

Population growth

[Sociology](#), [Population](#)



Population Growth Population Data The table below shows the population data for England and Wales between the years of 1801 and 1951. Census was not taken in 1941 because of the Second World War. | Year | Population |

1801	8, 892, 536
1811	10, 164, 256
1821	12, 000, 326
1831	13, 896, 797
1841	15, 914, 148
1851	17, 927, 609
1861	20, 066, 224
1871	22, 712, 266
1881	25, 974, 439
1891	29, 002, 525
1901	32, 527, 843
1911	36, 070, 492
1921	37, 866, 699
1931	39, 952, 377
1951	43, 757, 888

The graphics display calculator (GDC), Ti-84 Plus, was used to make a plot of population against time. The years were entered into the L1 column to represent the x-axis and the population was entered into the L2 list for the y-axis. I was now able to create a graph to show the data. The print screens of the GDC are shown below: The images taken from the GDC clearly shows a distinctive relationship between population and time. The image on the left shows the data entered into the GDC in a table format and the graph on the right shows the data graphically. The graph displays a positive correlation suggesting that the population increases as time progresses. I have calculated the change in population over the time interval of every 10 years to show how much the population increases with time. Change in Population 1801-1811: $10, 164, 256 - 8, 892, 536 = 1, 271, 720$ 1811-1821: $12, 000, 236 - 10, 164, 256 = 1, 835, 980$ 1821-1831: $13, 896, 797 - 12, 000, 236 = 1, 896, 561$ 1831-1841: $15, 914, 148 - 13, 896, 797 = 2, 017, 351$ 1841-1851: $17, 927, 609 - 15, 914, 148 = 2, 013, 461$ 1851-1861: $20, 066, 224 - 17, 927, 609 = 2, 138, 615$ 1861-1871: $22, 712, 266 - 20, 066, 224 = 2, 646, 042$ 1871-1881: $25, 974, 439 - 22, 712, 266 = 3, 262, 173$ 1881-1891: $29, 002, 525 - 25, 974, 439$

= 3, 028, 086 1891-1901: 32, 527, 843 — 29, 002, 525 = 3, 525, 318 1901-1911: 36, 070, 492 — 32, 527, 843 = 3, 542, 649 1911-1921: 37, 866, 699 — 36, 070, 492 = 1, 796, 207 1921-1931: 39, 952, 377 — 37, 866, 699 = 2, 085, 678 1931-1951: 43, 757, 888 — 39, 952, 377 = 3, 805, 511

The graph on the GDC is not very detailed and gives a rough idea therefore I have constructed a more defined graph using the software ‘ Graphical Analysis 3’ which allows us to study the data in more depth. After analysing the change in population data and the graph, it is evident that there is steady growth of population during 1801 — 1951. The rate of change of population over every 10 years increases during the years of 1801 — 1911 which is supported by the data of change of population. This is also represented by the increase in gradient of the growth line up to 1911 which shows that there is exponential growth. This suggests that the greater the population, the more people there are to produce offspring therefore population growth is greater when there is a larger population. Population growth has probably occurred due to the increase in standard of life. As the country develops, there is an improvement in the economy over time. This in turn improves the availability and the quality of food, medicine and services which reduce death rates and increase birth rates. However after 1911, the growth shown on the graph slows down as the gradient decreases. There could be many reasons as to why this is the case. The obvious explanation is due to the effects of World War One as many people had suffered and died which would have slowed down the population growth rate. Because many people died, there would have been a smaller population to produce offspring which explains the slowing down of growth rates. This is evident in the data of change in

