# Finite element analysis and analytical method



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Stone columns are widely used as a ground improvement technique especially in construction of shallow foundations. The main concern in the application of stone columns rely on how well it performs, which involves reducing the overall settlement of the stone column.

This project mainly investigates the comparison and contrast between finite element analysis and analytical method in modelling stone columns, whereby settlements of the stone columns are checked whether it is consistent. Finite element analyses were carried out by axisymmetric modelling of the stone column using 15-noded triangular elements with the software package PLAXIS. A drained analysis was conducted using Mohr-Coulomb's criterion for soft clay, stones and sand. Analytical data used to compare the settlement was found according to the design method published by Heinz J. Priebe (1995). Both methods were compared by varying parameters such as modulus of deformation of the column to sand ratio, area ratio, stress, diameter, and friction angle of stone column that signifies different soil conditions.

It is challenging to find a site with acceptable ground conditions for construction of structures such as buildings, bridges, etc. Often the bearing capacity of the soil would not be sufficient to support the loads of the structures nor would it be in a workable condition for the employees to build the structure. The need for the use of such land with weak cohesive soil strata has been a challenge for design engineers. Although the design of piles foundation can meet all the design necessities, extensive lengths of piles needed eventually results in vast increase of cost of the overall project. Therefore, it is a necessity that the ground conditions must be improved to allow the buildings and heavy construction.

A number of ground improvement techniques have been developed over the past fifty years. Main concern of these techniques includes creating stiff reinforcing elements to the soil mass, which results in a soil that has a higher bearing capacity. Out of the various techniques available for ground improvement, the stone column has been widely used.

Stone columns (also known as granular columns, granular piles or sand columns) are used to improve soft ground by increasing the load bearing pressure of the soil and reducing settlement of the foundation of structures, embankments, etc. Although these structures are permissible for a relatively large settlement, it is necessary that the settlement be minimized for maximum safety.

There have been several ways for installing stone columns depending on the design, local practice and availability of equipment. Among which, the most general methods are the vibro-replacement method and vibro-displacement or vibro-compaction methods. Vibro-replacement technique of stone column is a process whereby large sized columns of compacted coarse aggregates are installed through the weak soil by means of special in-depth vibrators. This can be carried out either with the dry or wet process. In the dry process, a hole of desired depth is drilled down in to the ground by jetting a vibroflot. Upon extraction of the vibroflot, the borehole must be able to stand open. The densification of the soil will be a result of the vibrator near the bottom of the vibroflot. In the wet process, the vibroflot will form a borehole that is of

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larger diameter than the vibrator and it requires continuous supply of water. As a result the uncased hole is flushed out and filled with granular soil. The main difference between wet and dry process is the absence of continuous jetting water during the initial formation of the borehole in the dry process.

The performance of the stone columns is not measurable by simple investigations. However, analytically, the efficiency of this composite system that consists of stone column and soil interactions can be assessed by separate consideration of significant parameters as proposed by Priebe (1995) [1].

Stone column technique has proven successful in improving many applications. Such applications include slope stability of both natural slopes and embankments. Construction of such embankments can commence immediately after the installation of stone columns (Vibro Stone Columns, 2009) [2]. Other advantages include increasing bearing capacity of ground, reducing total and differential settlements, reducing the liquefaction potential of sands. The main disadvantage of the stone column technique is its ability to induce bulging failure on the upper part of the stone column.

In-situ field tests (cone penetration test and full scale footing test) before construction and after construction of stone columns have shown significant improvements in the soil (J. T. Blackburn, J. K. Cavey, K. C. Wikar, and M. R. Demcsak., 2010) [3]. In a study of the behaviour of stone columns, (Mitchell J. K., and Huber T. K., 1985) [4], by using finite element analysis, had proved that the installation of stone columns leads to a 30-40% reduction in settlement of the values expected that of an untreated ground.

## 1. 2 Objectives

The main objective of this project is to show that the analytical method used to design stone columns and the finite element method used to model the stone column numerically, has comparable total and differential settlement. The analysis also provide the understanding of the influence on settlement by varying parameters such as modulus of deformation of the column to sand ratio (Ec/Es), Area ratio (Ac/A), stress ïf0, diameter D, and friction angle of stone column Φ'c, and finally comparing them against the Priebe analytical approach.

