Saddle point approximation research papers example

Business, Strategy

Executive Summary

Saddle point approximation implementation and derivations have typically relied on tools such as Edgeworth expansions, exponential tilting, complex integration, Hermite polynomials, and other advanced notions. Saddle point approximations are very important tools used in obtaining accurate expressions for distribution functions and densities. It has been a very valuable tool in asymptotic analysis. Since the seminal paper by Daniels (1954), several techniques of accurate approximations that rely on the saddle point approximation on one way or another have been developed. The saddle point approximations, however, derived its concept from the game theory, which involved two suspected criminals under police investigations. The prisoners were given offers separately, unanimous of each other's choices, and were told to confess against each other. This paper uses the example of two political parties seeking to rally support ahead of a forthcoming elections on a controversial issue such as the legalization marijuana, same sex marriages, and abortion in the United States. Assuming that either party chooses an option either to support, oppose, or evade the issue, the other party would have to take the opposite opinion of the former. The saddle point in this case is either obtained by ascertaining the miniax and the maximin values, or through ascertaining the values at which the parties have maximum and minimum support, which in this case is 30 percent for the former method and 30 percent to 70 percent for the latter method. This research paper indicates the details about the saddle point approximation and its significance in different situations. It indicates the origin of the saddle point through theory and its application through practice.

Introduction

A saddle point is described as the point where a function of two variables has partial derivatives equal to zero, but at which the function has neither a maximum nor a minimum value. A saddle point is the outcome that rational players would choose in a two-person constant sum game. The name saddle point derives from its being the minimum of the row that is also the maximum of the column in a payoff matrix, which correspond to the shape of a saddle. This point always exists in games of perfect information. Nevertheless, it may or may not exist in games of imperfect information (Agarwal, Dey, & Juneja, 2013). Each player in the game obtains an amount at least equal to his payoffs at that outcome by choosing a strategy associated with the outcome irrespective of the choices made by the other players in the game. In this case, the payoff is called the value of the game since in the perfect information games, the players' choices of strategies associated with the saddle point pre-ordinate the value of the game. This therefore makes these games firmly determined.

The modern statistical methods use sophisticated, complex models that can result in intractable computations. However, the saddle point approximations can provide a solution to these challenging situations of the modern statistics, especially to junior statistics students and researchers. The saddle point simplifies the statistical data into the forms that different learners of different levels can easily understand, comprehend, tractate. This makes the approximation method the most reliable method is drawing conclusions and making decisions. Most statistical methods have implemented the use of the saddle point approximation, and have since been used simultaneously in research and education. In mathematics, however, a saddle point refers to the point in the distribution of a function that is a motionless point, but not a local extremum (Butler, 2007). The name saddle point is mathematically derived due to the fact that the prototypical illustration in two proportions is a surface curving up in one direction, which resembles a saddle or a mountain pass. The saddle usually gives a contour in when represented in two dimensions appearing to intersect itself. This is the same principle that has been used in the saddle point approximations.

The approximation of distributions of statistics is important for those distributions whose exact distributions cannot be easily determined. If the first few moments of the distribution are provided, a common procedure is to fit the Edgeworth or the Pearson’s laws having similar moments provided that they are given (Butler, 2007). Despite the satisfaction of these two mtods in practice, they have the drawback that errors in the tail sections of the distribution are sometimes comparable with the frequencies. In particular, the Edgeworth approximation infamously can assume negative values in the tail regions (Kamalakis & Sphicopoulos, 2002). Even though the characteristic function of the statistic may be known, the difficulty becomes the analysis of inverting the Fourier transformation explicitly. The saddle point approximation however, has a number of desirable features. This latter method of approximation can be used to accurately approximate the probability of the occurrence of a given event, the mean of a given sample size, or the ratio of such means.

