Factors for asset price increase

Introduction

In finance, we buy securities such as stocks or commodities hoping that the asset will increase in value. This is known as a long position. In mathematics, we usually use a positive number to signify this. For example, if there is a share of +1, that means there is a long one share.

However, sometimes we borrow a security or commodity from others and sell it hoping that the asset will decrease in price. This is so that the borrower can buy the asset back at a lower price than they initially bought it with. In mathematics, we usually use a negative number to signify this. For example, if there is a share of -1, that means there a short one share.

There is a financial instrument called a derivative security which is when its price is dependent on one or few financial assets such as stocks. Derivative security is an agreement that two parties make to buy or sell an asset for a fixed price on or before a specific date. There are few examples of derivative securities such as forward contract, European call and put option.

A forward contract is a contract that is non-standard between two parties which is either to sell or buy an asset at a specified time in the future at a price which is agreed today. The specified time in the future is known as maturity T and the price they will buy or sell it for is known as strike price (K). S_T is the price of the asset at time T and therefore $(S_T - K)$ is the value of the contract at time T.

An option contract is similar to a forward contract but the only difference is that an option contract gives the party who buys the option the right but not an obligation to sell or buy a certain asset at a specific time during the life of the contract i. e. on or before maturity T.

There are two types of option; call option and put option. A European call option is the type of option that gives the party who buys the option the right but not the obligation to purchase a certain asset at a strike price K on maturity T. At maturity T the value of the contract is $(S_T - K)^+ = \max(S_T - K; 0)$ where S_T is the price of the asset at time T .

On the other hand an European put option is the type of option that gives the party who buys the option the right but not the obligation to sell the certain asset at a strike price K on maturity T . At maturity T the value of the contract is $(K - S_T)^+ = \max(K - S_T; 0)$ where S_T is the price of the asset at time T .

Note that, we have a put-call parity:

$$(S_T K)^+ (K \ S_T)^+ = S_T \quad (1)$$

K:

In words, the time-T value of a call less the time-T value of a put (with same strike/maturity) is equal to the time-T value of a forward contract (with same strike/maturity).

How to give a fair price of these securities before maturity? In what sense is it fair?

To analysis it, one should first rule out arbitrage. An arbitrage is a trading strategy which begins with no money, has a zero probability of losing money, and has positive probability of making money. An arbitrage may exist in the real market, but it is necessarily fleeting, as smart traders discover it, they quickly take advantage of it and remove it.

We will assume no arbitrage henceforth. And the pricing methodology under this assumption is called no-arbitrage pricing. The first example to illustrate this methodology is a binomial model.

One Period Model

One of the key factors in modern finance is the lack of arbitrage in the financial market. Arbitrage is the profit that is made from the difference in price when buying and selling an asset. The no-arbitrage condition which is also known as NA condition is essential when it comes to developing pricing methodologies. The consequences of this condition will be shown using a one period model sample.

Let’s imagine a simple market where there are three securities and there are only three possible outcomes in the next time period. These possible outcomes are also known as the states of nature. Security 1 has a current price of £60. If state 1 takes place then the price will decrease to £50, if state 2 occurs then the price will stay at £60 and if state 3 takes place then the price will increase to £80. However security 2 has a current price of £105 and similarly if state 1 occurs then its price will be £90, if state 2 occurs then it price will stay at £105 and if state 3 occurs then the price will increase to £130.

At the current time the state that will occur in the next time period is not known. However it is only known that one of the three states possible will take place. It is not possible to make an assumption on the probability of the chances of what state will occur. However it is known for sure that the probability of each state occurring is positive. They are known as states because the price in the next time period is uncertain.

The value of the third security does not depend on the state of nature in the next time period. It is a risk-free security or a risk-free bond. The value of the third security will be £110 in the next time period no matter which state of nature will occur and the current price of this security is £100. Hence, it is known that the interest rate in this simple model is 10%. Table 1. 1 summarizes the prices and payoffs.