The objectives of the project are to:

study the existing analytical and numerical theories related to stone column modelling

develop an axisymmetric simulation of the stone columns by using finite element method, and

compare the settlement difference with the analytical results by altering various parameters related to settlement change.

This project uses the finite element software package PLAXIS to simulate the stone column numerically and the design method proposed by Heinz J. Priebe (1995) [1] for the analytical results.

## 1.3 Organization of the research paper

In addition to the abstract, list of figures and notation, acknowledgement, and table of contents, this dissertation is divided to six chapters:

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The first chapter consists of introduction and background of stone columns where it briefly summarizes the installation methods, some of the advantages and disadvantages of the stone columns.

The second chapter describes the study of existing analytical and numerical theories regarding modelling stone columns. In this chapter, other than the main findings from the theories, the full procedure of Priebe (1995) method of modelling stone column has been reviewed.

Third chapter describes how the stone column was modelled using the PLAXIS software, including the assumptions made and technical data used in different models.

The fourth chapter shows the results obtained from the analysis compared to the analytical method proposed by Priebe (1995). The results are presented using necessary graphs and charts.

The fifth chapter includes the conclusion of the project and provides recommendations for further studying.

The final chapter lists out the references used in this project.

The Appendix contains documents such as the Risk Assessment, Diary of the work progress, and the any additional tables and figures of the analysis.

## **CHAPTER TWO**

## **2. LITERATURE REVIEW**

Many researchers in this field have made their effortless contribution studying the behaviour of stone columns numerically and analytically. Most of the numerical analyses were conducted using finite element analysis, whereas analytical method is derived from a series of equations. Some of the main findings from researchers related to this study are reviewed below.

## 2.1 Analytical Models

#### 2. 1. 1 Alamgir, Miura, Poorooshasb, and Madhav, (1996)

Alamgir et al. (1995) proposed a simple theoretical approach to evaluate the deformation behaviour of uniformly loaded ground reinforced by columnar inclusions. The displacements of the soil and stone columns are obtained by considering the elastic deformation of both soil and column. A typical column-reinforced ground and column soil unit (Fig. 2. 1) where the column is considered to be cylinder, of height H and diameter of dc (= 2a where a is the radius)

The deformation at a cross section within the column, wcz, is assumed to be constant throughout whereas the deformation of the surrounding soil, wrz, increases from the soil column surface towards the outer boundary of the unit cell (Fig. 2. 2). This denotes that since the column soil interface is elastic and no slip occurs, the displacements of the soil and the column at interface can be assumed to be equal. The deformation of the surrounding ground, wrz, is assumed to follow:

where wrz is the displacement of the soil element at a depth z and at a radial distance r, wcz is the displacement of the column element at a depth z,  $\hat{1}\pm cz$  and  $\hat{1}^2c$  are the displacement parameters, a and b are the radii of column and unit cell, respectively, r is the radial distance measured from the center of the column.

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The column and the surrounding soil were discretized in to a number of elements as shown in Fig. 2. 3. The interaction shear stresses and stresses on the column and the soil were obtained by using equilibrium of vertical forces within the medium (Fig. 2. 4).

Successively the displacement of the column and soil were obtained by solving equations by applying the linear deformation characteristics of the soil. Therefore, the deformation of the jth element of the column, Wcj was obtained as:

where  $\hat{a}^{+}H$  is the height of a single element, Es and Ec are the modulus of deformations of soil and column material respectively, vs is the Poisson's ratio of the soil, and Ïfcj is the normal stress acting at the top of the jth element of the column.

Due to the symmetry of load and geometry, the shear stress at the outside boundary of the unit cell is zero, which subsequently leads to an equation for  $\hat{l}^2c$ 

Furthermore, the compression of the soil element adjacent to the boundary of unit cell (N, jth element of the soil), wsNj was derived as:

where ÏfsNj is the normal stress acting at the top of the element, n is the spacing ratio b/a, â^†R is â^†r/a and â^†r is (b-a)/n.

