History of maclaurin series

MACLAURIN series is the expansion of Taylor series about 0. So we can say that it is a special case of 'Taylor Series'.

Where f '(0) is the first derivative evaluated at x = 0, f ''(0) is the second derivative evaluated at x = 0, and so on.

Maclaurin series is named after the Scottish mathematician Maclaurin.

In mathematics, the Taylor series is a representation of a function as an infinite sum of terms calculated from the values of its derivatives at a single point. The Taylor series was formally introduced by the English mathematician Brook Taylor in 1715. If the series is centered at zero, the series is also called a Maclaurin series, named after the Scottish mathematician Colin Maclaurin who made extensive use of this special case of Taylor's series in the 18th century. It is common practice to use a finite number of terms of the series to approximate a function. The Taylor series may be regarded as the limit of the Taylor polynomials.

HISTORY

Colin Maclaurin

Born: Feb 1698 in Kilmodan (12 km N of Tighnabruaich), Cowal, Argyllshire, Scotland

Died: 14 June 1746 in Edinburgh, Scotland

Colin Maclaurin was born in Kilmodan where his father, John Maclaurin, was the minister of the parish. The village (population 387 in 1904) is on the river Ruel and the church is at Glendaruel.

EXPANSION

Suppose that f is a real function, all of whose derived functions f (r)(r= 1, 2, …) exist in some interval containing 0. It is then possible to write down the power series

This is the Maclaurin series (or expansion) for f. For many important functions, it can be proved that the Maclaurin series is convergent, either for all x or for a certain range of values of x, and that for these values the sum of the series is f(x). For these values it is said that the Maclaurin series is a 'valid' expansion of f(x). The function f, defined by f(0)= 0 and for all x ≠ 0, is notorious in this context. It can be shown that all of its derived functions exist and that f (r)(0)= 0 for all r. Consequently, its Maclaurin series is convergent and has sum 0, for all x. This shows, perhaps contrary to expectation, that, even when the Maclaurin series for a function f is convergent, its sum is not necessarily f(x).

The Maclaurin series of a function f(x) up to order n may be found using series [f(x, 0, n)]. The nth term of a Maclaurin series of a function f can be computed in mathematics using series coefficient [f(x, 0, n)] and is given by the inverse Z transform.

Maclaurin series are the type of series expansion in which all the terms are non negative integer powers of the variable. Other more general types of series include the Laurent series.

Calculation of Taylor series

Several methods exist for the calculation of Taylor series of a large number of functions. One can attempt to use the Taylor series as-is and generalize the form of the coefficients, or one can use manipulations such as substitution, multiplication or division, addition or subtraction of standard Taylor series to construct the Taylor series of a function, by virtue of Taylor series being power series. In some cases, one can also derive the Taylor series by repeatedly applying integration by parts. Particularly convenient is the use of computer algebra systems to calculate Taylor series.

Maclaurin series for common functions:

for -1

1

2

For -∞ to + ∞

1

2

The explicit forms for some of these are

1

2

Maclaurin series of sin x

(where x is in radians)

Let f(x) = sin x so f(0)= sin0= 0

So f '(x)= cos x so f '(0)= cos0= 1

And f '(x)= - sinx so f '(0)= -sin0= 0

So f ''(x)= - cos x so f ''(0)= -cos 0= -1

And f ''''(x)= sinx so f''''(0)= sin 0= 0

Note that the fourth derivative takes us back to the start point, so these values repeat in a cycle of four as 0, 1, 0, -1 0, 1, 0, -1 0, 1, 0, -1 0, 1, 0, -1 etc

Note that the function is infinitely differentiable and that it, and all its derivatives, exist at x = 0.

Substitution of these values back into the Maclaurin Series gives

F(x)= sin x = 0 + 1. x + 0. x2+ -1. x3+…………

Or

When you think about it, this result is amazing! A transcendental, trigonometrical function being represented by an algebraic, polynomial function! Almost more amazing is how the series comes out in a regular form in which there is a pattern. There are no prizes for guessing that the next term would have been x9 / 9! and the next - x11 / 11!

