Mean variance optimization essay sample

Finance, Investment

Mean-variance portfolio theory is based on the idea that the value of investment opportunities can be meaningfully measured in terms of mean return and variance of return. Markowitz called this approach to portfolio formation mean-variance analysis. Mean-variance analysis is based on the following assumptions: 1. All investors are risk averse; they prefer less risk to more for the same level of expected return. 2. Expected returns for all assets are known.

3. The variances and covariances of all asset returns are known. 4. Investors need only know the expected returns, variances, and covariances of returns to determine optimal portfolios. They can ignore skewness, kurtosis, and other attributes. 5. There are no transaction costs or taxes.

The Mean-Variance Approach

The mean-variance theory postulated that in determining a strategic asset allocation an investor should choose from among the efficient portfolios consistent with that investor's risk tolerance amongst other constraints and objectives. Efficient portfolios make efficient use of risk by offering the maximum expected return for specific level of variance or standard deviation of return. Therefore, the asset returns are considered to be normally distributed. Efficient portfolios plot graphically on the efficient frontier, which is part of the minimum-variance frontier (MVF). Each portfolio on the minimum-variance frontier represents the portfolio with the smallest variance of return for given level of expected return. The graph of a minimum-variance frontier has a turning point that represents the Global Minimum Variance (GMV) portfolio that has the smallest variance of all the minimum-variance portfolios.

Economists often say that portfolios located below the GMV portfolio are dominated by others that have the same variances but higher expected returns. Because these dominated portfolios use risk inefficiently, they are inefficient portfolios. The portion of the minimum-variance frontier beginning with and continuing above the GMV portfolio is the efficient frontier. Portfolios lying on the efficient frontier offer the maximum expected return for their level of variance of return. Efficient portfolios use risk efficiently: Investors making portfolio choices in terms of mean return and variance of return can restrict their selections to portfolios lying on the efficient frontier. This reduction in the number of portfolios to be considered simplifies the selection process. If an investor can quantify his risk tolerance in terms of variance or standard deviation of return, the efficient portfolio for that level of variance or standard deviation will represent the optimal mean-variance choice. Because standard deviation is easier to interpret than variance, investors often plot the expected return against the standard deviation rather than variance.

The standard deviation is often plotted on the x-axis and the expected return on the y-axis. The trade-off between risk and return for a portfolio depends not only on the expected asset returns and variances, but also on the correlation of asset returns. The mean-variance theory can be extended to included nominally risk-free asset, where the theory points to choosing the asset allocation represented by the Tangency Portfolio given the investors can borrow or lend money at the risk free rate. The portfolio with the highest Sharpe Ratio amongst the efficient portfolio is called the tangency portfolio. The investor can then borrow money to increase the amount of leverage in tangency portfolio to achieve a higher expected return than the tangency portfolio, or split the money between the risk free asset and tangency portfolio to achieve a lower risk level than the tangency portfolio. The investor's portfolio will fall on the so called Capital Allocation Line (CAL) that describes the combination of expected return and standard deviation available to an investor from combining risk free asset with optimal portfolio of risky asset.

Once an efficient portfolio with the desired combination of expected return and variance has been identified, the portfolio constituent weights should be determined. To do so, the mean-variance optimization (MVO) process is used. We have used the Unconstrained MVF that places no constraints on portfolio constituent weights except that the weights sum to 1. Since, Black's two-fund theorem states that the asset weights of any minimum-variance portfolio are a linear combination of the asset weights of any other two minimum-variance portfolios, the determination of weights of two minimum variance portfolios is required to know the weights of any other minimum-variance portfolio. The Sign-constrained optimization includes the constraints that the asset-class weights sum to 1 and places no constraints on portfolio constituent weights, which implies that it can be sold short. The Minimum-Variance Frontier

An investor's objective in using a mean-variance approach to portfolio selection is to choose an efficient portfolio. An efficient portfolio is one offering the highest expected return for a given level of risk as measured by variance or standard deviation of return. Thus if an investor quantifies her tolerance for risk using standard deviation, she seeks the portfolio that she expects will deliver the greatest return for the standard deviation of return consistent with her risk tolerance. To begin the process of finding an efficient portfolio, we must identify the portfolios that have minimum variance for each level of expected return. Such portfolios are called minimum-variance portfolios. The set of efficient portfolios is a subset of the set of minimum variance portfolios. The minimum variance frontier shows the minimum variance that can be achieved for a given level of expected return. Modeling Optimization in Excel

Excel has several built in array formulas that can perform basic matrix algebra operations, which are key to determining the optimal portfolio weights for each stock. The main functions used in the model are given below: MINVERSE- Compute the inverse of matrix

MMULT- Compute matrix multiplication

TRANSPOSE- Compute transpose of matrix.

A key stroke combination of was used to evaluate an array function in Excel. In the Actual Data tab of the Excel spreadsheet is the example of daily stock quotes on 10 different stocks: Manulife Financial (MFC), Barrick Gold (ABX), Royal Bank of Canada (RY), Enbridge Inc (ENB), BCE Inc (BCE), Imperial Oil (IMO), Aecon Group Inc (ARE), Potash Corp (POT), CGI Group (GIB-A) and Maple Leaf Foods (MFI). The stocks were carefully selected from different industry sectors to highlight the effects of diversification by holding stocks with different returns, standard deviations and correlations and the model assumes that the returns are normally distributed. 10 year historical stock prices going as far back as Sep/30/2002 was downloaded from Yahoo Finance and daily returns were calculated by using the adjusted stock price for each closing day.

