

# Surface areas and volumes

[Science](#), [Physics](#)



## Question Bank In Mathematics Class X (Term-II) 13 SURFACE AREAS AND

VOLUMES A. SUMMATIVE ASSESSMENT (c) Length of diagonal =  $\sqrt{2}l$  (a)Lateral surface area =  $4l^2$  (b) Total surface area =  $6l^2$  (c) Length of diagonal=  $3l$  3. Cylinder : For a cylinder of radius  $r$  and height  $h$ , we have : (a) Areaof curved surface =  $2\pi rh$  (b) Total surface area =  $2\pi r(r + h)$  2. Cube : For a cube of edge  $l$ , we have : (a)Lateral surface area =  $4l^2$  (b) Total surface area =  $6l^2$  (c) Length of diagonal =  $\sqrt{3}l$ 3. Cylinder : For a cylinder of radius  $r$  and height  $h$ , we have : (a) Areaof curved surface =  $2\pi rh$  (b) Total surface area =  $2\pi r(r + h)$  4. Cone : For a cone of height  $h$ , radius  $r$  and slant height  $l$ , we have :(a) Curved surface area =  $\pi rl$  (b) Total surface area =  $\pi r(r + l)$  5. Sphere : For asphere of radius  $r$ , we have : Surface area =  $4\pi r^2$  6. Hemisphere (solid) : Fora hemisphere of radius  $r$  we have : (a) Curved surface area =  $2\pi r^2$  (b) Totalsurface area =  $3\pi r^2$  7. Hollow cylinder : For a hollow cylinder of height  $h$ , outer radius  $R$  and inner radius  $r$ , we have :(a) Lateral surface area =  $2\pi h(R + r)$  (b) Total surface area =  $2\pi h(R + r) + 2\pi(R^2 - r^2)$ 4. Cone : For a cone of height  $h$ , radius  $r$  and slant height  $l$ , we have :(a) Curved surface area =  $\pi rl$  (b) Total surface area =  $\pi r(r + l)$  Sol. Let the side of cube =  $y$  cm Volume of cube =  $64$  $\text{cm}^3$  Then, volume of cube =  $\text{side}^3 = y^3$  As per condition  $y^3 = 64$   $\therefore y^3 = 4^3$  $\therefore y = 4$  13. SURFACE AREA OF A COMBINATION OF SOLIDS 1. Cuboid :For a cuboid of dimensions  $l$ ,  $b$  and  $h$ , we have : (a) Lateral surface area =  $2h(l + b)$  (b) Total surface area =  $2(lb + bh + lh)$ (c) Curved surface area of hollow cylinder =  $2\pi h(R - r)$ , where  $R$  and  $r$  are outer and inner radii2. A toy is in the form of a cone of radius  $3.5$  cm mounted on a hemisphere of same radius. The totalheight of the toy is  $15.5$  cm. Find the total surface area of the toy. [2011 (T-II)]Sol. Radius of the cone = Radius of hemisphere =  $3.5$  cm Total height ofthe toy =  $15.5$  cm  $\therefore$  Height of the cone =  $(15.5 - 3.5)$  cm =  $12$  cm Slantheight of the cone ( $l$ ) =  $\sqrt{r^2 + h^2} = \sqrt{3.5^2 + 12^2} = 12.5$  cmDiameter of the hollow cylinder =  $14$  cm

Radius of the hollow hemisphere =  $r$  cm = 7 cm  
 Radius of the base of the hollow cylinder = 7 cm  
 Total height of the vessel = 13 cm  
 Height of the hollow cylinder =  $(13 - 7)$  cm = 6 cm  
 Inner surface area of the vessel = Inner surface area of the hemisphere + Inner surface area of the hollow cylinder  
 $= 2\pi(7)^2 \text{ cm}^2 + 2\pi(7)(6) \text{ cm}^2 = 98\pi \text{ cm}^2 + 84\pi \text{ cm}^2 = \pi(98 + 84) \text{ cm}^2$   
 $= 182\pi \text{ cm}^2 = 182 \times \frac{22}{7} \text{ cm}^2 = 26 \times 22 \text{ cm}^2 = 572 \text{ cm}^2$ .  
 PR AK = Q. 2. A vessel is in the form of a hollow hemisphere mounted by a hollow cylinder. The diameter of the hemisphere is 14 cm and the total height of the vessel is 13 cm.

Find the inner surface area of the vessel. [2011 (T-II)] Sol. Diameter of the hollow hemisphere = 14 cm  $\therefore$  Radius of the hollow hemisphere =  $\frac{14}{2}$  cm = 7 cm  
 Total surface area of the toy = Curved surface area of the hemisphere + Curved surface area of the cone  
 $= 2\pi(7)^2 \text{ cm}^2 + \pi(7)(12) \text{ cm}^2 = 294\pi \text{ cm}^2 + 84\pi \text{ cm}^2 = 378\pi \text{ cm}^2$   
 $= 378 \times \frac{22}{7} \text{ cm}^2 = 5544 \text{ cm}^2$   
 O TH ER YA L BR S Q. 4. A cubical block of side 7 cm is surmounted by a hemisphere. What is the greatest diameter the hemisphere can have? Find the surface area of the solid. [2011 (T-II)] Sol. Side of cubical block = 7 cm  
 Side of cubical block = Diameter of hemisphere = 7 cm  $\therefore$  Radius of hemisphere =  $\frac{7}{2}$  cm  
 Surface area of solid = Surface area of the cube - Area of base of hemisphere + C. S. A. of hemisphere  
 $= 6 \times (\text{side})^2 - \pi r^2 + 2\pi r^2$   
 $= 6 \times (7)^2 - \pi \left(\frac{7}{2}\right)^2 + 2\pi \left(\frac{7}{2}\right)^2$   
 $= 294 - \frac{49\pi}{4} + \frac{49\pi}{2}$   
 $= 294 + \frac{49\pi}{4}$   
 $= 294 + \frac{49 \times 22}{4}$   
 $= 294 + 270.25$   
 $= 564.25 \text{ cm}^2$   
 7. A cuboid of length 10 cm, breadth 4 cm and height 4 cm is surmounted by a hemisphere. Find the surface area of the solid. Sol. Length of cuboid = 10 cm, breadth = 4 cm, height = 4 cm  
 Diameter of hemisphere = breadth of cuboid = 4 cm  
 Radius of hemisphere =  $\frac{4}{2}$  cm = 2 cm  
 Surface area of solid = Surface area of cuboid - Area of base of hemisphere + C. S. A. of hemisphere  
 $= 2(lb + bh + hl) - \pi r^2 + 2\pi r^2$   
 $= 2(10 \times 4 + 4 \times 4 + 4 \times 10) - \pi(2)^2 + 2\pi(2)^2$   
 $= 2(44) - 4\pi + 8\pi$   
 $= 88 + 4\pi$   
 $= 88 + 4 \times \frac{22}{7}$   
 $= 88 + \frac{88}{7}$   
 $= \frac{616 + 88}{7}$   
 $= \frac{704}{7} \text{ cm}^2$   
 $= 100.57 \text{ cm}^2$

+  $4 \times 8$ )  $\text{cm}^2 = 2(32 + 6 + 32) \text{ cm}^2 = 2(80) \text{ cm}^2 = 160 \text{ cm}^2$ . N Q. 5. A hemispherical depression is cut out from one face of a cubical wooden block such that the diameter  $l$  of the hemisphere is equal to the edge of the cube. Determine the surface area of the remaining solid. Sol. Diameter of the hemisphere =  $l$  = Side of the cube = 45 mm  
 $45^2 \text{ mm}^2 + 25\pi \text{ mm}^2 = ??$   $(45 + 25) \text{ mm}^2 = 70\pi \text{ mm}^2$   $22 = 70 \times \pi \text{ mm}^2 = 220 \text{ mm}^2$ . 7 Hence, surface area of capsule =  $220 \text{ mm}^2$  O Q. 6. A medicine capsule is in the shape of a cylinder with two hemispheres stuck to each of its ends (see figure below). The length of the entire capsule is 14 mm and the diameter of the capsule is 5 mm.