Maclaurin series of ex

let f(x)= so f(0)= 1

so f'(x)= so f'(0)= 1

and f''(x)= so f''(0)= 1 etc

of course, the function and all its successive derivatives are the same, so these values repeat

indefinitely as 1, 1, 1, 1, 1, 1, 1, 1, etc

Note again that the function was infinitely differentiable and that it, and all its derivatives, exist at x = 0.

Then substitutes these values back to the maclaurin series gives

F(x)=

Another truly amazing result! The exponential function can also be represented by a polynomial expansion with, again, a regular pattern. Again, there are no prizes for guessing subsequent terms.

MACLAURIN SERIES OF ln x

Let f(x)= ln x so f(0)=?

So f'(x)= 1/x so f'(0)= 1/0=?

And f'(x)= -1/x2

Neither the function nor any of its derivatives exist at x = 0. So there is no polynomial.

Maclaurin expansion of the natural laogarithm fn is ln x.

Not wishing to be put off by this, is it possible to produce a Maclaurin series for any natural logarithm function at all? The answer is 'yes'. All that has to be done is to shift the function/curve left by 1 unit and an expansion for logarithm can be found since the new function and all its derivative now all exist at x = 0.

MACLAURIN SERIES of ln (1+x)

Let f(x)= ln (1+x) so f(0)= ln (1+0)= 0

So f'(x)= 1/1+x so f'(0)= 1/1+0= 1

And f''(x)= -1/1+x so f''(0)= -1

Substitute these values back to the maclaurin series

MACLAURIN SERIES OF COS x

This series, as with the sine series, is valid for all values of x (unlike the ln (1 +x) series), as is exhibited when using the accompanying Maclaurin Series applet.

Note, though, that there is a quick way of deriving the series for cos x if the series for sin x is already known. Since both sin x and cos x are both infinitely differentiable and their function and differential values all exist at x = 0, the Maclaurin Series for cos x could have been found by differentiating both sides of the series expansion for sinx term by term.

Note that, by applying the same procedure, differentiating the series for ln (1 + x) term by term results in the series for 1 / (1 + x).

Example: Write the Maclaurin series of f(x) = sin x.

We see that all of the even order derivatives at x = 0 will be zero. The odd multiple derivatives will alternate between 1 and −1. Hence, we can write the Maclaurin series of f(x) = sin x as

Basic properties

The only function which is both even and odd is the constant function which is identically zero (i. e., f(x) = 0 for all x).

The sum of an even and odd function is neither even nor odd, unless one of the functions is identically zero.

The sum of two even functions is even, and any constant multiple of an even function is even.

The sum of two odd functions is odd, and any constant multiple of an odd function is odd.

The product of two even functions is an even function.

The product of two odd functions is an even function.

The product of an even function and an odd function is an odd function.

The quotient of two even functions is an even function.

The quotient of two odd functions is an even function.

The quotient of an even function and an odd function is an odd function.

The derivative of an even function is odd.

The derivative of an odd function is even.

The composition of two even functions is even, and the composition of two odd functions is odd.

The composition of an even function and an odd function is even.

The composition of any function with an even function is even (but not vice versa).

The integral of an odd function from −A to +A is zero (where A is finite, and the function has no vertical asymptotes between −A and A).

The integral of an even function from −A to +A is twice the integral from 0 to +A (where A is finite, and the function has no vertical asymptotes between −A and A).

Series

The Maclaurin series of an even function includes only even powers.

The Maclaurin series of an odd function includes only odd powers.

The Fourier series of a periodic even function includes only cosine terms.

The Fourier series of a periodic odd function includes only sine terms.

Also, if all the derivatives of an analytic function at a point are zero, the function is constant on the corresponding connected component.

These statements imply that while analytic functions do have more degrees of freedom than polynomials, they are still quite rigid.

List of Maclaurin series of some common functions

Exponential function:

Natural logarithm:

Finite geometric series:

Infinite geometric series:

Variants of the infinite geometric series:

Square root:

Trigonometric functions:

Hyperbolic functions: