Stochastic applications of actuarial models with r coding

Technology, Computer

Credit risk refers to the probability that the counterparty may not meet its contractual obligations. Bond rating is a good indicator of credit risk of a sovereign issuing the bonds. Thus it is important to analyse the possible future rating movements of a sovereign due to credit events. 1. Probability of Default Rating To model the credit risks of bonds over time, a continuous time Markov model is implemented to calculate the probability of credit migration. In using a continuous time Markov model, the Kolmogorov forward equations are used:

where $P_{ij}(t)$ is the transition probability from rating i to rating j at time t, and r_{kj} refers to the transition rates from rating k to rating j based on the annual transition rates matrix estimated. The above equation can be rewritten in matrix form, where $P(t)$ is the matrix of transition probabilities over time t, and R is the annual transition rate matrix estimated.

A possible solution to the above equation is, where $P(0)$ is the matrix of initial transition probabilities. It is assumed that the sovereign will remain in their current rating initially (at t= 0), so for all ratings the probability of transition from one rating to another will be zero at time 0, while probability of remaining in the same rating is 1 (implying that $P(0)$ is the identity matrix). To observe the probability of default over time for sovereign rated AAA, AA, A and B, the probability transition matrix will be calculated for 100 equally spaced time frames over a 10 year period (starting from beginning of year 1995 to end of year 2004), The probability transition matrix can be calculated using approximation techniques, however, MATLAB is able to calculate the matrix more precisely by taking exponential of matrices, and

extract the appropriate probabilities of default ratings from sovereign rated AAA, AA, A and B over time. MATLAB was used to perform this procedure and plot the results in the below graphs. The code can be located in the Appendix 1. 1 attached.

The green, black, blue and red lines indicate the probability for default rating for a sovereign rated BBB, A, AA, AAA respectively at the beginning of 1995. It can be noted that the probability for default rating for sovereigns rated BBB, which is around 3. 4 10-3 after 10 years, exceeds the probability of default ratings for sovereigns of other ratings by a significant amount over the 10 years. Thus it is very difficult to determine the probability of default ratings for sovereigns of other ratings from the above graph. Thus another graph for sovereigns rated A, AA, AAA and their probability of default rating over time is plotted below.

It is noted that the probability of default rating for A rated sovereigns (represented by the black line), which is at approximately 4*10-4 after 10 years, is significantly greater than that of AA and AAA rated sovereigns over time. Another plot is made based on probability of default rating over time on just sovereigns with AA and AAA ratings. The above plot shows the probability of default rating over time for AAA (red line) and AA (blue line) rated sovereigns. The AA rated sovereigns have a significantly higher probability of default than AAA rated sovereigns, being at around 2. 7*10-5 at t= 10. This value is still very small meaning that sovereigns rated AA are not likely to enter default ratings, indicating they are a relatively safe investment.

The above plot shows the probability of default rating over time for AAA rated sovereigns, being at around 3. 6*10-7, which is the lowest probability out of all the ratings. This means that AAA rated bonds are the safest investment, as it is not likely to enter default rating. It can be observed that over time, the probability of entering a default rating from any other rating will increase, which is intuitive as larger time periods presents greater opportunities for ratings movement. It is also interesting to note that one level higher in credit rating corresponds to the probability of default rating reducing by a factor of 10, except in the case of an AAA rated sovereign; whose probability of default rating is around 100 times less than that of an AA rated sovereign.

2. Long run proportion of time in each rating To determine the long run proportion of time within each rating, we note that the net transition rate into one category is equal to the net transition rate out of the same category. This can be translated into the following equations: and where are the transition rates from rating k to rating j, is the transition rate out of rating j, while is the long run proportion of time in rating k. The values of and for every rating can be found in the estimated transition rate matrix. The second equation is from noting that the proportion of time in each rating should add up to 1. These equations can be solved simultaneous using matrix operations, and MATLAB is once again used to obtain the results.