Surface of revolution given by rotating an arc of ellipse essay

History, Revolution

Introduction

Ellipse is the locus of points of Euclidean plane for which the sum of the distances to two data points F1 and F2 (called foci) is constant and greater than the distance between the foci.

Bulk body is a body of revolution generated by the rotating flat geometrical figure bounded by a curve about an axis lying in the same plane.

Volume of the solid formed by rotation around the x axis of the figure bounded function f (x) on the interval [a; b], the x-axis and direct x = a and x = b is equal to:

Volume of the solid formed by rotation around the y-axis of the figure bounded function f (x) on the interval [a; b], y-axis and the line y = a and y = b is equal to:

The purpose of this research work is to calculate the volume of figure given by rotating an arc of ellipse around the y-axis. Then, this volume must be maximized within the given number of limits (constraints).

Modeling

Consider the given equation of the ellipse:

x2+a2y2= 1The ellipse is rotated around the y-axis, with the given constraint:

$x > 0$

According to the conditions, we need the tank to have the maximum volume possible, for storage efficiency. However, it costs a certain number of dollars per square inch of surface area to build it. Therefore, we are given a fixed budget, hence a fixed surface area S.

Hence,

x= fy= 1-a2y2f'y=-2a2y21-a2y2S= 2π01a1-a2y2∙1-2a2y21-a2y2dy= const

So, we have to find the maximum of the function:

V= 2π01a1-a2y2dy

When

S= 2π01a1-a2y2∙1-2a2y21-a2y2dy= const

Calculation

V= 2π01a1-a2y2dy= 2π1a-13a= 4π3a

This volume must be maximized with the given fixed surface area S:

S= 2π01a1-a2y2∙1-2a2y21-a2y2dy

x2a2+y2b2= 1

The surface area of the oblate spheroid is:

Here a and b are the following:

a= 1 b= 1a in our termse= 1-1a2

Hence, the surface area is:

S= 2π(1+1-1+1a21-1a2tanh-11-1a2= 2π(1+1a2-1tanh-11-1a2

This expression can be solved for “ a” for any fixed S given.

The obtained volume of our oblate spheroid is a function of a, which can be maximized according to the given constraints on surface area S.

Interpretation and Conclusion

The surface area of an additive numerical characteristic surface. The easiest way to determine the area of polygonal surface : as the sum of squares of their flat faces .

Most often, the surface area is determined for the class of piecewise smooth surfaces with piecewise smooth boundary (or without boundary) . This is usually done by the following construction . The surface is broken into smaller pieces with piecewise smooth boundaries : in every part of choosing the point at which there is a tangent plane and orthogonal design is frequently considered on the tangent plane of the surface at the selected point , the area of the obtained planar projections summarize , and finally pass to the limit as more and more small tessellations (such that the largest diameter parts of the partition tends to zero)

The obtained expression for volume is a function of a. The surface area is fixed, hence, a can be determined as a function of S. For doing

And the volume is the composite function of S. So, it is possible to maximize V as a function of one variable.

The resulting expression for the volume allow us to conclude that the smaller the parameter " a", the greater the amount of rotation of the body. For each fixed value of the surface area " S" parameter " a" is also fixed.

Hence, the tank with the given surface area could be constructed according the formula:

V= 4π3a

Final Section

Integral concept is widely applicable to life. Integrals used in various fields of science and technology. Integral Calculus has extensive application in practice. With the definite integral calculated area hypograph functions square shapes and surface area of solids of revolution. With multiple integrals calculated volumes of three-dimensional figures.

In this research paper we have solved the real world problem – we have found the volume of figure given by rotating an Arc of Ellipse. The figure is oblate spheroid. It was determined that the surface area of this figure is given by the following expression:

S= 2π(1+1a2-1tanh-11-1a2

This equation can be solved for parameter “ a” with the given fixed value of surface area S.

With the determined parameter “ a”, the volume of figure given by rotating an Arc of Ellipse can be found by using the following formula:

V= 4π3a

Where “ a” is a parameter determined in the equation of surface area.

Problem of calculating the volumes of the figures have an important application in physics, chemistry and biology.

Works Cited

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