By using the displacement compatibility and substituting  $r/a = n-\hat{a}^{+}R/2$ , Eq. [2. 1] can be written as:

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Finally, solving the equations 2. 2, 2. 3, 2. 4, and 2. 5 can lead to the displacement parameter

The settlement profiles, the shear stress distribution, and the load sharing from the above mention method was compared against a simple finite element analysis as shown in Fig. 2. 5, Fig. 2. 6, and Fig. 2. 7. It is seen that the results obtained shows a reasonable agreement between the two methods and can be used as a useful method to determine the settlement of the stone columns.

#### 2. 1. 2 Priebe (1995)

Priebe (1995) proposed a design method to assess the behaviour of stone columns that uses an improvement factor which stone columns improve the performance of the subsoil in comparisons to the state without columns. The above statement was best described using the following relationship:

According to this improvement factor, the deformation modulus of the composite system is increased respectively settlements are reduced.

A unit cell of area A is considered which consists of a single column with the cross section area Ac. Calculation of the improvement factor was done by assuming that:

The stone column to be of incompressible material

The stone column is installed within a rigid layer

The bulk densities of the stone column and soil are also neglected.

Hence, according to Priebe's approach, column cannot fail in end bearing and any settlement of the load area results in a bulging of the column, which remains constant all over its length.

The improvement of a soil achieved by the presence of stone columns is evaluated based on the assumption that the column material shears from the beginning whilst the surrounding soil reacts elastically. Additionally, the coefficient of earth pressure amounts to K=1 by assuming that the soil to be displaced already during the column installation to such a degree that its preliminary resistance corresponds to the liquid state. Using the above criterion the basic improvement factor n0 is expressed as:

#### where

= Improvement factor

Ac = Area of the stone column

- A = Grid area of the single unit
- = Poisson's ratio
- = Coefficient of active earth pressure for the stone column material
- = Friction angle of the stone column material

Since a Poisson's ratio of 1/3 is adequate for the state of final settlement in most cases, the results of the evaluation is expressed as basic improvement factor n0 and substituting 1/3 as Poisson's ratio, which leads to the following equation. The relation between the improvement factor n0, the area ratio A/Ac and the friction angle of the backfill material is illustrated in figure 2. 8 below.

The compacted backfill material of the stone column is still compressible. Due to this reason, applied load of any amount will lead to settlements that are unconnected with bulging of the columns. Subsequently, compressibility of the column is integrated by adding up an additional area ratio (A/Ac) as a function of the constrained moduli of the columns and soil Dc/Ds and is provided in the Fig. 2. 9.

The improvement factor as a result of the consideration of the column compressibility is represented by n1, as shown in the equation:

where

and

Furthermore, for = 1/3 can be found using the equation below

The additional loads due to the bulk densities of the soil and columns decrease the pressure difference asymptotically and reduce the bulging correspondingly. Subsequently, multiplying the basic improvement factor by a depth factor could incorporate the effect of the bulk density, which is given by:

where,

fd = Depth factor

K0C = Coefficient of earth pressure at rest for stone column material

= Bulk density of the soil

#### = Layer thickness

Pc = Pressure within the column along the depth

Figure 2. 10 shows the influence factor y as a function of the Area ratio A/Ac and can be used to approximate the depth factor. The figure considers the same bulk density for the columns and soil, which may not be true in most cases. Therefore as a safety measure, the lower value of the soil should be always considered.

Using the above depth factor fd, a more enhanced improvement factor can be defined that considers the effects of the overburden pressure, and therefore is represented by n2 where it can be related by the following equation:

The depth factor is limited so that the settlement of the columns resulting from their inherent compressibility does not exceed the settlement of the composite system. This is because as the depth increases, the support by the soil reaches such an extent that the column do not bulge anymore. The first compatibility control where the depth factor is limited is applied when the existing soil is stiff or dense and is given by:

The second compatibility control is required since should not be considered even if it may result from the calculation. This second control relates to the maximum value of the improvement factor nmax and is applied when the existing soil is loose or soft. Finally, the total settlement of a single or a strip footing can be assessed using the above series of equations. The design results from the performance of an unlimited column grid below an unlimited load area. For the unimproved ground, the settlement can be found using the equation:

where,

- sâ<sup>\*</sup>ž = Total settlement
- p = Pressure exerted by the above structure
- d = Depth of the stone column
- Ds = Constrained modulus of the soil

Similarly, the total settlement of the improved ground, where the improvement factor is incorporated, can be found by dividing the settlement by n2, which is shown below:

This method is one of the most common and well-known method of designing stone columns and has been widely used all over the world because of its simplicity. Moreover, in comparison with the other methods, it shows a much wider behaviour of the stone column by assuming the stone column and surrounding soil as a composite system.