Find its surface area. TH ER YA L BR Sol. Diameter of capsule = Diameter of hemisphere = Diameter of cylinder = 5 mm  
 Radius of the hemisphere =  $r = \frac{5}{2} \text{ mm}$   
 Height of the cylinder =  $[14 - (2 \times \frac{5}{2})] \text{ mm} = 9 \text{ mm}$   
 Surface area of the capsule = Surface area of cylinder + 2 Surface area of hemisphere  
 $G O S = l^2 \times \pi \times 24 \times \frac{1}{4} = 2\pi (2) (2.1) \text{ m}^2 + 2\pi (2) (2.8) \text{ m}^2 = (8.4\pi + 5.6\pi) \text{ m}^2$   
 $22 = 14\pi \text{ m}^2 = 14 \times \pi \text{ m}^2 = 44 \text{ m}^2$  7 ? Cost the canvas of the tent at the rate of Rs 500 per  $\text{m}^2 = \text{Rs } 44 \times 500 = \text{Rs } 22000$  Hence, cost of the canvas is Rs 22000. Q. 8. From a solid cylinder whose height is 2.4 cm and diameter 1.6 cm, a conical cavity of the same height and same diameter is hollowed out. Find the total surface area of the remaining solid to the nearest  $\text{cm}^2$ . Sol. Height of cylinder = 2.4 cm  
 Height of cone = 2.4 cm  
 Radius of cylinder =  $r$  = Radius of cone = 0.8 cm  
 Slant height, of the cone  $l = \sqrt{r^2 + h^2} = \sqrt{0.8^2 + 2.4^2} = \sqrt{0.64 + 5.76} = \sqrt{6.4} = 2.5 \text{ cm}$   
 Radius of the cylinder = 0.8 m  
 Total surface area of the tent = Curved surface area of the cylinder + Curved surface area of the cone  
 $AK \times l \times \pi \times 2 = 2\pi \times 0.8 \times 2.4 + 2\pi \times 0.8 \times 2.5$  AS  $l^2$

Surface area of the remaining solid = Surface area of hemisphere + Surface area of cube – Area of base of hemisphere ?

Radius of the hemisphere = Q. 7. A tent is in the shape of a cylinder surmounted by a conical top. If the height and diameter of the cylindrical part are 2.1 m and 4 m respectively and the slant height of the top is 2.8 m, find the area of the canvas used for making the tent. Also, find the cost of the canvas of the tent at the rate of Rs 500 per m<sup>2</sup> (note that the base of the tent will not be covered with canvas. ) Sol. Radius of the cone = 2 m ? ? 5? 2 ? ? 5? 2 2 = 2? ? ? (9) mm + 2 ? 2? ? ? ? mm ? 2? ? 2? ? ? ? ? (0.7)2 ? (2.4) 2 cm = 2.5 cm HA 1.4 cm = 0.7 cm 2 N Q. 9.

A wooden article was made by scooping out a hemisphere from each end of a solid cylinder, as shown in figure. If the height of the cylinder is 10 cm, and its base is of radius 3.5 cm, find the total surface area of the article. Sol. Height of cylinder = 10 cm Total surface area of the remaining solid = C. S. A. of cylinder + C. S. A. of cone + Area of base = 2? rh + ? rl + ? r<sup>2</sup> = ? r (2 h + l + r) 22 = ? 0.7 ? (2 ? 2.4 + 2.5 + 0.7) cm<sup>2</sup> 7 22 7 = ? (4.8 + 3.2) cm<sup>2</sup> 7 10 22 7 = ? ? 8.0 cm<sup>2</sup> 7 10 176 = cm<sup>2</sup> = 17.6 cm<sup>2</sup> 10 Hence, total remaining surface area = 17.6 cm<sup>2</sup> = 18 cm<sup>2</sup>. Radius of cylinder = 3. cm Total surface area of the article = C. S. A of cylinder + 2 C. S. A. of hemisphere = 2? (3.5 (10) cm<sup>2</sup> + 2 [2? (3.5)<sup>2</sup>] cm<sup>2</sup> = 70?? cm<sup>2</sup> + 49? cm<sup>2</sup> = ?? (70 + 49) cm<sup>2</sup> 22 2 = 119? cm<sup>2</sup> = 119 ? cm 7 = 17 ? 22 cm<sup>2</sup> = 374 cm<sup>2</sup>. OTHER IMPORTANT QUESTIONS Q. 1. A cylindrical pencil sharpened at one edge is the combination of : (a) a cone and a cylinder (b) frustum of a cone and a cylinder (c) a hemisphere and a cylinder (d) two cylinders Sol. (a)

The given shape is a combination of a BR O TH ER S PR AK Its surface area =

6 ? YA L AS Increase in surface area = ? Per cent increase = cone and a cylinder. G O Q. . If each edge of a cube is increased by 50%, the percentage increase in the surface area is : (a) 25% (b) 50% (c) 75% (d) 125% Sol. (d)

Let the edge of the cube be  $a$ . Then, its surface area =  $6a^2$  150a 3a New edge =  $\frac{3}{2}a$  . 100 2 4 Q. 3. The total surface area of a hemisphere of radius 7 cm is : [2011(T-II)] (a)  $447\pi$  cm<sup>2</sup> (b)  $239\pi$  cm<sup>2</sup> (c)  $147\pi$  cm<sup>2</sup> (c)  $174\pi$  cm<sup>2</sup>

Sol. (c) Total surface area of the hemisphere =  $3\pi r^2 = 3\pi \times 7^2 = 147\pi$  cm<sup>2</sup> =  $147\pi$  cm<sup>2</sup> Q. 4. If two solid hemispheres of same base radius  $r$  are joined together along their bases, HA  $9a^2$   $27a^2 = 4^2$   $27a^2$   $15a^2 - 6a^2 = 2^2$   $15a^2$   $2$   $100\pi^2 = 125\%$   $6a^2$  N hen curved surface area of this new solid is : (a)  $4\pi r^2$  (b)  $6\pi r^2$  (c)  $3\pi r$  (d)  $8\pi r^2$  Sol. (a) The resulting solid will be a sphere of radius  $r$ . Its curved surface area =  $4\pi r^2$ .

Q. 9. The total surface area of a top (lattu) as shown in the figure is the sum of total surface area of hemisphere and the total surface area of cone. Is it true? Sol. No, the statement is false. Total surface area of the top (lattu) is the sum of the curved surface area of the hemisphere and the curved surface area of the cone. Sol. (d) We have  $\pi^2$   $6a^2$   $2$   $6a^2$   $a^2$   $3 = AS$   $4$   $64$   $a^2$   $\pi = 3$   $a^2$   $27$  HA

Q. 5. Volumes of two cubes are in the ratio  $64 : 27$ .

The ratio of their surface areas is : (a)  $3 : 4$  (b)  $4 : 3$  (c)  $9 : 16$  (d)  $16 : 9$  Q. 10.

Two cones with the same base radius 8 cm and height 15 cm are joined together along their bases. Find the surface area of the shape so formed. N 32 Sol. True. Since the curved surface area taken together is same as the sum of curved surface areas measured separately. G O  $\pi r$   $r^2$   $\pi h^2$   $2\pi rh$  . Is it true? YA L Q. 7. If a solid cone of base radius  $r$  and height  $h$  is placed over a solid cylinder having same base radius and height as that of the cone, then

the curved surface area of the shape is  $BR \dots$  Radius of the hemispherical toy,  $r = 3$  cm. Curved surface area of the toy  $= 2\pi r^2 = 2\pi (3)^2 \text{ cm}^2 = 77 \text{ cm}^2$ . Total surface area of the toy  $= 3\pi r^2 = 3\pi (3)^2 \text{ cm}^2 = 115.50 \text{ cm}^2$ .

