Pre calculus

Technology, Information Technology

Pre Calculus Part 1. The Product Rule Solutions The derivative of f(x) = g(x) h(x) is given by f (x) = g(x)h (x) + h(x) g (x) (Larson, 2012).

a. f(X) = (7X + X-1)*(3X + X2)

Let g(x) = (7X + X-1) and h(X) = (3X + X2)

Therefore, f (X) = (7X + X-1) (3 + 2X) + (3X + X2) (7 -X-2) =

21x + 14x2 + 3x-1 + 2 + 21x – 3x -2 + 7x2 – 1

f (X) = 42x + 21x2 +3x-1 -3x -2 +1

b. . f(X) = (X0. 5)*(5-X)

Let g(x) = (X0. 5) and h(x) = (5-x)

f (x) = (x1/2) (1) + (5-x) (1/2x -1/2)

f (x) = x1/2 + 2. 5x -1/2 – x3/2

c. f(X) = (X3 + X4)*(50 + X2)

Let g(x) = (X3 + X4) and h(x) = (50+x2)

Therefore, f (X) = (X3 + X4) (2x) + (50 + x2) (3x2 + 4x3) =

2x4 + 2x5 + 150x2 + 200x3 + 3x4 +4x5

f (X) = 5 x4 + 6 x5 + 150x2 + 200x3

2. Use the Chain Rule to find the derivatives of the following functions:

a. f(X) = (1- X2)5

b. f(X) = (7X + X-1)-1

c. f(X) = (5-X) 2

d. f(X) = (X3 + X4)3

Solutions

Let f(x) = (g o h) (x) = g (h(x)) and u = g(x)

Let y = f (u); the derivative of f with respect to x, f (x) is given by:

f (x) = (dy / du) (du / dx) (Larson & Edwards, 2012).

Or simply, f (x) = (derivative of outside) • (inside) • (derivative of inside).

a. f(X) = (1- X2)5

Let u = 1- X2 and y = u5

Therefore, du/dx = -2x and dy/du = 5u4

using the chain rule, f (x) = (dy / du) (du / dx) = (5u4) (-2x)

substituting U = 1- X2 into the above equation,

f (x) = 5(1- X2)4(-2x)

b. f(X) = (7X + X-1)-1

From f (x) = (derivative of outside) • (inside) • (derivative of inside),

f (x) = -1. (7X + X-1)-1. (7- X-2)

c. f(X) = (5-X) 2

f (x) = 2. (5-X) 2(-1)

d. f(X) = (X3 + X4)3

f (x) = 3. (X3 + X4)3. (3x2 + 4x3)

3. Use the Quotient Rule to find the derivatives of the following functions:

a. f(X) = 700/X4

b. f(X) = 1/ (2X + X2)

c. f(X) = 50/ (1-X)

Solutions

According to (Diefenderfer & Nelsen, 2010), derivative of f(x) ÷ g(x) equals

(f (x) • g(x)) - (f(x) • g(x))

$g^2(x)$

a. f(X) = 700/X4

f (x) = 0. (x4) – 700 (4x3)/x4

f (x) = -2800x3/x4

b. f(X) = 1/ (2X + X2)

f (x) = 0. (2X + X2) – 1 (2 + 2x)/ (2X + X2)

f (x) = (-2 - 2x)/ (2X + X2)

c. f(X) = 50/ (1-X)

f (x) = 0. (1-X) – 50. (-1)/ (1-X)

f (x) = 50/ (1-X)

4. For each of the following functions find the 1) first and second derivative, 2) explain whether or not the function has a maximum or a minimum, and how you reached that conclusion, and 3) the value of the maximum or minimum

a. f(X) = 7X2 - 2X

b. f(X) = 800X - X2

c. f(X) = 7X3 - 4X2

Solutions

For a maxima point, d2y/dx2 < 0 or –ve

For a minima point, d2y/dx2 > 0

Let f(x) = y

a. f(X) = 7X2 - 2X

First order derivative = dy/dx = 14x – 2

Second order derivative = d2y/dx2 = 14

Therefore, the function has a minima value (+14).

b. f(X) = 800X - X2

First derivative dy/dx = 800 – 2x

second derivative, d2y/dx2 = -2 (Maxima)

c. f(X) = 7X3 - 4X2

dy/dx = 21x2 -8x

d2y/dx2 = 42x -8

at maximum, dy/dx = 0

therefore, 21x2 -8x = 0

x (21x – 8) = 0

either x = 0 or x = 8/21

when x = 0, d2y/dx2 = (42 × 0) – 8 = -8 (Maxima)

when x = 8/21, d2y/dx2 = (42 × (8/21)) – 8 = 16 – 8 = 8 (minima).

Part 2

In the module twos part2 assignment you maximized revenue. In this module, you will be maximizing profit. In order to maximize profit, you have to take the cost of your lemonade and your cups into account. Suppose it costs you a total of 50 cents to make a cup of lemonade.

The assignment instructions are as follows:

A. Write a function for your profits for each price you charge. This is done by multiplying (P-. 5) times your function from Module 1. I. e. if your function is Cups Sold = 1000 - 100P, your profit function would be (P - . 5)*(1000 - 100P).

Solution

Function of cup sold (Y) = 256 -101. 0909P

profit (л) = total revenue – total cost = TR – TC

л = (p – 0. 5). (256 -101. 0909P)

= 256P -101. 0909P2 - 128 + 50. 9545P

= 306. 9545P- 101. 0909P2 -128

or л = 101. 0909P2 -306. 9545P + 128

B. Calculate the first derivative of your profit function, and create another table with the price, profit, and value of the first derivative at the prices from Module 1. Can you tell what your profit maximizing price is from this table?

dл/dp = 306. 9545 – 202. 1818P

Price

Profit ($)

Value of first derivative

0. 25

-57. 5796

256. 4091

0. 5

0. 2045

205. 8636

0. 75

45. 3523

151. 3182

1

77. 8636

104. 7727

1. 25

97. 7386

54. 2273

1. 5

104. 9773

3. 6818

1. 75

99. 5795

-46. 8637

2

81. 5454

-97. 4091

2. 25

50. 8749

-147. 9546

2. 5

7. 5682

-198. 5000

Profit maximizing price is $1. 5

C. Calculate the second derivative, and also use the first derivative to find the profit maximizing price. What is the price, and what does the second derivative tell you?

d2 л/dp2 = – 202. 1818

Using the first derivative, profit maximizing price is $1. 5

The second derivative indicates that profit begin to decline if more than the revenue maximizing price is charged for a cup of lemonade.

References

Diefenderfer, C. L & Nelsen, R. B. (2010). The calculus collection: a resource for AP and beyond. Washington, DC: Mathematical Association of America.

Larson, R. (2012). Calculus: an applied approach. Boston: Brooks/Cole.

Larson, R. & Edwards, B. H. (2012). Calculus I with precalculus: a one-year course. Boston, MA. : Brooks/Cole, Cengage Learning.