Taxicab geometry research paper

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Introduction

Taxicab geometry is a better idealized model for urban geography as compared to Euclidean geometry. This is very applicable in urban planning where distances within the roads are divided within rigid blocks and the shortest distance cannot be the hypotenuse. This is very helpful in establishing the shortest distance between any two points. (Surhome, Timpledon & Susan).

Geometry can be traced back over 2000 years. This is solely refers to Euclidean geometry introduced by Euclid. A German mathematician Hermann Miknowski proposed taxicab geometry. He had the proposition that distances in taxicab geometry is different than in Euclidean geometry. His aim was to prove that the hypotenuse is not always the shortest distance between two points. This is because it is not always the possible to use the hypotenuse. (Krause pp. 1-8). The word Taxicab geometry was first used by Karl Menger in 1952 in his book “ you will like geometry”. Taxicab geometry shares several aspects with Euclidean geometry. However, there exists fundamental distinction between the two types. (Sexton).

Taxicab geometry assumptions

1. The grids Vertical and Horizontal lines represents streets
2. Points are only positioned at grid intersections
3. Numerical coordinates are always set as integers
4. The taxicab distances between point A and point B is the minimum imaginary number of grid units which an imaginary taxi can travel between this two points. (Sexton).

Taxicab geometry: Distance

Taxicab and Euclidean geometry coordinate is very identical. The lines, angles and points are similar and measured in the same way. The difference comes in terms of distances. The taxicab geometry, distances referred is the distance that a taxicab would cover from point A to B as indicated below. (Martin 78)

The path indicated by the red color is the taxicab distance, while the blue path is the Euclidean path; all distances measured from point A to B. (Krause pp. 1-8).

The taxicab geometry affects the shape of circles, however, in both Euclidean and taxicab geometry the circle is defined as the set of all points equidistant from a given single point. The shape of the circle in taxicab geometry looks like as shown below.

In the taxicab geometry the shape of the eclipse looks very different form the Euclidean geometry. Generally the eclipse is a set all points in which the sum of the distances between two fixed points is fixed. In Taxicab geometry the eclipse is defined as a " set of all points where Taxicab distance AC +Taxicab distance CB" is kept constant. For example in the diagram shown below.

The idea of distances affects the shape of many geometrical figures. However, the geometry of the figures is the same. In this case, the parabola is set of all points positioned at the same distance from a fixed line and point.

Taxicab geometry defines a parabola distances AC= taxicab distances from the line to point A and taxicab distance from line to B. for example the parabola looks as shown below.

Sexton (2006) defines the Perpendicular bisectors of a line as " the set of all point equidistant from the end of the line segment". In a taxicab geometry the perpendicular bisector of AB line segment is the set of points that lie within the condition that the taxicab distances CB= AC.

An angle is defined according to the inner product or the unit circle. The definitions agree in a Euclidean space unlike in taxicab metric where the where the angles cannot be defined in terms of inner product. This is because the parallelogram law is not satisfied by the natural norm from the taxicab metric. The angles in a unit circle in taxicab geometry are defined as below. (Thompson & Dray).

Thompson and dray defines a t-radian as " an angle whose vertex is at the center of a unit taxicab circle and intercepts an arc of length L" the number of t-radians gives the subtended angle by the unit circle about the vertex. In this case, the unit circle has a circumference of 8 and hence is considered to have 8 t-radians. (Sexton).

Trigonometry

Sine and cosine functions can be defined in terms of taxicab geometry. The terminal side intersects with the taxicab unit circle at (Cos Ɵ, Sin Ɵ), where Ɵ is the taxicab angle of the terminal side. The sine and cosine do not agree with the Euclidean values. These values vary piecewise and linearly. Therefore are defined by the following equations. (Sexton).

The trigonometric graphs are constructed similarly to Euclidean geometry, for example the graphs of sin and cosine are as shown in the diagram below.

Similarity and congruence

Unlike the Euclidean geometry where the conditions SAS, AAS and ASA ensure triangles are congruent, taxicab geometry only allows triangles to be congruent if and only if they meet the condition SASAS. For example if a triangle defined by the points (0, 0), (2, 2) AND (2, O), the size of the triangle are 2 by 2 by 4 and of angles of t-radians 1, 1 and 2. Rotating the triangle 1 t-radian in the clockwise direction we obtain a triangle of dimensions 2*2*2, and angles 1, 1, 2 t-radians as shown below. (Greenspan).

The condition ASASA is not satisfied by the two triangles and hence not congruent this also eliminates the possibility of SAS, ASA, AAS, AAA or SSA conditions being satisfied.(Thompson and Dray).
If the condition SASAS is considered, it can be proofed by the fact that the sum of all angles in the triangle is constant totaling to 4 t-radians. This is based on the relationship between parallel lines and transverse line. There is congruence in the alternating angles. This leads to the lemma that alternating interior angles are congruent whenever we have a transverse in two parallel lines. (Thompson and Dray).

Considering parallel lines and a transverse line below

The angles α and β are said to be congruent

From above, it can be shown that the sum of angles in the triangle is constant. The proof transforms the angle γ at point Q to point R; this is done by the congruence by alternate interior angles. It seeks to show that the total of the sum off angles is 4 t-radians in the triangle and this is constant. (Thompson and Dray).

When two triangles where any two angles and the three sides being congruent, then it follows that the triangles are similar. This is the only relation in which the triangle is similar in taxicab geometry. (sexton).

Conclusion

Taxicab geometry is an idealized and essential model for urban planning. It renders itself very practical in urban planning where structuring of roads is very important. It enables considering practical distances in which a taxi would cover rather than considering a theoretical hypotenuse which cannot be applicable in some cases like in urban centres.

References

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