population as the increase of population changes from 3, 542, 649 to 1, 796, 207 hence the growth rate had reduced. Other reasons may have been due to the shortages of food or even economic problems where families would not want to produce more offspring as they would not be able to support themselves. There is no data recorded for 1941 due to WW2 which would have had similar effects as WW1 by keeping the growth rate down. Lines of Regression Having studied the graph, no particular type of regression can be associated with the graph since it has two distinct parts. The first part from 1801- 1911 consists of exponential growth as the gradient is constantly increasing. However from 1911 onwards, due to the world wars, this trend changes as the line straightens. To determine which type of regression this graph follows, the GDC is used to carry out regression tests in order to see which type of regression best fits the data. The tests will be carried out in two parts, first testing the data of 1801-1901 and then testing the data of 1911—1951. The correlation coefficient of the regression lines will be calculated in order to see which line best fits the data and this is represented by ' r '. This has a range from -1 to 1, where -1 is perfect negative, 1 is perfect positive correlation and 0 would indicate no correlation. The following regression lines are calculated during the period: 1801-1901. During the period of 1801-1901, all regression lines have a correlation coefficient close to that of 1, hence showing there is a strong positive correlation. The regression line that best fits the data in this time period is the quadratic regression since its ' r ' = 0. 9995769014 which is very close to 1. The equation of the quadratic regression line is: $y = 44.78909091x^2 + 20331.48364x - 166382856.5$ During the period of 1911-1951, the regression line

that best fits the data in this time period is the linear regression because its ' $r' = 0.9997845752$ which is again very close to one hence it almost represents perfect positive correlation. The equation of the linear regression line is: $y = 193370.9371x - 333503988.3$ Therefore the regression lines that best fit the population data are: $y = 44.78909091x^2 + 20331.48364x - 166382856.5$; x^0 is a constant. The solution of this differential equation can then be used to calculate population. [pic] [pic] [pic] [pic] [pic] [pic]

Where A = constant of integration k = constant of proportionality This formula forms the exponential model which can be used to predict populations however the values of A and k need to be calculated first in order to use it. By using the population data of years 1911 and 1921, the values of A and k can be calculated. First, the constant of proportionality k needs to be calculated. 1911: $P = 36070492$ $t = 1911$ 1921: $P = 37866699$ $t = 1921$ 1911: $36070492 = Ae^{1911k}$ 1921: $37866699 = Ae^{1921k}$ To combine the two formulae, divide 1921 by 1911. $37866699 = Ae^{1921k}$ $36070492 = Ae^{1911k}$ $1.049797131 = e^{10k}$ $\ln(1.049797131) = 10k$ [pic] [pic] $0.0048596937 = k$ $k = 0.00486$ Now the constant of integration A needs to be calculated. [pic] [pic] 1911: [pic] To double check by substituting values of 1921: 1921: [pic] In both cases, the value of $A = 3339.33$ Therefore $A = 3339.33$ and $k = 0.00486$ so the exponential model is represented by this equation: [pic] In order to see how useful the exponential model is, it is best to compare it with the estimates of population found using the average growth rates. Estimates of 1941, 1971, 1991 and 2001 were previously found therefore I will use the exponential model to predict values for the following years. 1941: $P = 3339.33e^{0.00486 \times 1941}$ $P = 41732279$ 1971: $P = 3339.33e^{0.00486 \times 1971}$ $P = 47893449$ 1991: $P = 3339.33e^{0.00486 \times 1991}$ $P = 55244249$ 2001: $P = 3339.33e^{0.00486 \times 2001}$ $P = 63999999$

$33e0.00486 \times 1971 P = 48282776$ 1991: $P = 3339.33e0.00486 \times 1991 P =$
 53211519 2001: $P = 3339.33e0.00486 \times 2001 P = 55861471$ The data has