**Q. 8.** Two identical solid cubes of side  $a$  are joined end to end. Then find the total surface area of the resulting cuboid. **Sol.** The resulting solid is a cuboid of dimensions  $2a \times a \times a$ . Total surface area of the cuboid  $= 2(lb + bh + hl) = 2(2a \times a + a \times a + a \times 2a) = 10a^2$ .

**Q. 6.** The diameter of a solid hemispherical toy is 7 cm. Find its curved surface area and total surface area. **Sol.** Diameter of the hemispherical toy  $= 7$  cm.

**Q. 11.** A tent of height 8.5 m is in the form of a right circular cylinder with diameter of base 30 m and height 5.5 m, surmounted by a right circular cone of the same base. Find the cost of the canvas of the tent at the rate of Rs 45 per  $\text{m}^2$ . **Sol.** Height of the tent  $= 8.5$  m. Height of the cylindrical part  $= 5.5$  m. Height of the conical part  $= (8.5 - 5.5) \text{ m} = 3$  m. Base radius of the tent  $= 15$  m. Slant height of the conical part  $= \sqrt{(15)^2 + (3)^2} \text{ m} = 15.25$  m. Surface area of the resulting shape  $= 2\pi rh + \pi r^2 = 2\pi (15)(5.5) + \pi (15)^2 = 1237.50 \text{ m}^2$ . Curved surface area of the tent  $=$  curved surface area of the cylindrical part  $+ \text{curved surface area of the conical part} = 2\pi rh + \pi rl = \pi r(2h + l) = \pi (15)(2 \times 5.5 + 15.25) \text{ m}^2 = 1237.50 \text{ m}^2$ . Rate of the canvas  $=$  Rs 45 per  $\text{m}^2$ . Cost of the canvas  $=$  Rs  $(1237.50 \times 45) =$  Rs 55687.50.

**Sol.** Slant height of the cone  $=$  AS and height of the cone  $= 14$  cm. Total surface area of the cone  $= \pi rl +$

$\pi r^2 = \pi r (l + r)$  2 2 ER Slant height of the cone =  $r^2 + h^2 = 154 (5 + 1)$   
 $\text{cm}^2$  Surface area of the cube =  $6 \times 14^2 \text{ cm}^2 = 1176 \text{ cm}^2$  Surface area of  
 the remaining solid left out after the cone is carved out = surface area of the  
 cube – area of base of the cone + curved surface area of the cone 2 2 ? ?  
 $= 1176 - \pi \times 7^2 + \pi \times 154 \times 5 \text{ cm}^2$  7 ? ? YA L =  $1022 + 154 \times 5 \text{ cm}^2$ . ? ? Q. 13. A  
 toy is in the form of a cone mounted on a hemisphere of common base  
 radius 7 cm. The total height of the toy is 31 cm. Find the total surface area  
 of the toy. [2007, 2011 (T-II)] 6 G O S Q. 14. A solid is in the form of a right  
 circular cylinder with hemispherical ends.

The total height of the solid is 58 cm and the diameter of the cylinder is 28  
 cm. Find the total surface area of the solid. [2006] Sol. Q. 15. A toy is in the  
 shape of a right circular cylinder with a hemisphere on one end and a cone  
 on the other. The radius and height of the PR Q. 12. A cone of maximum size  
 is carved out from a cube of edge 14 cm. Find the surface area of the cone  
 and of the remaining solid left out after the cone carved out. Sol. Diameter of  
 the cone = 14 cm =  $625 \text{ cm} = 25 \text{ cm}$  Total surface area of the toy =  
 Curved surface area of the hemisphere + Curved surface area of the cone =  
 $2\pi r^2 + \pi rl = \pi r (2r + l) =$

Radius of the each hemisphere = base radius of the cylinder = 14 cm Total  
 height of the toy = 58 cm Height of the cylinder =  $[58 - (14 + 14)] \text{ cm} =$   
 $30 \text{ cm}$  Total surface area of the solid =  $2\pi r^2 + 2\pi rh + 2\pi r^2 = 2\pi r (2r + h)$   
 $22 = 2 \times \pi \times 14 (2 \times 14 + 30) \text{ cm}^2$  7 =  $88 \times 58 \text{ cm}^2 = 5104 \text{ cm}^2$ . AK 22 ? 7 (14  
 + 25) cm =  $858 \text{ cm}^2$ . 7 HA N Height of the toy = 31 cm Base radius of the  
 cone = radius of the hemisphere = 7 cm Height of the cone =  $(31 - 7) \text{ cm}$   
 $= 24 \text{ cm}$   $r^2 + h^2$  72 ?  $24^2 + 7^2 = 576 + 49 = 625 \text{ cm}^2$  cylindrical part are 5 cm and 13



cm respectively. The radii of the hemispherical and conical parts are the same as that of the cylindrical part.

Find the surface area of the toy if the total height of the toy is 30 cm. [2002]

Sol.  $= 2\pi r^2 + 2\pi rh + \pi r^2 = \pi r (2r + 2h + r) = \pi \times 5 (2 \times 5 + 2 \times 13 + 5) \text{ cm}^2$

$= 770 \text{ cm}^2$

TH PRACTICE EXERCISE 13. 1 A Choose the correct option (Q 1 – 7) : 1. A funnel is the combination of : (a) a cone and a cylinder (b) frustrum of a cone and a cylinder (c) a hemisphere and a cylinder (d) a hemisphere and a cone. 2. A plumbline (shahul) is the combination of : (a) a cone and a cylinder (b) a hemisphere and a cone (c) frustrum of a cone and a cylinder (d) a sphere and a cylinder

OR  $ER = 144 \times 25 \text{ cm} = 13 \text{ cm}$ . Total surface area of the toy = curved surface area of the hemisphere + curved surface area of the cylinder + curved surface area of the cone BR 3. A shuttle cock used for playing badminton has the shape of the combination of : [2011 (T-II)] (a) a cylinder and a cone (b) a cylinder and a hemisphere (c) a sphere and a cone (d) frustrum of a cone and a hemisphere 4. The height of a conical tent is 14 m and its floor area is  $346.5 \text{ m}^2$ . The length of 1.1 m wide 7 G O Y A L S canvas required to built the tent is : (a) 490 m (b) 525 m (c) 665 m (d) 860 m 5.

The ratio of the total surface area to the lateral surface area of a cylinder with base diameter 160 cm and height 20 cm is : (a) 1 : 2 (b) 2 : 1 (c) 3 : 1 (d) 5 : 1 6. The radius of the base of a cone is 5 cm and its height is 12 cm. Its curved surface area is : (a)  $30\pi \text{ cm}^2$  (b)  $65\pi \text{ cm}^2$  (c)  $80\pi \text{ cm}^2$  (d) none of these 7. A right circular cylinder of radius  $r$  cm and height  $h$  cm ( $h > 2r$ ) just encloses a sphere of diameter : (a)  $r$  cm (b)  $2r$  cm (c)  $h$  cm (d)  $2h$  cm 8. Two identical solid hemispheres of equal base radius  $r$  cm are stuck together

along their bases. The total surface area of the combination is  $6\pi r^2$ . Is it true? PR Slant height of the cone =  $12^2 + 5^2 = 13$  cm.  $\therefore$  Total surface area of the combination =  $2\pi r^2 + \pi r^2 + \pi r^2 + \pi r^2 + \pi r^2 + \pi r^2 = 6\pi r^2$ .  $\therefore$  Required cost of painting = Rs 5.25  $\times$   $6\pi r^2 = 6\pi \times 5.25 \times 12^2 = 1010.38$ . AK Radius of the cone = Radius of the cylinder = radius of the hemisphere = 5 cm. Total height of the toy = 30 cm Height of the cylinder  $h = 13$  cm Height of the cone =  $[30 - (13 + 5)]$  cm = 12 cm. Internal radius ( $r$ ) of the vessel = 12 cm Total surface area of the vessel =  $2\pi R^2 + 2\pi r^2 + \pi(R^2 - r^2) = \pi[2R^2 + 2r^2 + (R^2 - r^2)]$  cm<sup>2</sup> =  $\pi[3R^2 + r^2]$  cm<sup>2</sup> AS HA Q. 16. The internal and external diameters of a hollow hemispherical vessel are 24 cm and 25 cm respectively.

