Math portfolio

MATH PORTFOLIO NUMBER OF PIECES Kanishk Malhotra 003566-035 (May 2012) In physics and mathematics, the ‘ DIMENSION’ of a space or object is informally defined as the minimum number of coordinates needed to specify each point within it. Thus a line has a dimension of one because only one coordinate is needed to specify a point on it. A surface such as a plane or the surface of a cylinder or sphere has a dimension of two because two coordinates are needed to specify a point on it (for example, to locate a point on the surface of a sphere you need both its latitude and its longitude). The inside of a cube, a cylinder or a sphere is three-dimensional because three coordinates are needed to locate a point within these spaces. *taken from www. Wikipedia. com AIM: - TO INVESTIGATE THE MAXIMUM NUMBER OF PEICES OBTAINED WHEN AN ‘ N’ DIMENSIONAL OBJECT IS CUT. 1st - DIMENSION The first dimension is best and the easiest to start with. Lets us take an example of a line segment which is a finite one dimensional object. Now if there is 1 cut on the line segment, then The number of pieces obtained are 2 If there are two cuts on a line segment, then 2 The number of pieces obtained are 3 If there are three cuts on the line segment, then The number of pieces obtained are 4 If there are four cuts on the line segment, then Then the number of pieces obtained are 5 If there are five cuts in the line segment The number of pieces obtained are 6. This information is represented in a tabular form below: - 3 Number of cuts 1 2 3 4 5 Pieces 2 3 4 5 6 Thus while observing the pattern it has been seen that for the number of cuts ‘ N’ the pieces obtained has been ‘ N+1’ Hence the conjecture for the number of pieces obtained for one dimensional object is ‘ N+1’ which means that the number of pieces for a particular number of cuts will be equal to the number of cuts + 1. This conjecture according to the question has to be defined as (S). 2nd - DIMENSION Now let us observe a finite two dimensional figure which is a circle. Now if there is 1 cut in the circle, then 4 The number of pieces obtained are 2 If there are two cuts in the circle, then The number of pieces obtained are 4 If there are 3 cuts in the circle, then Then the number of pieces obtained are 7 If there are 4 cuts in the circle, then 5 The number of pieces obtained are 11 If there are 5 cuts in the circle, then The number of pieces obtained are 16 This information has been represented in a tabular form below: - 6 Number of Pieces for Pieces for 2-D (R) cuts 1-D (S) 1 2 + 2 2 3 + 4 3 4 + 7 4 5 + 11 5 6 16 By analysing the above table, the pattern can be found which is recursive so the Recursive formula for 2-D figures for ‘ n’ number of cuts is Rn= S(n-1) + R(n-1) We form the conjecture for the above by a particular method which is — Consider the general sequence with terms U1, U2, U3, U4 and U5, and then the difference array is as follows: Sequence U1 U2 1st difference U3 1U1 2nd difference U4 1 U2 U5 1U3 2U1 2U2 1U4 2U3 We can now form a conjecture that an expression for Un in terms of U1 is given byUn= U1 + (n- 1) 1U1 + 2U1+......+ rU1+.....+ n-1U1 Applying this for the sequence 2, 4, 7, 11, 16 Sequence 1st difference 2nd difference 2 4 7 2 3 1 11 16 4 1 5 1 7 U1= 2, 1U1= 2, 2U1= 1 Un= 2+ 2(n-1) + 1 = 2+ 2(n-1) + = = Hence this is the conjecture for the sequence. Thus the number of pieces made by the cuts in a second dimensional object (which is a circle) can be found out by the conjecture . This conjecture according to the question has to be defined as (R). Thus writing it as Rn= . The above conjecture can hence be proved by the principle of Mathematical Induction. P(n): the number of pieces made by n cuts can be denoted as Rn= . Thus P(1) = = 2. Hence P(1) is true. Now assume that P(k) is true. Therefore Rk= To prove: - P(k+1) is true Proof: For this part we would have to use recursive rule: Rn= R (n-1) + n 8 Substituting (k+1) in the place of n, R (k+1) = Rk + (k+1) R (k+1) = + k+1 = = = = P (k+1) Hence by the principle of Mathematical Induction P (n) is true for all value of Z+ Hence the conjecture can be written in the form of: = (R) + n+1 (X) .... equation (1) (S) (X) can also be written as Therefore equation (1) becomes = (R) + n+1 (X) (S) 3rd - DIMENSION Now let us observe a finite three dimensional object which is a cube. 9 Now if there is one cut in the cube, then Number of pieces obtained are 2 If there are two cuts in the cube, then Number of pieces obtained are 4 10 If there are three cuts in the cube, then Number of pieces obtained are 8 If there are four cuts in the cube, then Number of pieces obtained are 15 11 If there are five cuts in the cube, then Number of pieces obtained are 26 This information has been represented in a tabular form below: Number of cuts 1 2 3 4 5 Pieces for 1-D (S) 2 3 4 5 6 Pieces for 2-D (R) Pieces for 3-D (P) 2 4 7 11 16 2 4 8 15 26 + + + + By analysing the above table, the pattern can be found which is recursive, so the Recursive formula for 3-D figures for ‘ n’ number of cuts is Pn = Pn-1 + Rn-1 12 The conjecture for the sequence above is formed by a particular pattern which is: Consider the general sequence with terms U1, U2, U3, U4 and U5, and then the difference array is as follows: Sequence U1 U2 1st difference U3 1U1 U4 1 U2 2nd difference U5 1U3 2U1 2U2 1U4 2U3 We can now form a conjecture that an expression for Un in terms of U1 is given byUn= U1 + (n- 1) 1U1 + 2U1+......