been tabulated below in order to compare the two models with the national

census. | | Average Growth | Exponential model | Difference | | Year | Rates

model | | | | 1941 | 42149757 | 41732279 | 417478 | | 1971 | 48571255 |

48282776 | 288479 | | 1991 | 53914093 | 53211519 | 702574 | | 2001 |

56879368 | 55861471 | 1017897 | | | Average Growth | Exponential model |

National Census | | Year | Rates model | | | | 1941 | 42149757 | 41732279 |

N/A | | 1971 | 48571255 | 48282776 | 49000000 | | 1991 | 53914093 |

53211519 | 49718300 | | 2001 | 56879368 | 55861471 | 52041916 | By

graphing the data of the two models and the national census, we can

compare and test how accurate the models are. [pic] It is evident that the

two models produced are similar as there is not much difference between

them. This is supported by the first table which shows the difference

between the values of the two models, however over the years the gap

between the models keeps on increasing. Although there may be a large

difference shown on the graph between the models and the national census,

in reality there is not much difference between them. A range has been set

on the graph which enlarges the difference however if the graph zoomed out

where the graph starts from zero, very little difference would be seen which

is what the figures show. Thus the two models are quite accurate however

they may not be very reliable. The average growth rates model has a fixed

average proportionate growth rate and therefore does not take into account

the changes in population (especially those caused by events and changes in

lifestyle). The average proportionate growth rates had been changing during

the years of 1801-1951 however it is then assumed that this remains constant from year 1921 onwards and this makes the model unreliable. A similar problem exists with the exponential model because the model is based on the values of 1911 and 1921. The constants were determined using the values of 1911 and 1921 and therefore the constants would be different if calculated using other values from other years. I shall now base the exponential model on the years 1801 and 1811.

1801: $P = 8892536$ $t = 1801$
 1811: $P = 10164256$ $t = 1811$

1801: $8892536 = Ae^{1801k}$ 1811: $10164256 = Ae^{1811k}$

To combine the two formulae, divide 1811 by 1801. $10164256 = Ae^{1811k}$ $8892536 = Ae^{1801k}$

$1.143009823 = e^{10k}$ $\ln(1.143009823) = 10k$

$0.0133664979 = k$ $k = 0.0134$

Clearly there is a large difference in comparison to the previous constant of proportionality calculated which was $k = 0.00486$. Therefore if this new constant based on 1801 and 1811 is used, it will form a completely different exponential model which would model the population during the years of 1801-1901. To show this, the constant of integration A must be calculated.

1801: $P = 2.94 \times 10^{-4}$

To check the constant with 1811: 1811: $P = 2.94 \times 10^{-4}$

Therefore the integration constant $A = 2.94 \times 10^{-4}$ which is again different to the constant calculated using 1911 and 1921 data. The exponential model based on the years 1801-1811 is: $P = 2.94 \times 10^{-4} e^{0.0134 t}$

Using this formula, I can now find the predictions of the population using this model and then compare to the actual population over the years and test its accuracy.

The Two Exponential Models	Year	1801 Model	Actual value	Difference
	1801	8899176	8892536	6640
	1811	10175254	10164256	10998
	1821	11634312	12000236	-365924
	1831			

13302589 | 13896797 |-594208 | | 1841 | 15210084 | 15914148 |-704064 | |
 1851 | 17391101 | 17929609 |-538508 | | 1861 | 19884860 | 20066224 |-
 181364 | | 1871 | 22736206 | 22712266 | 23940 | | 1881 | 25996415 |
 25975439 | 20976 | | 1891 | 29724114 | 29002525 | 721589 | | 1901 |
 33986339 | 32527843 | 1458496 | | 1911 | 38859736 | 36070492 | 2789244 |
 | 1921 | 44431943 | 37866699 | 6565244 | | 1931 | 50803165 | 39952377 |
 10850788 | | 1951 | 66417372 | 43757888 | 22659484 | The 1801

exponential model is based on 1801 and 1811 and therefore the difference between the model's value and the actual value of 1801 and 1811 is small in comparison to the rest of the data. There are negative and positive differences which show that the 1801 models the data accurately up to the year 1901. | Year | 1911 Model | Actual Value | Difference | | 1801 |

21133826 | 8892536 | 12241290 | | 1811 | 22186298 | 10164256 | 12022042
 | | 1821 | 23291184 | 12000236 | 11290948 | | 1831 | 24451093 | 13896797
 | 10554296 | | 1841 | 25668766 | 15914148 | 9754618 | | 1851 | 26947079 |
 17929609 | 9017470 | | 1861 | 28289053 | 20066224 | 8222829 | | 1871 |
 29697858 | 22712266 | 6985592 | | 1881 | 31176821 | 25975439 | 5201382 |
 | 1891 | 32729438 | 29002525 | 3726913 | | 1901 | 34359375 | 32527843 |
 1831532 | | 1911 | 36070484 | 36070492 |-8 | | 1921 | 37866806 | 37866699
 | 107 | | 1931 | 39752586 | 39952377 |-199791 | | 1951 | 43810561 |

