rate changes have to affect all rates of maturities by exactly the same amount. Generally speaking, the slope of the yield shouldn’t be affected at all and the yield curve is assumed to be flat. However, the shape of the yield curve fluctuates over the business cycle and consequently this expected slope change has to be considered. The mentioned “ partial duration” and “ spread duration” try to handle this shortcoming.

Further problems involve uncertainty over the proportion of assets and liabilities. Estimates have to consider for example prepayment of loans, customer shifts out of deposits and uncertain cash payments due to default risk according to Fooladi and Roberts (2004). As Bierwag and Kaufman (1988) showed, default alters bond’s cash flows and their timing.

Additionally, one has to predict the stochastic process governing interest rate movements to value options. This can create a stochastic process risk which can be quantified by approaches to to measure interest rate volatility risk. Ho (2007) states that practitioners tie duration and vega measures which specify the sensitivities to the shift in the swap curve and the volatility surface, respectively. In his approach volatility risk is measured by the value sensitivity of an option to the change in the implied volatility function at the key rate points on the curve.

Ingersoll, Skelton, and Weil (1978) argue that the assumed stochastic process to develop duration models is inconsistent with equilibrium conditions. Occurring large shocks to interest rates, riskless arbitrage became possible, but on the practical side the riskless-arbitrage argument seemed hypothetical. To overcome these weaknesses, Mishkin and Eakins (2006) mentions more sophisticated approaches such as scenario analysis and value-at-risk analysis and convexity which is a second-order Taylor series approximation and can be used as a correction measure. Paroush and Prisman (1997) strengthen this assumption and show that convexity (second-order) can be more important than the duration (first order).

To put it in a nutshell, with increasing complexity of securities, myriad extensions have been added to the former duration analysis founded by Macaulay to handle the occurring risks. Furthermore, different duration measures face different assumptions about slope and shape of the yield curve or the stochastic process driving interest rates. One has to take in mind how accurate these assumptions are, because as seen in the recent financial crisis failures affect the entire economy, according to my starting statement. However, duration analysis is an adaptable framework and used carefully, a tool to get a first impression of interest-rate risk.

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