The adjusted stock prices are an excellent tool when examining historical returns because it gives the investors the true value of firm equity by accounting for corporate actions such as dividends and distributions, stock splits and rights offerings. The daily return stocks was calculated by subtracting previous day's adjusted stock price from the current adjusted stock price and dividing the result by the previous day's adjusted price. Exactly 2, 515 observations were collected over a 10 year period and returns were aggrandized in a separate tab entitled Actual Returns. The transpose of these returns were transferred into another tab called Transpose Actual Returns to facilitate convenient computation of summary statistics. The Output tab includes mean, standard deviation, variance, maximum return and minimum return using historical returns. In addition, correlations and covariances amongst each stock were calculated to form 11X11 matrixes of percentage returns. Descriptive statistic involves the use of sample mean, standard deviation and correlations as inputs to determine the optimal combination of stocks in a mean-variance model.

A serious limitation of this approach is that it assumes past winners will future winners and past losers will be future losers. Therefore, the results of the optimization tend to overweigh the portfolio towards past winners and short-sell past losers. Given that future returns are independent of past returns, this is a flawed strategy to determine optimal portfolio. A better approach is to use an asset pricing model to predict expected stock returns and use these forecasted mean in portfolio optimization as inputs. This approach has the benefit of mitigating idiosyncratic realizations from future forecasts. It is particularly important to eliminate unique realizations at three levels: firm specific, industry or sector specific and country specific. The country specific idiosyncrasies do not apply to our model, since all stocks are from Canadian firms listed on the Toronto Stock Exchange (TSX).

The asset pricing model used in our model is the widely used Capital Asset Pricing Model (CAPM). The CAPM is an equation describing the expected return on any asset or portfolio as a linear function of its Beta, which is a measure of the asset's sensitivity to movements in the market. The CAPM assumes that the expected return has two components: the risk free rate and an extra return equal to the market risk premium. Since, all stocks included in our model is listed on the TSX, we have assumed it to be the market portfolio. The covariance between market portfolio and individual stock divided by the variance of the market portfolio was calculated to determine the Beta value for each stock. With the Beta and risk free rate from the Actual Returns tab as input, the expected return for each stock was calculated using the CAPM formula below: $E(Ri)= Rf+\beta(Rm-Rf)$

The beta and expected turn for each stock is given in the Output tab. The Global Minimum Variance portfolio was determined by solving the following optimization problem using the built in Excel function called Solver: Minimize $\sigma p= \sum i\sum j$ wiwjCov[E(Ri), E(Rj)] Subject to $\sum \times i= 100\%$, Where, $\sigma p=$ standard deviation of portfolio and $\times i=$ percentage allocated to each stock. Since, there were no constraints included to restrict short selling the optimal portfolio allocation could include negative values for some stocks. Solver was used to compute the Efficient Portfolio by solving the following optimization problem: Minimize $\sigma p=\sum i\sum jwiwjCov[E(Ri), E(Rj)]$, Subject to 1) $\sum \times i= 100\%$, 2) $E(Rp)= max(E(Ri))$, Where, $E(Rp)$= Expected return on portfolio and $E(Ri)$= expected stock return. Once again, there were no constraints for short selling and the covariance between the GMV portfolio and Efficient Portfolio was calculated using matrix multiplication. The Tangency Portfolio is determined by solving the following optimization problem in solver: Maximize $[E(Rp)- Rf]/\sigma p$, Subject to $\sum \times i= 100\%$

The objective function in this case is called the Sharpe Ratio or the ratio of the difference between the expected portfolio return and risk free rate, and portfolio standard deviation. By varying the portfolio weights between the GMV portfolio and efficient portfolio, and calculating the corresponding expected return and standard deviation an Efficient Frontier was plotted on the graph. Similarly, the CAL was plotted on the graph by varying the weights in the tangency portfolio and calculating the corresponding expected return and standard deviation. As visually depicted on the graph, the efficient portfolio is superior to all the other individual stocks that lie below it. Therefore, the risk return trade-off is better in the efficient portfolio than the individual stocks.

This phenomenon iterates the advantages of portfolio diversification. Since the GMV portfolio exhibits the minimum level of risk on the efficient frontier, it should allocate the maximum percentage to stocks with the minimum variance. We notice that this is true for the result we have obtained with Solver. Enbridge and BCE have the smallest variance and have the largest weights in the GMV portfolio. While Royal Bank exhibited the third smallest variance, its covariance with Enbridge and BCE is higher than that of Maple Leaf. This resulted in a higher portfolio weight for Maple Leaf, which makes intuitive sense. The stock weights in the efficient and tangency portfolios consider both the expected returns and the variance of stocks. Manulife, Potash and Imperial Oil that carried negative weights in the GMV portfolio have higher expected returns. Therefore, they were included in efficient portfolio to meet the required return at minimum risk. However, their weights were reduced in the tangency portfolio to account for their higher volatility.

References:

[1]. Quantitative Investment Analysis by Richard A. DeFusco, Dennis W. McLeavey, Jerald E. Pinto, David E. Runkle. [2]. Quantitative Investment Analysis by Richard A. DeFusco, Dennis W. McLeavey, Jerald E. Pinto, David E. Runkle.