If the cost of painting 1 cm<sup>2</sup> of the surface area is Rs 5.25, find the total cost of painting the vessel all over. [2001] Sol. External radius ( $R$ ) of the vessel = 12.5 cm N ER 16. A rocket is in the form of a cone of height 28 cm, surmounted over a right circular cylinder of height 12 cm. The radius of the bases of cone and cylinder are equal, each being 21 cm. Find the total surface area of the rocket.  $\therefore$  =  $\pi r^2 + \pi r^2 + \pi r^2 + \pi r^2 + \pi r^2 + \pi r^2 = 6\pi r^2$  7 22 G 13. 2 VOLUME OF A COMBINATION OF SOLIDS 1. Volume of a cuboid of dimensions  $l$ ,  $b$  and  $h$  =  $l \times b \times h$ . 2. Volume of a cube of edge  $l$  =  $l^3$ . 3. Volume of a cylinder of base radius  $r$  and height  $h$  =  $\pi r^2 h$ . O YA L BR 4. Volume of a cone of base radius  $r$  and height  $h$  =  $\frac{1}{3}\pi r^2 h$ . 3 4 3 5. Volume of a sphere of radius  $r$  =  $\frac{4}{3}\pi r^3$ . 3 2 6. Volume of a hemisphere of radius  $r$  =  $\frac{2}{3}\pi r^3$ . 3 TEXTBOOK'S EXERCISE 13. 2 22 . 7 O TH Unless stated otherwise, take  $\pi = \frac{22}{7}$ . Q. 1. A solid is in the shape of a cone standing on a hemisphere with both their radii being equal to 1 cm and the height of the cone is equal to its radius. Find the volume of the solid in terms of  $\pi$ . 8 S PR Sol. AK 9. A solid cylinder of radius  $r$  and height  $h$  is

placed over other cylinder of same height and radius. The total surface area of the shape formed is  $4\pi h + 4\pi r^2$ . Is it true? 10. A solid ball is exactly fitted inside the cubical box of side  $a$ . Surface area of the ball is  $4\pi a^2$ . Is it true? 11. From a solid cylinder whose height is  $2.4$  cm and diameter  $1.4$  cm, a conical cavity of the same height and same diameter is hollowed out. Find the total surface area of the remaining solid to the nearest  $\text{cm}^2$ . 12. A decorative block shown below, is made of two solids – a cube and a hemisphere. The base of the block is a cube with edge  $5$  cm, and the hemisphere fixed on the top has a diameter  $4.2$  cm. Find the total surface area of the block. 22 ? ? ?? = ? . ? 7? [2011 (T-II)] 3. A tent of height  $3.3$  m is in the form of a right circular cylinder of diameter  $12$  m and height  $2.2$  m, surmounted by a right circular cone of the same diameter. Find the cost of canvas of the tent at the rate of Rs 500 per  $\text{m}^2$ . 15. A solid is composed of a cylinder with hemispherical ends. If the whole length of the solid is  $108$  cm and the diameter of hemispherical ends is  $36$  cm, find the cost of polishing the surface at the rate of  $7$  paise per  $\text{cm}^2$ . AS HA 14. Three cubes each of side  $5$  cm are joined end to end. Find the surface area of the resulting cuboid. N O YA L BR O 1 ? 3 ? 2 ??  $\text{cm} = ? \text{cm}^3$ . 3 3 ? Q. 2.

Rachel, an engineering student, was asked to make a model shaped like a cylinder with two cones attached at its two ends by using a thin aluminium sheet. The diameter of the model is  $3$  cm and its length is  $12$  cm. If each cone has a height of  $2$  cm, find the volume of air contained in the model that Rachel made. (Assume the outer and inner dimensions of the model to be nearly the same. ) Sol. = ? ?+ ? TH For conical portion : Radius of the base ( $r$ ) =  $1.5$  cm Height of cone ( $h_1$ ) =  $2$  cm  $3 \text{ cm} = 1.5 \text{ cm}$   $2 \text{ cm} = 2$  We know

[illegible]

$28 \times \text{cm}^3 \times 100 = 338.184 = 338 \text{ cm}^3$  (approximately)  $11 \text{ cm}^3 \times 30$  ? Volume of four conical depressions  $11 \times 3 \times 22 \times 3 \text{ cm} = \text{cm} = 1.7 \text{ cm}^3 \times 30 \times 15$  ? Volume of the wood in the pen stand  $= (525 - 1.47) \text{ cm}^3 = 523.53 \text{ cm}^3$ . = 4? S PR Q.  
 5. A vessel is in the form of an inverted cone. Its height is 8 cm and the radius of its top, which is open, is 5 cm. It is filled with water up to the brim. When lead shots, each of which is a sphere of radius 0.5 cm are dropped into the vessel, one-fourth of the water flows out. Find the number of lead shots dropped in the vessel. Sol. Radius of cone = 5 cm Height of cone = 8 cm Volume of cone = = AK = = O YA L BR Q.  
 4. A pen stand made of wood is in the shape of a cuboid with four conical depressions to hold pens.

The dimensions of the cuboid are 15 cm by 10 cm by 3.5 cm. The radius of each of the depressions is 0.5 cm and the depth is 1.4 cm. Find the volume in the entire stands. (See figure). TH O Radius of spherical lead shot,  $r_1 = 0.5 \text{ cm}$  ? Volume of a spherical lead shot G Sol. Length of cuboid,  $l = 15 \text{ cm}$  Width of cuboid,  $b = 10 \text{ cm}$  Height of cuboid,  $h = 3.5 \text{ cm}$  Volume of the cuboid  $= 15 \times 10 \times 3.5 \text{ cm}^3 = 525 \text{ cm}^3$  Volume of a conical depression  $= \frac{1}{3} \times \pi \times r^2 \times h$   $= \frac{1}{3} \times \pi \times (0.5)^2 \times 1.4 \text{ cm}^3 = \text{cm}^3$  1 3 6 ? Volume of water that flows out  $= \frac{1}{4} \times (0.5)^2 \times 1.4 \text{ cm}^3$  3 10 AS 1 ? volume of the cone 4 1 ? 200? ? 50?  $\text{cm}^3$  ? ? = 4? 3 ? 3 HA 2 1 ?  $r h = ? (5)^2 \times 8 \text{ cm}^3$  3 3 200 ?  $\text{cm}^3$  3 N Let the number of lead shots dropped in the vessel be  $n$ . Volume of  $n$  lead shots = As per condition,  $n \times \text{cm}^3 \times 6 \times n \times 50? = 6 \times 3 = 31680? \text{ cm}^3 + 3840?? \text{ cm}^3 = 35520?? \text{ cm}^3 = 35520 \times 3.14 \text{ cm}^3 = 111532.8 \text{ cm}^3$  ? Mass of the pole  $= 111532.8 \times 8 \text{ g} = 892262.4 \text{ g} = 892.26 \text{ kg}$  Hence, the mass of the pole is 892.26 kg (approximately). BR O TH ER S Sol. Diameter of cylinder ABCD = 24 cm  $24 \text{ cm} \div 2 = 12 \text{ cm}$  Height of cylinder ABCD ( $h$ ) = 220 cm ? Volume of

cylinder ABCD =  $\pi r^2 h = \pi (12)^2 (220) \text{ cm}^3 = 31680\pi \text{ cm}^3$  Base radius of cylinder A? B? C? D? , R = 8 cm Height of cylinder A? B? C?