## 2. 2 Numerical Models

# 2. 2. 1 A. P. Ambily and Shailesh R. Gandhi (2007)

Ambily and Shailesh (2007) studied the behaviour of stone columns by comparing experimental and Finite Element analysis on a single stone column and a group of 7 columns.

Laboratory experiments were carried out on a stone column of 100mm diameter surrounded by soft clay in cylindrical tanks of 500mm high with diameter varying from 210 to 420 mm for a single column test and from 210 – 835 mm for a group of 7 columns. This represents the required unit cell area of soft clay around each stone column. Pressure cells attached to the loading plate were used to measure the stress intensity of the column and the soil as shown in figures 2. 12 and 2. 13. Furthermore, it is also assumed the stone columns are installed in a triangular pattern.

The load deformation behaviour of the column/treated soil was studied by applying vertical load for both cases; column only loading and entire area loading, and observed for equal intervals of settlements until failure occurs. After a series of procedure, the shapes of the tested columns are obtained. It is clearly seen in Fig. 2. 14 that bulging mode of failure only occurs in the case of column alone loaded, and not in the case of entire area loaded.

Finite Element analysis was conducted using 15-noded triangular elements with the software package PLAXIS, to compare the load-settlement behaviour with the model test and the laboratory experiment. The analysis was carried out using a stone column of diameter 25 mm and 225 mm high, which was made at the center of the clay bed and loaded with a plate of diameter two times the diameter of the stone column. The axisymmetric finite element mesh to represent the single stone column and the group of stone columns are shown in Fig. 2. 15 and Fig. 2. 16 respectively.

Likewise the laboratory experiment, finite element analyses were done for column alone loaded and entire area loaded case for s/d=3. The results of these simulations (Fig. 2. 17) shows that failure by bulging occurs in column alone loaded case, which also agrees with the results from laboratory experiment.

The comparison of the experimental results and finite element analysis data shows significant consistency in both methods. The comparisons made by A. P. Ambily and Shailesh R. Gandhi include the effect of shear strength, Cu (Fig. 2. 18) and the effect of s/d (Fig. 2. 19) on the behaviour of stone columns. Additionally, the effect of surcharge on stress settlement behaviour (Fig. 2. 20) and effect of s/d and  $\hat{l}_i$  on the stiffness improvement factor (Fig. 2. 21) was compared between both methods. These tests have also shown similar behaviour. The stiffness improvement factor ( $\hat{l}^2$ ) was calculated as the ratio of the stiffness of treated and untreated ground, and beyond s/d = 3, it shows no significant improvement.

The analysis was extended to study the effect of the angle of internal friction of stones by varying the  $\hat{l}$  as 35, 40, 43, and 450 for varying values of s/d ranging from 1. 5 – 4. From the results shown in Fig. 2. 22, it is confirmed that this relationship is valid for any shear strength values of surrounding soil. Furthermore, the comparisons between a single column and group of 7 columns were found as in Fig. 2. 23.

Both experimental and finite element method results reveal comparable behaviour regarding the ultimate load and load deformation relationship. To ensure that this proposed design method agrees with the existing theories, this study was compared with the existing theories as shown in Fig. 2. 24 and Fig. 2. 25. The result shows a slightly higher stiffness improvement factor ( $\hat{I}^2$ ) for an area ratio more than 4 and a lower value for an area ratio less than 4 compared to Priebe (1995).

#### 2.3 Summary

The studies mentioned above show comparable results and have been adopted by many engineers and contractors. However, not many researchers had compared Priebe's analytical model with finite element method. Therefore, the finite element analysis carried out in this project will be compared to the design method proposed by Priebe (1995), since it gives a much broader overview of the composite system consisting of the stone column and soil interactions and moreover it is the most common and improved analytical method used by the design engineers around the globe.