The Game Theory and the Saddle Point Approximation

The game theory derived its name from a hypothetical situation as described below. Suppose two criminals are arrested for suspiciously committing a criminal offense together. Nevertheless, the police lack the sufficient evidence to prove that these people are responsible for the crimes they are accused of them and intend to have them convicted. Therefore, the police would isolate the prisoners from each other. The criminals would later be visited separately and offered a deal. The deal would be that the person who offers evidence against the other would be freed at the expense of his mate. Inferably, if none of the criminal suspects accepts the offer, it would be suggested that they are cooperating against the police. Consequently, they would receive little punishments for lack of sufficient evidence. Therefore, they both gain (Jun, Sheng, Gang, & Jian, 2006). On the contrary, if either of the suspects betrays the other, and confesses to the police, the defector stands to gain more because his promise will be honored, and his mate who chose to remain silent would be receive the full punishment for committing the offense. This is because he refused to assist the police retrieve the vital information they require for their investigations. However, if they both decide to open up to the police and confess, they will both receive punishments for their offenses, but the punishment would be less severe compared to if they decided to remain silent. The dilemma therefore, in this case, is the fact that both suspects have to choose between two options. Nevertheless, they are uncertain of making the good decision because of the uncertainty in the other's decision. One might fear opening up and his partner remains silent.

Determining the Saddle Point

Maximin criterion For a two-person, in a zero sum game each player rationally chooses the strategy that they are sure that if they implement would maximizes the minimum payoff, and the pair of strategies and payoffs such that each player maximizes her minimum payoff is the " solution to the game." Strictly determined games usually have at least one saddle point to meet this definition (Jun, Sheng, Gang, & Jian, 2006). According to the strictly determined games, the choice of the column and row through a saddle point provides the most favorable strategy for both players. Additionally, all saddle points in a specific strictly determined game have similar payoff values, which is usually zero in a fair game, unless the game is biased or unfair (Jun, Sheng, Gang, & Jian, 2006).

Illustration

Assuming that each party in needs to maximize the votes they receive in the forthcoming elections, then it is important that they make the wisest decisions. Party A seems to have a difficult time in making the decision because it relies on party B's choice of strategy. If party B evades the issue, party A automatically supports it, and opposes the same if Party B supports it. Form table 1, it can be noted that party B obtains the largest percentage of the votes notwithstanding what the former party does through opposing the issue rather than evading or supporting it. Upon realizing that this strategy is not working for the party, party A obviously needs to evade the issue, thereby settling at 30 percent of the total votes cast, which implies that party B would receive 70 percent. Therefore, a 30 to 70 percent allocation of the votes to Party A and Party B becomes the saddle point of this game.

Nevertheless, determining the minimax and the maximin values also present a more systematic way of finding a saddle point. While still using the example in the table 1, party A first determines the minimum percentage vote that it can possibly obtain while it uses any of its strategies. Since there are three possibilities, which include supporting, opposing, or evading the issue, the party A will have three minimum values. The party will then find the maximum values of these three values, giving the maximum value. From table 1, the minimum percentages that party A will get if it opposes, supports, or evades the issue are 25 percent, 20 percent, and 30 percent respectively. The largest of these three values is 30 percent, and therefore the maximum value. In the same way, for every strategy that Party B chooses to implement on the issue, it determines the maximum percentage of the votes that Party A will receive, and therefore the minimum that Party B can win. Therefore, in this case, if party B supports the issue, evades it, or opposes it, the maximum votes that party A is likely to receive is 80 percent, 80 percent, and 30 percent respectively. Therefore, party B will ensure it maximizes party A’s minimum percentage of the total votes cast in order to obtain the largest percentage of the votes, which gives the minimax. Consequently, considering that the smallest minimum values for party A is 30 percent, this therefore, becomes the minimax value for party B. since both the maximax and the maximin values coincide at 30 percent, the 30 percent, therefore, becomes the saddle point. From the political perspective, the two parties used in this example can decide to announce their strategies in advance because the other party is unlikely to gain from this knowledge.