D? (H) = 60 cm ? Volume of cylinder A? B? C? D? =  $\pi R^2 h = \pi (8)^2 (60) \text{ cm}^3 = 3840\pi \text{ cm}^3$  ? Volume of solid iron pole = Volume of the cylinder ABCD +

Volume of the cylinder A? B? C? D? Base radius of cylinder ABCD, r = YA L PR

Q. 6. A solid iron pole consist of a cylinder of height 220 cm and base diameter 24 cm, which is surmounted by another cylinder of height 60 cm and radius 8 cm. Find the mass of the pole, given that 1  $\text{cm}^3$  of iron has approximately 8 g mass. (Use  $\pi = 3.14$ ) Radius of the cone OAB (r) = 60 cm

Height of cone OAB (h<sub>1</sub>) = 120 cm ? Volume of cone OAB  $\frac{1}{3} \pi r^2 h_1 = \frac{1}{3} \pi (60)^2 (120) \text{ cm}^3 = 144000\pi \text{ cm}^3$

Radius of the hemisphere (r) = 60 cm

= ? Volume of hemisphere =  $\frac{2}{3} \pi r^3 = \frac{2}{3} \pi (60)^3 \text{ cm}^3 = 144000\pi \text{ cm}^3$

Radius of the cylinder (r) = Height of cylinder (h<sub>2</sub>) = ? Volume of cylinder =  $\pi r^2 h_2 = \pi (60)^2 (180) \text{ cm}^3 = 648000\pi \text{ cm}^3$

Hence, the number of lead shots dropped in the vessel is 100. Q. 7. A

solid consisting of a right circular cone of height 120 cm and radius 60 cm standing on a hemisphere of radius 60 cm is placed upright in a right circular cylinder full of water such that it touches the bottom. Find the volume of

water left in the cylinder, if the radius of the cylinder is 60 cm and its height

is 180 cm. Sol. HA N 2 3 ? r 3 2 ? (60)<sup>3</sup>  $\text{cm}^3 = 144000\pi \text{ cm}^3$  60 cm 180 cm ?

$r^2 h_2$  So, r = OTHER IMPORTANT QUESTIONS Q. 1. Volume of the largest right

circular cone that can be cut out from a cube of edge 4.2 cm is : (a) 9.7

$\text{cm}^3$  (b) 77.6  $\text{cm}^3$  (c) 58.2  $\text{cm}^3$  (d) 19.4  $\text{cm}^3$  O TH YA L BR O Sol. (d)

Radius of the cone = 4.2 cm = 2.1 cm. 2 ER 8.5 cm 2 S Sol. Diameter of

sphere = 8.5 cm  $\therefore \frac{4}{3} \pi (4.25)^3 \text{ cm}^3 + \frac{4}{3} \pi (4.25)^3 \text{ cm}^3 = 321.39 \text{ cm}^3 + 25.12 \text{ cm}^3 = 346.51 \text{ cm}^3$

= Hence, she is correct. The

correct volume is 346. 51 cm<sup>3</sup>. remains unfilled. Then the number of marbles that the cube can accommodate is : (a) 142296 (b) 142396 (c) 142496 (d) 142596 Sol. a) Volume of the cube = 223 cm<sup>3</sup> = 10648 cm<sup>3</sup> Space which remains unfilled G Height of the cone = 4. 2 cm. 1 ? Volume of the cone = ?  $r^2 h \div 3 = \frac{1}{3} \pi r^2 h$  PR Q. 8. A spherical glass vessel has a cylindrical neck 8 cm long, 2 cm in diameter; the diameter of the spherical part is 8. 5 cm. By measuring the amount of water it holds, a child finds its volume to be 345 cm<sup>3</sup>. Check whether she is correct, taking the above as the inside measurements, and ? = 3. 14. Amount of water it holds = 4 ? 8. 5 ? ? ? ? cm<sup>3</sup> + ? 12 (8) cm<sup>3</sup> 3 ? 2 ? 10648 cm<sup>3</sup> = 1331 cm<sup>3</sup> 8 Remaining space = (10648 – 1331) cm<sup>3</sup> = = 9317 cm<sup>3</sup> 1 22 ? ? 2. 1 ? 2. 1 ? . 2 cm<sup>3</sup> = 19. 404 cm<sup>3</sup>. 3 7 Q. 2. A hollow cube of internal edge 22 cm is filled with spherical marbles of diameter 0. 5 cm and it is assumed that 1 space of the cube 8 12 4 ? (0. 25)<sup>3</sup> cm<sup>3</sup> 3 Let n marbles can be accommodated. Volume of 1 marble = Then, n ? AK 3 4 22 ? ? (0. 25)<sup>3</sup> = 9317 3 7 AS HA = ? (60)<sup>2</sup> (180) cm<sup>3</sup> = 648000? cm<sup>3</sup> ? Volume of water left in the cylinder = Volume of the cylinder – [Volume of the cone + Volume of the hemisphere] = 648000? cm<sup>3</sup> – [144000? + 144000? ] cm<sup>3</sup> = 648000?? cm<sup>3</sup> – 288000? cm<sup>3</sup> = 360000?? cm<sup>3</sup> 360000? = m<sup>3</sup> = 0. 36? m<sup>3</sup> 100 ? 100 ? 100 22 3 = 0. 36 ? m = 1. 131 m<sup>3</sup> (approx. 7 Radius of cylindrical neck = 1 cm Height of cylindrical neck = 8 cm N ? n = 9317 ? 3 ? 7 4 ? 22 ? (0. 25) 3 = 142296. Q. 3. A medicine capsule is in the shape of a cylinder of diameter 0. 5 cm with two hemispheres stuck to each of its ends. The length of entire capsule is 2 cm. The capacity of the capsule is : (a) 0. 36 cm<sup>3</sup> (c) 0. 34 cm<sup>3</sup> Sol. (a) (b) 0. 35 cm<sup>3</sup> (d) 0. 33 cm<sup>3</sup> Q. 5. The volume of a sphere (in cu. cm) is equal to its surface area (in sq. cm). The

diameter of the sphere (in cm) is : [2011 (T-II)] (a) 3 (b) 6 (c) 2 (d) 4

Sol. (a)  $d = 2r = 2 \times 3 = 6$  cm

Sol. (b)  $BR = 22 \frac{1}{2}$  cm

Height of the cylindrical part =  $(2 - 0.5)$  cm = 1.5 cm

Radius of each hemispherical part = Radius of the cylindrical part = 0.5 cm

Capacity of the capsule =  $\frac{4}{3}\pi r^3 + \pi r^2 h = \frac{4}{3}\pi (0.5)^3 + \pi (0.5)^2 \times 1.5 = 0.36\pi$  cm<sup>3</sup>

Q. 7. The ratio between the radius of the base and the height of the cylinder is 2 : 3. If its volume is 1617 cm<sup>3</sup>, the total surface area of the cylinder is : [2011 (T-II)] (a) 208 cm<sup>2</sup> (b) 77 cm<sup>2</sup> (c) 707 cm<sup>2</sup> (d) 770 cm<sup>2</sup>

Sol. (d) Let the radius and height of the cylinder be  $2x$  and  $3x$  respectively. Then, volume of the cylinder =  $\pi r^2 h = 1617$

$\pi (2x)^2 \times 3x = 1617$

$12\pi x^3 = 1617$

$x^3 = \frac{1617}{12\pi} = 42$

$x = 3.5$  cm

Total surface area of the cylinder =  $2\pi r(h + r) = 2\pi \times 7 \times (10.5 + 7) = 770$  cm<sup>2</sup>

Q. 8. On increasing each of the radius of the base and the height of a cone by 20%, its volume will be increased by : (a) 25% (b) 40% (c) 50% (d) 72.8%

Sol. (d) Let the original radius and height be  $r$  and  $h$  respectively. Then, original volume =  $\frac{1}{3}\pi r^2 h$

New radius =  $1.2r$  and New height =  $1.2h$

New volume =  $\frac{1}{3}\pi (1.2r)^2 (1.2h) = 1.728 \times \frac{1}{3}\pi r^2 h$

Increase in volume =  $1.728 - 1 = 0.728$

Percentage increase =  $\frac{0.728}{1} \times 100 = 72.8\%$

Q. 6. The ratio of the volumes of two spheres is 8 : 27. The ratio between their surface areas is : [2011 (T-II)] (a) 2 : 3 (b) 4 : 27 (c) 8 : 9 (d) 4 : 9

Sol. (d) Let the radii of the two spheres be  $r_1$  and  $r_2$  respectively.