# **CHAPTER THREE**

# **3. METHODOLOGY**

#### 3.1 Introduction

Different methods of modelling stone columns numerically have been implemented in the past. Among those, the most simplest and common type of numerical modelling is using finite element method. In fact, studies have shown that the settlements predicted from the finite element analysis shows comparable results that of the values gained from actual field tests (Kirsch, F. 2009). Numerical calculations are usually complex and most of the time is impossible to conduct without means of dedicated software. Likewise, in this research project, PLAXIS software is used to carry out the finite element analyses.

#### 3. 2 PLAXIS software

The main computer software used in this investigative project is PLAXIS Professional Version 8. 2. PLAXIS is a comprehensive package for finite element analyses for geotechnical applications. It allows simulating the soil behaviour by using soil models. The software employs a graphical user interface that makes it simple to use and also provide the ability to input the necessary parameters such as different soil layers, structural elements, variety of loadings, and boundary conditions through CAD drawing procedures. It allows discretizing the soil component into either 6-noded or 15-noded triangular elements whereby 15-noded triangles provides high stress results for complex problems. The software also allows automatic generation of 2D finite element meshes that can be further refined according to the choice of analysis. In addition to that, the software comes with a very useful feature named Staged Construction. This feature allows the models to be simulated at different stages by activating and deactivating clusters of elements, application of loads, etc. One of the advantages of this software is the ability to generate the results quickly with minimum errors. The output results include values for stresses, strains, settlements, and structural forces

together with the plots of different curves such as, load-displacement curve, stress-strain diagrams, and time-settlement curve.

#### 3.3 Finite Element Modelling

Finite element analysis was conducted to compare the load-settlement behaviour of the stone column. A two dimensional axisymmetric analysis was carried out since the investigation concerns a single unit of stone column using Mohr-Coulomb's criterion for clay and stone column. 15-noded discretization was used for more precise results. The initial vertical stress due to gravity has been considered in this analysis. Similarly, the stress due to column installation, which often depends on the method of construction, is also considered in this analysis.

Assumptions made in the finite element modelling:

The soil is assumed to be homogenous, infinite and behaves as Mohr-Coulomb model.

The ground water table is at the same level as the stone column and clay layer, meaning the stone column and clay layer is submerged in the water. Hence, effect of ground water condition should be taken into account.

The base of the clay layer is rigid, i. e., full fixity at the base of the geometry (ux = 0, uy = 0) and roller conditions at the vertical sides (ux = 0, uy = free) – boundary conditions are shown in Figure 3. 1(a).

Assumed that deformation of the column is mainly by radial bulging and no significant shear is possible. Therefore, interface element between stone column and clay has not been used.

Mitchell, J. K., and Huber, T. R. (1985) also carried out similar type of finite element analysis without the inclusion of the interface element.

#### **3. 4 Geometrical Parameters**

The dimensions of the PLAXIS model are shown in Figure 3. 1(b). H is the height of the column, which varies between 10m, 20m, and 30m. D is the diameter of the stone column, which has a typical value of 1m, in all the models except for the model to check the influence of diameter and spacing. Equivalent diameter De depends on the spacing between stone columns as well as the arrangement pattern of the columns. The value of De was calculated by considering the following Influence Area methods.

#### 3. 4. 1 Influence Area Methods

There are several methods for calculating the equivalent diameter around the stone column, which depends greatly on the spacing, diameter, and pattern of installation of the stone column. Two methods were considered in this investigation.

#### 3. 4. 1. 1 Equivalent Area method

The equivalent area method simply equates the area of the grid spacing with that of the cross sectional area of column to find the influence area around the stone column. The following example gives a better understanding of the above statement.

Example:

Grid spacing of the column =  $1.5 \times 1.5$  meters (square grid)

#### Therefore, Diameter of stone column =

Finally,

Where, De is the equivalent diameter around the stone column.