Inferences

As witnessed in the game theory, making the decision to confess to the police would be very difficult because the suspects are unaware of the possible actions of their counterparts. For instance, if the first suspect refuses to confess with the aim of protecting the other suspect, and the latter confesses to the police considering the offer to be attractive, he will lose. However, even if they both decide to confess, they will still loose. Therefore, getting the minimax and the maximin values of the other party in an upcoming election, in determining the prices to sell the products, or rallying support for controversial situations without prior knowledge of the actions of the counterparts at times becomes challenging to the decision makers (Carr & Madan, 2009). Saddle point approximations however help in ascertaining the possibilities of maximizing on what the opponents are unable utilize. Consequently, through estimating the minimum percentage that the opponent can acquire, the competitors can use such approximations to reduce these minimum values as possible as they can manage to ensure that the latter receives much of the allocations – be it market share, or voter support (Zulehner, 2011). It is however assumed that in this case, as the police already had their options, notwithstanding the responses of the suspects, they would still have an option to execute. The voters and customers in the political field and the market respectively already have made their decisions on whom to support or at what prices to buy. Therefore, it eventually depends on the actions of the service or product providers and the political parties to support the agenda that is appealing to the majority in order to win the support of the subordinates.

Saddle point approximations are very important tools used in obtaining accurate expressions for distribution functions and densities. It has been a very valuable tool in asymptotic analysis. Since the seminal paper by Daniels (1954), several techniques of accurate approximations that rely on the saddle point approximation on one way or another have been developed. Saddle point approximation implementation and derivations have typically relied on tools such as Edgeworth expansions, exponential tilting, complex integration, Hermite polynomials, and other advanced notions (Yuen, Wang, & Siu-Kui, 2007).

Saddle point approximations are equally applied in probability, mathematical physics, and complex analysis. The idea of the saddle point approximation is to change the path of integration in a specific integral of the Laplace of Fourier type in order to acquire better convergence (Butler, 2007). Seemingly, the optimal path should pass through the saddle point of the integrand. After ascertaining the optimal path, one can either approximate expansions or apply some numerical integration routine efficiently. Saddle point can be used to approximate certain phenomena such as the European call/put prices in the Heston model. In some cases, the global parameterization of the optimal path can be found.

Option pricing formula

Consider European put option with strike price X, maturity T, riskless rate r, written on an asset with price S. Its price at the moment t may be written as where τ = T - t is the time to maturity, and the expected value and probability are both conditioned on the current value St of the underlying.

Let Y t = ln St be a diffusion process and let K denote the conditional cumulant generating function of Y t, that is,

We assume that for each (τ, x), K exists in some interval (-c, d) with c ≥ 0, d ≥ 0 and c + d > 0.

Notice that E Pr(ST < X) = eK(1)(S T < X) where the probability is defined by

The cumulant generating function of is defined by

Therefore, provided that the function K is known we can express the option price as

and all that remains to be done is to approximate the cumulant probabilities Pr(Y T < ln X) and (Y T < ln X).

The saddle point approximation can be used as above in assisting in the determination of the best price to buy the products is a specific distribution. The saddle point assists in this case to help the European prices to be determined based on their minimum values and maximum values, which include approximation the opportunity costs of the available prices in the market. It also helps the producers to set their prices in relation to the prices of their competitors, while considering the best method of maximizing the profits from the prices that they set. In making such decisions, the companies usually consider the actions of the competitors through approximating their best abilities and inabilities, and majoring in the weaknesses of the latter.

Conclusion

The saddle point approximation is a very important tool for measuring different statistical phenomena. Saddle point approximations are equally applied in probability, mathematical physics, and complex analysis. The idea of the saddle point approximation is to change the path of integration in a specific integral of the Laplace of Fourier type in order to acquire better convergence. The modern statistical methods use sophisticated, complex models that can result in intractable computations. However, the saddle point approximations can provide a solution to these challenging situations of the modern statistics, especially to junior statistics students and researchers. The saddle point simplifies the statistical data into the forms that different learners of different levels can easily understand, comprehend, tractate. This makes the approximation method the most reliable method is drawing conclusions and making decisions.

The game theory explains the concept of the saddle point approximations since it indicates the relationship between two or more criminal suspects, which represent the parties used in this research paper. The uncertainty in the decision-making in both cases rely on approximations of the other counterparts decision. However, without making such approximations, a win-win situation will always arise, with Party B winning the elections against the Party B based on their strategies. Therefore, it is very important to understand saddle point approximations to understand the forces playing in the market and the political arena.

Appendix:

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