$\frac{\frac{4}{3}\pi r_1^3}{\frac{4}{3}\pi r_2^3} = \frac{8}{27}$

$\frac{r_1^3}{r_2^3} = \frac{8}{27}$

$\frac{r_1}{r_2} = \frac{2}{3}$

Ratio between surface areas =  $\frac{4\pi r_1^2}{4\pi r_2^2} = \left(\frac{r_1}{r_2}\right)^2 = \left(\frac{2}{3}\right)^2 = \frac{4}{9}$



of the volume of sphere to that of cube = cm. Then, volume of the metallic solid cylinder of 91 2 ? r h. 375 ? Per cent increase in volume = AK ? 216 ? 125 ? 2 = ? ? ? r h ? 375 ? height 10 = BR Q. 9. A sphere and a cube have the same surface. Show that the ratio of the volume of sphere to that of the cube is 6: ? O 91? 100 ? 3 = 72. 8%. 375 TH ER = 91 2 100 ? r h ? 1 2 375 ? r h 3 2 cm. 3 = Volume of the metal in the spherical shell 32 4 2 = ? 53 ? 33 ? ?? r ? 3 3 32 2 4 r = (125 ? 27) ? 3 3 3 4 ? ? 98 ? r<sup>2</sup> = 32 3 49 7 ? r = cm ? r<sup>2</sup> = 4 2 Hence, the diameter of the base of the cylinder AS ( Increase in volume = 72 2 1 ? r h - ? r<sup>2</sup>h 3 125 2011 (T-II)] Sol. Let the radius of the sphere be r and the edge YA L O of the cube be x. Whole surface area of sphere = 4? r<sup>2</sup> and whole surface area of cube = 6x<sup>2</sup>. According to question, ? S Q. 11. A solid ball is exactly fitted inside the cubical box of side a. The volume of the ball is 4 3 ? a . Is it true? 3 PR = 7 cm. Sol. Diameter of the ball = side of the cube ? Radius of the ball = ? Volume of the ball = G 4? r<sup>2</sup> = 6x<sup>2</sup>. r<sup>2</sup> x 2 = 6 3 r = ? = 4? 2? x 3 2? 4 3 ? r Volume of sphere 3 Now, = Volume of cube x<sup>3</sup> = Hence, the statement is false. 4 ? r? 4 ? r? r ?? ? = ?? ? ? 3 ? x? 3 ? x? x 3 2 Q. 12.

From a solid cube of side 7 cm, a conical cavity of height 7 cm and radius 3 cm is hollowed out. Find the volume of the remaining solid. 14 HA ) a 2 1 ? 6r ? 6h New volume = ? ? ? ? 3 ? 5 ? 5 72 2 = ? r h. 125 Q. 10. The internal and external radii of a hollow spherical shell are 3 cm and 5 cm respectively. If it is melted to form a solid 2 cylinder of height 10 cm, find the diameter of 3 the cylinder. [2011 (T-II)] Sol. Let the radius of the base of the cylinder be 4 a<sup>3</sup> ? a<sup>3</sup> ?? = 3 8 6 N Sol. Volume of the cube = 7<sup>3</sup> cm<sup>3</sup> = 343 cm<sup>3</sup> Sol. 1 ? ?



the difference of the volumes of cube and the toy. Also, find the total surface area of the toy. Sol.

Volume of the toy = Volume of the cone + Volume of the hemisphere =  $\frac{1}{3} \pi r^2 h + \frac{2}{3} \pi r^3 = \frac{1}{3} \pi r^2 (h + 2r)$   $= \frac{1}{3} \times \frac{22}{7} \times 15^2 \times (10 + 2 \times 15)$   $= \frac{1}{3} \times \frac{22}{7} \times 225 \times 40$   $= 14080 \text{ cm}^3$  Sol. Capacity of the box =  $16 \times 8 \times 8 \text{ cm}^3 = 1024 \text{ cm}^3$  Volume of the 16 glass spheres =  $16 \times \frac{4}{3} \pi r^3 = 16 \times \frac{4}{3} \times \frac{22}{7} \times 2^3 = 16 \times \frac{4}{3} \times \frac{22}{7} \times 8 = 11264 \text{ cm}^3$  Volume of water filled in the box =  $11264 - 10240 = 1024 \text{ cm}^3$  A cube circumscribes this toy, hence edge of the cube = 8 cm. Volume of the cube =  $8^3 \text{ cm}^3 = 512 \text{ cm}^3$  Required difference in the volumes of the toy and the cube =  $14080 - 512 = 13568 \text{ cm}^3$  Total surface area of the toy = curved surface area of the cone + curved surface area of the hemisphere =  $\pi r l + \pi r^2 = \pi r (l + r) = \frac{22}{7} \times 15 \times (25 + 15) = 1408 \text{ cm}^2$  diameter of the dome is equal to its total height above the floor, find the height of the building. [2001] Sol. Let the internal height of the cylindrical part be h and the internal radius be r. Then, total height of the building = h+r Also,  $2r = h + r \Rightarrow h = r$ . Now, volume of the building = Volume of the cylindrical part + Volume of the hemispherical part =  $\pi r^2 h + \frac{2}{3} \pi r^3 = \pi r^2 (h + \frac{2}{3} r) = \frac{22}{7} \times 4^2 \times (4 + \frac{2}{3} \times 4) = \frac{22}{7} \times 16 \times \frac{10}{3} = 171.47 \text{ cm}^2$  Q. 16. 16 glass spheres each of radius 2 cm are packed into a cubical box of internal dimensions 16 cm × 8 cm × 8 cm and then the box is filled with water. Find the volume of water filled in the box. Sol. Volume of 16 spheres =  $16 \times \frac{4}{3} \pi r^3 = 16 \times \frac{4}{3} \times \frac{22}{7} \times 2^3 = 11264 \text{ cm}^3$  Volume of the box =  $16 \times 8 \times 8 = 1024 \text{ cm}^3$  Volume of water filled in the box =  $11264 - 10240 = 1024 \text{ cm}^3$  Q. 17. A building is in the form of a cylinder surmounted by a

hemispherical valuted dome 19 m<sup>3</sup> of air. If the internal 21 2 880 = ? r<sup>3</sup> + ? r<sup>3</sup> [? r = h] 3 21 5? r 3 880 = 21 3 AS 2 19 = ? r<sup>2</sup>h + ? r<sup>3</sup> 3 21 HA N Q. 18. A godown building is in the form as shown in the figure.

