History of mathematics

ASSIGN BUSTER

Biographical information Evariste Galois was born on 25th October, 1811 in a village called Bourg-la-Reine not far from Paris. His father, Nicholas Gabriel Galois, was an important man in their village and in 1815 was appointed the mayor of the village. He was a supporter of Napolean and it is believed that Galois inherited his liberal ideas from his parents. His mother, Adelaide Marie Demante, was an educated woman and served as Galois's sole teacher until he entered school at the age of 12. She gave him a solid background in Greek and Latin, but also passed on her own skepticisms towards religion. Although there was no record of any mathematical talent in his family, both Galois's parents were intelligent and well-read in subjects considered important at the time: philosophy, classical literature and religion.

In October 1823, Galois was enrolled at the Lycee of Louis-le-Grand and his initial years in school were marked with many successes and several prizes. During his first term, however, there was a rebellion by forty students who were all expelled. Although Galois was not involved in the rebellion, the severe attitude taken by the school against those students left an impression on his already developing political outlook.

By his third year, he became bored with the classical studies and his interests were soon absorbed by Legrendre's Geometry, a book understood by very few. He regularly failed his exams and was asked to repeat the year due to poor work in rhetoric. He faced severe criticisms from his rhetoric teachers and soon his own family started to think of him as strange.

February 1827 marked a turning point in his life, as he entered his very first mathematics course, taken by M. Vernier. Typical reports from his school described him as bizarre, singular, original and closed. His passion for mathematics dominated him and his director of studies advised his parents

to let him concentrate only on mathematics and forgo all others. His time in school apart from mathematics was described as a waste of time “ as he did nothing more than torment his teachers and overwhelm himself with punishments”. However, M. Vernier reported him as an intelligent student with zeal and success (Rothman, 1982).

M. Vernier constantly implored Galois to work more systematically, an advice which he ignored. Galois attempted the entrance examination to Ecole Polytechnique a year early and without the required course in mathematics. His failure to clear the exam and the subsequent failures left a deep gash on him and embittered him for life (Rothman, 1982).

Instead of giving up though, Galois enrolled in the course of Louis-Paul-Emile Richard, a distinguished instructor of mathematics. Richard encouraged Galois immensely, and even remarked that Galois should be offered admission to Polytechnique without the examination. This encouragement reaped its fruits as Galois published his first minor paper in 1829.

In 1929, while still only 17, he submitted to the Academy his first researches on the solubility of equations of prime degree. A. L. Cauchy was appointed as the referee. The year marked two major disasters in his life. Cauchy’s delay in presenting his paper resulted in a burial of his genius which he claimed as an injustice. The biggest tragic blow came within a month of this submission: his father’s suicide. He loved his father dearly and his death wounded the young man profoundly. A few days later, he failed his examination for Polytechnique a second time. Legend has it that Galois was so enraged by the stupidity of the question that when asked for the solution, he merely stated that the answer was completely obvious. And so was the result (Rothman, 1982).

Despite these disasters, Galois acquired both a Bachelor of Letters and a Bachelor of Science in 1829, and enrolled in the lesser prestigious Ecole Normale. However, in 1830 he was expelled due to improper behavior. Another important factor in Galois's life was his political interests. It is believed that Galois's interests in joining Polytechnique involved political aspirations as well. Once expelled, Galois threw himself into political agitation, ended up in conflict with the government and was sent to jail. Twice imprisoned due to his political outbursts, he also managed to fall in love with a woman he could not have. It is said that the robust life of Galois ended in a duel that revolved around this love life. The night before he died, Galois penned down his discoveries and theories and though his works were ignored during his life and for years succeeding that, they were later discovered and given adequate recognition.

His work

Although theories for solving quadratic equations were found as early as the 16th century, mathematicians struggled for another four centuries before discovering formula to solve the polynomial or fifth degree equations. Galois theory was originally motivated by the theory discovered by Abel-Ruffini concerning solution of polynomial equations using algebraic operations (such as addition, subtraction, multiplication and division) and application of radicals (such as square roots, cube roots, etc.).

Given a polynomial p(x) with rational coefficients, for example p(x)= a x2+b x+c= 0 using only the above mentioned operations, the solution gives:

x=

-b±

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√b2-4a c

2a

The coefficients a, b, c are all rational, and only the operations multiplication, division, addition, subtraction and square root have been utilized.

There are more complicated examples, suppose p(x)= x4-4x2+2. We can factorise this as p(x)=(x2-2)2-2. So the solutions will satisfy x2-2=±√2, or x2= 2±√2. Square rooting this we get,

. So, x4-4x2+2 can be solved in this way too (Goodman, 2003).

Galois's theory thus helped to prove that if the degree of p(x) is less than 5 then the polynomial is soluble by radicals and those of degree 5 and higher can be solved using his discoveries in number theory and group theory.

California Mathematics Standards – Algebra I

1. 0 Students identify and use the arithmetic properties of subsets of integers and rational, irrational, and real numbers, including closure properties for the four basic arithmetic operations where applicable:

2. 0 Students understand and use such operations as taking the opposite, finding the reciprocal, taking a root, and raising to a fractional power. They understand and use the rules of exponents.

11. 0 Students apply basic factoring techniques to second-and simple third-degree polynomials. These techniques include finding a common factor for all terms in a polynomial, recognizing the difference of two squares, and recognizing perfect squares of binomials.

14. 0 Students solve a quadratic equation by factoring or completing the square.

Galois’s theory is essentially meant for students who pursue mathematics in higher degrees. In its simple form, it is used to learn the solutions to algebraic quadratic and polynomial equations. Galois’s work emphasized on solving algebraic equations radically, with the use of algebraic operations (such as addition, subtraction, multiplication and division) and application of radicals (such as square roots, cube roots, etc.).

Bibliography

Goodman, Dan. An Introduction to Galois Theory. University of Cambridge: 2003. http://nrich. maths. org/public/viewer. php? obj_id= 1422 – viewed on March 27, 2008.

Rothman, Tony. Genius and Biographers: The Fictionalization of Evariste Galois. American Mathematical Monthly: 1982. http://www. physics. princeton. edu/~trothman/galois. html - viewed on March 27, 2008.

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