43757888 | 52673 | Since the 1911 exponential model is based on 1911 and 1921, the difference again between the model and the actual data of 1911 and 1921 is minute however there are larger differences between the two prior to 1911. [pic] [pic] I have used the GDC to graph the two tables where the graph on the left shows the 1801 exponential model with the actual data

and the graph on the right shows the 1911 exponential model with the actual graph. It is evident that the 1801 model corresponds well with the actual data up to the year 1901 and this is shown on the table where the difference between the two sets of data is minimal up to the year 1901. After 1911, the difference increases rapidly as the 1801 model fails to represent the actual data. It is also evident that the 1911 model represents the actual data accurately after the year 1911 and this is again supported by the data table which shows a very small difference between the two after 1911. Before 1911, the differences are large hence the 1911 model cannot be used to show pre-1911 data. To form the ideal model that represents the population in England and Wales during 1801-1951, we can combine the two exponential models. By setting the range of 1801 exponential model to 1801-1911 and setting the range of the 1911 exponential model from 1911-1951, the two exponential models can be combined to represent the actual data. Below I have combined the two exponential models in one graph and have compared it to the actual data to show the correlation. After comparing the two exponential models to the actual set of data, the model that fits the actual data with the most accuracy is: $P = 3339.33 e^{0.00486t}$; $t < 1911$ + $P = 2.94 \times 10^{-4} e^{0.0134t}$; $t \geq 1911$ [pic] [pic] The graph on the right with the two exponential models demonstrates how similar the two models together are with the actual data. Once the specific ranges are set to their exponential models, the two can be combined together to form a single model that best represents the actual data. Conclusion The graphs clearly show that the best model that can be used to represent and predict the population in England and Wales is the combination of the 1801 and the

1911 exponential models. In terms of reliability, the exponential model is far more reliable than the average proportionate growth rate since the proportionate growth rate model was based on a fixed value which in reality would change over the years. This makes the exponential more reliable however this model is unreliable to a certain extent as it is too based on a few specific values such as the data from 1801-1811 and 1911-1921. This in turn affects the accuracy of the model since it predicts the actual population accurately during the years 1801-1811 and 1911-1921 however it loses accuracy over the other years. Although the model is formulated from certain years, this technique does improve its reliability and accuracy because it will take into account of any major events that affect the population such as the world wars. As long as the model is updated on a regular interval, where a new exponential model is formulated after a certain amount of time, it would allow us to accurately predict the population. To improve the current exponential model which consists of the two exponential models formed from a differential equation, a third exponential model could have been combined with the other two. Since there is a large time interval from 1801 to 1911, the data shows that the differences between the model and the actual population increase over time until the second model is introduced in 1911. To reduce this difference, a third exponential model based on the years 1851 and 1861 could be combined to make sure the difference does not increase. This would increase the accuracy of the model. Ideally many exponential models based on every ten years (every census released) could be combined together to form a single exponential model that would almost perfectly match the actual population, however this would

make the model very complicated. It would not be a great model because it would simply be showing the actual population data and would not be efficient in predicting long term populations since it is based on every census taken. I found the exponential model formulated from the two sets of years to be accurate as it modelled the actual population accurately and it gives great scope in prediction future populations. However, since the model is based on the data from England and Wales, it would not be accurate to predict the populations in other countries. For example, this model would fail to show the populations in countries that did not have much of an impact from the two world wars since the population in these countries would have continued to rise at their proportionate rate. Other countries may be affected by other factors such as weather conditions and disasters such as earthquakes which England and Wales may not have. Thus the model would not take these things into account and therefore it would not be useful in other countries. If the model is based on a specific country, it will be useful to predict the population in that country and other similar countries. Ideally a model can be made for every country which would allow us to compare population growth in various countries and continents which would enable us to predict the total population of the world. In general, a perfect model cannot be made because the world is controlled by a large number of factors that constantly change it. We can formulate models such as the exponential model to predict the population in a certain country or even the world however we can never predict the occurrence of certain events which have a huge impact on the total population. ----- Linear Regression

Exponential Regression Quadratic Regression The following regression lines are calculated during the period: 1911-1951