The vertical cross section parallel to the width side of the building is a rectangle 7 m ? 3 m, mounted by a semicircle of radius 3. 5 m. The inner measurements of the cuboidal portion of the building are 10 m ? 7 m ? 3 m. Find the volume of the godown and the total interior surface area excluding the floor 22 ? ? (base). ? ? = ? . ? 7 ? ? 1 2? = 2 ? ? r ? = ? r<sup>2</sup> ? 2 ? 22 ? (3. 5) 2 m<sup>2</sup> = 38. 5 m<sup>2</sup> 7 Total interior surface area excluding the base floor = area of the four walls = = 250. 5 m<sup>2</sup>. Sol. The godown building consists of cuboid at the bottom and the top of the building is in the form of half of the cylinder.

Length of the cuboid = 10 m, Breadth of the cuboid = 7 m Height of the cuboid = 3 m Volume of the cuboid = lbh = 10 ? 7 ? 3 m<sup>3</sup> = 210 m<sup>3</sup>. Radius of the cylinder = 3. 5 m Length of the cylinder = 10 m 1 2 Volume of the half of the cylinder = ? r h 2 1 22 = ? ? (3. 5)<sup>2</sup> ? 10 m<sup>3</sup> 2 7 = 192. 5 m<sup>3</sup> Volume of the godown = volume of the cuboid + volume of the half cylinder = (210 + 192. 5) m<sup>3</sup> = 402. 5 m<sup>3</sup> Interior surface area of the cuboid = Area of four walls = 2 (l + b) h = 2(10 + 7) 3 m<sup>2</sup> = 102 m<sup>2</sup> Interior curved surface area of half of the cylinder 22 = ? rh = ? 3. 5 ? 10 m<sup>2</sup> = 110 m<sup>2</sup> 7 YA L BR O TH ER Q. 19.

A tent is in the shape of a cylinder surmounted by a conical top. If the height and diameter of the cylindrical part are 2. 1 m and 4 m respectively and the slant height of the top is 2. 8 m, find the area of canvas used for making the tent. Find the cost of the canvas of the tent at the rate of Rs 550 per m<sup>2</sup>. Also, find the volume of air enclosed in the tent. [2008C] Sol. O S G PR

Height of the cone,  $H = \sqrt{2.8^2 + 2^2} = 3.46 \text{ m}$

Area of canvas required for making the tent = Curved surface area of the tent = Curved surface area of the cylindrical part + curved surface area of the conical part

$$= 2\pi rh + \pi l = \pi r(2h + l) = \pi \times 2.8(2 \times 2 + 3.46) = 102.7 \text{ m}^2$$

Interior area of two semicircles

$$= 2 \times \frac{1}{2} \pi r^2 = \pi r^2 = \pi \times 2.1^2 = 110.7 \text{ m}^2$$

Area of the tent (curved surface area of the cylinder) + 2 (area of the semicircle) =  $(102.7 + 110.7 + 38.5) \text{ m}^2 = 251.9 \text{ m}^2$

Cost of canvas = Rs 500  $\times$  251.9 = Rs 125950.

Volume of the air enclosed in the tent = Volume of the cylindrical part + Volume of the conical part

$$= \pi r^2 h + \frac{1}{3} \pi r^2 H = \pi \times 2.8^2 \times 2 + \frac{1}{3} \pi \times 2.8^2 \times 3.46 = 88.25 \text{ m}^3$$

ER Q. 20. From a solid cylinder whose height is 8 cm and radius 6 cm, a conical cavity of height 8 cm and of base radius 6 cm, is hollowed out. Find the volume of the remaining solid correct to two places of decimal.

Also find the total surface area of the remaining solid. (Take  $\pi = 3.14$ ) [2008, 2011 (T-II)]

Q. 21. A juice seller serves his customers using a glass as shown in the figure. The inner diameter of the cylindrical glass is 5 cm, but the bottom of the glass has a hemispherical portion raised which reduces the capacity of the glass. If the height of the glass is 10 cm, find the apparent capacity of the glass and its actual capacity. (Use  $\pi = 3.14$ ) [2009] Sol.

Radius of the cylindrical glass  $r = 2.5 \text{ cm}$

Radius of the cylinder = radius of the cone = 6 cm. Height of the cylinder = height of the cone = 8 cm.

Volume of the remaining solid

$$= \pi r^2 h - \frac{1}{3} \pi r^2 H = \pi r^2 h \left(1 - \frac{H}{3h}\right) = \pi \times 6^2 \times 8 \left(1 - \frac{8}{3 \times 8}\right) = 603.19 \text{ cm}^3$$

Slant height of the cone,  $l = \sqrt{6^2 + 8^2} = 10 \text{ cm}$

Q. 22. A cylindrical vessel with internal diameter 10 cm and height 10.5 cm is full of water. A solid cone of the diameter 7 cm and height of 6 cm is completely immersed in water. Find the volume of (i) water displaced out of

the cylindrical vessel. (ii) water left in the cylindrical vessel. [Take  $\pi = 18$  PR  
 Height of the glass = 10 cm Apparent capacity of the glass =  $\pi r^2 h = 3.14 \times$   
 $2.5 \times 2.5 \times 10 \text{ cm}^3 = 196.25 \text{ cm}^3$  Volume of the hemispherical portion  $\frac{2}{3}$   
 $= \frac{2}{3} \pi r^3 = \frac{2}{3} \times 3.14 \times 2.5 \times 2.5 \times 2.5 \text{ cm}^3 = 32.71 \text{ cm}^3$

Actual capacity of the glass =  $(196.25 - 32.71) \text{ cm}^3 = 163.54 \text{ cm}^3$ . AK AS  
 22 ] 7 HA 1.95  $\times 22 \times 22 \times 2.1 \text{ m}^3 = 7 \times 7 \times 7 \times 2.1 \text{ m}^3 = 12 \times 7 \times 7 \times 2.1$   
 $\text{cm}^3 = 36 \times 64 \text{ cm}^3 = 10 \text{ cm}^3$  Total surface area of the remaining solid  
 = curved surface area of the cylinder + area of top + curved surface area of  
 the cone =  $2\pi rh + \pi r^2 + \pi rl = \pi r (2h + r + l) = 3.14 \times 6 (16 + 6 + 10)$   
 $\text{cm}^2 = 18.84 \times 32 \text{ cm}^2 = 602.88 \text{ cm}^2$ . =  $r^2 \times h^2$  [2009] Sol. Radius of the  
 cylinder,  $r = 5 \text{ cm}$  Height of the cylinder,  $h = 10.5 \text{ cm}$  Capacity of the vessel  
 $= \pi r^2 h = \pi \times 5 \times 5 \times 10.5 \text{ cm}^3 = 825 \text{ cm}^3$  7 1 Volume of the cone =  $\frac{1}{3} \pi r^2 h$   
 $= \frac{1}{3} \times 3.14 \times 3.5 \times 3.5 \times 6 \text{ cm}^3 = 77 \text{ cm}^3$ . 7 (i) Water displaced out of the  
 cylinder = Volume of the cone =  $77 \text{ cm}^3$  (ii) Water left in the cylindrical  
 vessel = Capacity of the vessel - Volume of the cone =  $(825 - 77) \text{ cm}^3 =$   
 $748 \text{ cm}^3$ . 10 cm, 5 cm and 4 cm. The radius of each of the conical  
 depressions is 0.5 cm and depth is 2.1 cm. The edge of the cubical  
 depression is 3 cm. Find the volume of the wood in the entire stand. Sol.  
 Volume of a cuboid =  $10 \times 5 \times 4 \text{ cm}^3 = 200 \text{ cm}^3$ . Volume of the conical  
 depression Choose the correct option (Q 1 – 5) : 1. The surface area of a  
 sphere is  $154 \text{ cm}^2$ . The volume of the sphere is : 2 1 (a)  $179 \text{ cm}^3$  (b)  $359$   
 $\text{cm}^3$  3 2 2 3 1 (c)  $1215 \text{ cm}^3$  (d)  $1374 \text{ cm}^3$  3 3 2.