# 3. 4. 1. 2 Unit cell method (Balaam & Booker, 1981)

Unit cell consists of the column and the surrounding soil within the zone of influence of the column. The unit cell has the same area as the actual domain and its perimeter is shear free and undergoes no lateral displacement. Balaam & Booker (1981) relates the diameter of the unit cell to the spacing of the columns as:

where, De is the equivalent diameter

(for square grid) S is the spacing of the stone column

Similarly the different geometrical patterns due to column arrangements are shown in the Figure 3. 2.

Both methods reviewed above gives relatively similar magnitudes. However, Priebe's analytical method concerns more on unit cell area. Hence, for this investigation Equivalent Area method is used to model the influence are in PLAXIS.

# 3. 5 Mesh Refinement Test

Mesh generation has a great influence in the accuracy of the model. Generally, the finer the mesh the more accurate the result would be. However, this is not true for every case. Therefore a simple test using PLAXIS was conducted to check the effect of mesh refinement. Initially, mesh generation was set to coarse (around 100 elements), utilized as global coarseness of model. The test was carried out by comparing it with the refined mesh (around 500 elements). Moreover, the mesh is further refined which in PLAXIS is set to very fined (around 1000 elements). The generated meshes are shown in Figure 3. 3. followed by the timedisplacement graph showing the comparison between coarse, medium, fine and very fine mesh refinements. (Figure 3. 4)

From the above graph it can be seen that the four curves gives comparable results. However, the coarse, medium, and fine meshes give very similar results compared to the very fine mesh refinement. The objective here was to get the lowest value for the displacement since the improved ground due to the installation of stone column would eventually lead to a reduced settlement. Therefore, the finest mesh refinement gives the most precise result.

Even though it takes a substantial amount of time to simulate using the most finest meshing, for this investigation, models had been simulated using the very fine mesh option.

## 3. 6 Input Parameters

Varying the soil parameters can alter soil characteristics. Most important outcome by altering these parameters is deformation that leads to settlement. Such parameters that have major impact on settlement includes, material type, spacing of stone columns, diameter of influence area, diameter of stone column, elastic modulus of both column and soil, depth of the soil layer, Poisson's ratio for both column material and soil, Unit weights of the materials, cohesion, friction angle, etc. Soil and material properties are shown in Table 3. 1. Note that the effective stress cohesion, c' of the stone column is given a small nonzero value to avoid numerical complications.

The majority of the above parameters are considered for only one type of test model and are varied for different model tests. The varied parameters such as elastic modulus of soil and column, friction angle, spacing between columns and influence area around the stone column are reviewed in the following section.

## 3.7 Test Models

The main objective of this project is comparing both analytical and numerical method using Priebe's analytical approach and finite element analysis as numerical solution. This can only be achieved by developing multiple models and simulations to obtain a range of values to compare with, which would lead to a more solid conclusion.

Three constitutive models were considered for the representation of the following three cases.

A clay layer of 30 m, which has a stone column of height 10 m installed.

A clay layer of 30 m, which has a stone column of height 20 m installed.

A clay layer of 30 m, which has a stone column of height 30 m installed.

Note that 1 and 2 are floating columns that are not extended to bedrock or hard layer, which in stone column installation is a rare case, yet installed occasionally.

Each of the above tests was carried out by varying the spacing between columns, which would alter the s/d relationship together with the Ac/A ratio. Further tests were carried out to check the influence of stress Ïf0, diameter D, modulus of deformation of the column to sand ratio Ec/Es and friction angle of stone column Φ'c using the third case and compared them against the Priebe analytical approach.

The summary of test models is given in the Tables 3. 2. All the tests were carried out in 3 stages.

Install the stone column: Just after the stone column is installed

Apply Load: Just after the load is applied to the column

Consolidation: After the consolidation process completed to a minimum pore pressure of 1kPa

In the all cases the materials were idealized as the Mohr-Coulomb model with the characteristic linear-elastic-perfectly plastic behaviour and the failure criteria defined by the strength parameters given in tables below.

Table 3. 2 Summary of Model tests

Model Test

#### Description

#### Constants

#### Variables

#### 1

Influence of column height on settlement

(case 1, 2, and 3)

 $\ddot{I}f0 = 100 \text{ kPa}$ 

Ac/A = 0.2

 $\hat{I}_{1}'c = 40o$ 

Ec/Es = 20

Heigh