+ rU1+.....+ n-1U1 Applying this to the sequence 2, 4, 8, 15, 26 Sequence 2 4 1st difference 8 2 2nd difference 4 2 1 U1 = 2, 1 2 U1 = 2, 7 3 3rd difference U1 = 2, 15 3 26 11 4 1 U1 = 1 13 Un = 2 + 2(n-1) + 2 = 2+ 2(n-1) + 2 + + = 2n + = 2n + (n-1) (n-2) + = 2n + = = Un = Hence this is the conjecture for the sequence. Thus the number of pieces made by the cuts in a second dimensional object which is a circle can be found out by the conjecture . This conjecture according to the question had to be defined as (P). Hence the conjecture can be written as Pn = The above conjecture can be proved by the principle of Mathematical Induction. w(n): - the number of pieces obtained by n cuts on a cube which is denoted by Pn = w(1): - = 2, which is true. Hence w(1) is true. Assume that w(k) is true. P(k) = 14 To prove w(k+1) is true P(k+1) = For this part we would need to use the recursive rule to prove it further, which is: Pn = R(n-1) + P(n-1) Where P is the number of pieces in 3-D objects, R(n-1) is that value of R which is one less cut in a 2-D object. P (n-1) is that value of P which is one less cut in a 3-D object. Substituting n with (k+1) in the recursive rule: P (k+1) = Rk + Pk P(k+1) = = = P(k+1) = = w(k+1) Hence by the principle of mathematical induction w(n) is true for all values of + Z. Therefore this formula can be written as — .... Equation (2) P Thus Y X S can be written as Thus equation (2) can be written as = 15 4th dimension Observing the pattern of the parts obtained in 1-D, 2-D and 3-D we can also get the number of pieces obtained in 4th dimension by ‘ n’ cuts even if 4th dimension is unknown. The number of pieces can be found out by the rule mentioned in 3-D which is: Qn = P (n-1) + Q (n-1) Where Q is the number of parts in 4-D object, P (n-1) is that value of P which is one cut less in the 3-D object Q (n-1) is that value of Q which is one less in the 4-D object. DIMENSIONS CUTS 1 2 3 4 5 1 (S) 2 (R) 3 (P) 2 4 4 5 6 2 4 7 11 16 2 4 8 15 26 4 (Q) + + + + 2 4 8 16 31 4th dimension 1st cut — it will always remain 2 throughout in all the dimensions as it has remained constant in the first three dimensions. 2nd cut — As stated in the rule above that Qn = P (n-1) + Q (n-1), therefore the pieces obtained in the second cut will be P (n-1) which is 2 + Q (n-1) which is 2. Thus there will be four pieces obtained from two cuts. 3rd cut - Will be P (n-1) which is 4 + Q (n-1) which is also 4 and thus results to be 8 pieces. 4th cut- Will be P (n-1) which is 8 + Q (n-1) which is also 8 and thus results to be 16 pieces. 5th cut- Will be P (n-1) which will be 15 + Q (n-1) which will be 16 and thus will result to be 31. 16 The information about the pieces obtained by 4-D is represented in the table below: Cuts 1 2 3 4 5 Pieces 2 4 8 16 31 The conjecture for the sequence above is formed by a particular pattern which is: Consider the general sequence with terms U1, U2, U3, U4 and U5, and then the difference array is as follows: Sequence 1st difference U1 U2 U3 1U1 2nd difference U4 1 U2 2U1 U5 1U3 2U2 1U4 2U3 We can now form a conjecture that an expression for Un in terms of U1 is given byUn= U1 + (n-1) 1U1 + 2U1+......+ rU1+.....+ n-1U1 17 Applying this to the sequence 2, 4, 8, 16, 31 Sequence 2 4 1st Difference 8 2 4 2nd Difference 8 2 3rd Difference 1U1 = 2, 31 15 4 7 2 4th Difference U1 = 2, 16 3 1 2U1 = 2, 3U1 = 2, 4U1 = 1 Un = 2 +2 (n-1) + = 2n + (n-1)(n-2) + = 2n + = = Hence this is the conjecture for the sequence. Thus the number of pieces made by the cuts in a second dimensional object which is a circle can be found out by 18 the conjecture . This conjecture according to the question had to be defined as (Q). Hence the conjecture can be written as Qn = . The above conjecture can be proved by the principle of Mathematical Induction. M (n): - the number of pieces obtained by n cuts which is denoted by Qn = M (1) = = 2 Hence M (1) is true Assume that M (k) is true Qk = To prove M (k+1) is true Q (K+1) = = For this part we would need to use the recursive rule to prove it further, which is: Qn = P(n-1) + Q(n-1) Substituting n with (k+1) in the recursive rule: Q (k+1) = Pk + Qk = Q (k+1) = = M (k+1) 19 Hence by the principle of mathematical induction M (n) is true for all values of + Z. ... EQ 3 Q Z Y X S Can be written as Thus the equation (3) can be written as = Q Z Y X S CONCLUSION As per the combination rule when ‘ m’ things are selected out of ’n’ ways, it is denoted as: n Cm = Consider this example for the number of cuts and dimensions where ‘ n’ are the number of cuts and ‘ m’ are the dimensions Consider m = 2 n C2 = = = which is equal to X (Thus we can write equation (3) stated above as: Q = nC4 + nC3 + nC2 + nC1 +1 20 Q +1 This formula gives us the number of pieces obtained by ‘ n’ cuts in the fourth dimension. Therefore observing the pattern, the general formula for obtaining the number of pieces by ‘ n’ cuts when m dimension figure is involved is: - 21