The ratio of the volumes of two spheres is 8 : 27. The ratio between their  
 surface areas is : (a) 2 : 3 (b) 4 : 27 (c) 8 : 9 (d) 4 : 9 3. The curved surface  
 area of a cylinder is  $264 \text{ m}^2$  and its volume is  $924 \text{ m}^3$ . The height of the

cylinder is : (a) 3 m (b) 4 m (c) 6 m (d) 8 m 4. The radii of the base of a cylinder and a cone of same height are in the ratio 3 : 4. The ratio between their volumes is : (a) 9 : 8 (b) 9 : 4 (c) 3 : 1 (d) 27 : 16

TH ER PRACTICE EXERCISE 13. 2A 5. The capacity of a cylindrical vessel with a hemispherical portion raised upward at the bottom as shown in the figure is : (a)  $\frac{2}{3}\pi r^2 h$  (b)  $\frac{2}{3}\pi r^2 h + \frac{1}{2}\pi r^2 h$

(c)  $\frac{2}{3}\pi r^2 h + \frac{1}{2}\pi r^2 h$  (d)  $\frac{2}{3}\pi r^2 h + \frac{1}{2}\pi r^2 h$  6. Two solid cones A and B are placed in a cylindrical tube as shown in the figure. The ratio of their capacities is 2 : 1. Find the heights and capacities of the cones. Also, find the volume of the remaining portion of the cylinder. G O 7. Marbles of diameter 1.4 cm are dropped into a cylindrical beaker of diameter 7 cm containing 19

PR Q. 23. A pen stand made of wood is in the shape of a cuboid with four conical depressions and a cubical depression to hold pens and pins respectively. The dimensions of the cuboid are 4 cm  $\times$  2 cm  $\times$  2 cm. Volume of wood in the entire stand =  $[200 - (2 \times 2 + 27)] \text{ cm}^3 = 170.8 \text{ cm}^3$ . = (d)  $\frac{2}{3}\pi r^3$

$(3h + 4r) \times \frac{1}{3}\pi r^2 h = \frac{1}{3}\pi r^2 h (3h + 4r)$  11. An ice cream cone consists of a right circular cone of height 14 cm and the diameter of the circular top is 5 cm. It has a hemispherical scoop of ice cream on the top with the same diameter as of the circular top of the cone. Find the volume of ice cream in the cone. 12. A solid toy is in the form of a hemisphere surmounted by a right circular cone.

Height of the cone is 2 cm and the diameter of the base is 4 cm. If a right circular cylinder circumscribes the toy, find how much more space it will cover. [2011 (T-II)] 13. A cylindrical tub of radius 12 cm contains water to a depth of 20 cm. A spherical iron ball is dropped into the tub and thus the

Height of the cone is 2 cm and the diameter of the base is 4 cm. If a right circular cylinder circumscribes the toy, find how much more space it will cover. [2011 (T-II)] 13. A cylindrical tub of radius 12 cm contains water to a depth of 20 cm. A spherical iron ball is dropped into the tub and thus the

level of water is raised by 6.75 cm. What is the radius of the ball? 13.3  
 CONVERSION OF SOLID FROM ONE SHAPE TO ANOTHER TEXTBOOK'S  
 EXERCISE 13.3 22, unless stated otherwise. 7 Q. 1. A metallic sphere of  
 radius 4.2 cm is melted and recast into the shape of a cylinder of Take  $\pi =$   
 20 G O Y A L B R

O T H E R S 16. A heap of rice is in the form of a cone of diameter 9 m and  
 height 3.5 m. Find the volume of the rice. How much canvas cloth is  
 required to just cover the heap? 17. 500 persons are taking a dip into a  
 cuboidal pond which is 80 m long and 50 m broad. What is the rise of water  
 level in the pond, if the average displacement of the water by a person is 0.  
 04 m<sup>3</sup>. 18. A rocket is in the form of a right circular cylinder closed at the  
 lower end and surmounted by a cone with the same radius as that of the  
 cylinder. The diameter and height of the cylinder are 6 cm and 12 cm  
 respectively.

If the slant height of the conical portion is 5 cm, find the total surface area  
 and volume of the rocket. (Take  $\pi = 3.14$ ) radius 6 cm. Find the height of  
 the cylinder. Sol. Radius of sphere = 4.2 cm ? Volume of sphere = PR some  
 water. Find the number of marbles that should be dropped into the beaker so  
 that the water level rises by 5.6 cm. 8. A solid is in the form of a right  
 circular cone mounted on a hemisphere. The radius of the hemisphere is 3.5  
 cm and the height of the cone is 4 cm. The solid is placed in a cylindrical tub,  
 full of water, in such a way that the whole solid is submerged in water.

If the radius of the cylinder is 5 cm and height 10.5 cm, find the volume of  
 water left in the cylindrical tub. 9. The largest possible sphere is carved out  
 from a solid cube of side 7 cm. Find the volume of the sphere. 10. A



A building is in the form of a cylinder surmounted by a hemispherical dome as shown in the figure. The base diameter of the dome is equal 2 of the total height of the building. Find the height of the building, if it contains 67 1 m<sup>3</sup> of air. [2011 (T-II)]

**Sol.** Let the radius of the dome be  $r$  m. Then the height of the cylinder is  $2r$  m. The height of the building is  $3r$  m. The volume of the cylinder is  $\pi r^2 \cdot 2r = 2\pi r^3$  m<sup>3</sup>. The volume of the hemisphere is  $\frac{2}{3}\pi r^3$  m<sup>3</sup>. The total volume of the building is  $2\pi r^3 + \frac{2}{3}\pi r^3 = \frac{8}{3}\pi r^3$  m<sup>3</sup>. According to the question,  $\frac{8}{3}\pi r^3 = 671$ . Solving for  $r$ , we get  $r = 7$  m. The height of the building is  $3r = 21$  m.

(iii) Let the radius of the resulting sphere be  $R$  cm. Then volume of the resulting sphere is  $\frac{4}{3}\pi R^3$  cm<sup>3</sup>. The volume of the sphere of radius 8 cm is  $\frac{4}{3}\pi (8)^3$  cm<sup>3</sup>. The volume of the sphere of radius 10 cm is  $\frac{4}{3}\pi (10)^3$  cm<sup>3</sup>. According to the question,  $\frac{4}{3}\pi R^3 = \frac{4}{3}\pi (8)^3 + \frac{4}{3}\pi (10)^3$ . Solving for  $R$ , we get  $R = 12$  cm.

(iv) Let the radius of the well be  $r$  m. Then the depth of the well is  $14$  m. The volume of the well is  $\pi r^2 \cdot 14$  m<sup>3</sup>. The volume of the circular ring is  $\pi (R^2 - r^2) \cdot h$  m<sup>3</sup>, where  $R$  is the radius of the ring and  $h$  is the height of the ring. According to the question,  $\pi r^2 \cdot 14 = \pi (R^2 - r^2) \cdot h$ . Solving for  $h$ , we get  $h = \frac{14r^2}{R^2 - r^2}$  m.

Volume of earth taken out =  $\frac{4}{3}\pi r^2 h$  Radius of well (r) = AK H = Sol. We know that, volume of the sphere =  $\frac{4}{3}\pi r^3$  AS Q. 2. Metallic spheres of radii 6 cm, 8 cm and 10 cm, respectively, are melted to form a single solid sphere. Find the radius of the resulting sphere.

Diameter = 3 m  $\therefore$   $\frac{4}{3}\pi r^3 = \frac{4}{3}\pi (14)^3 \text{ m}^3 = \frac{4}{3}\pi \times 2744 \text{ m}^3$  Width of the  
 embankment = 4 m Let the height of the embankment be H m.  $\therefore$  Radius of  
 the well with embankment, R  $\therefore$   $\frac{4}{3}\pi R^3 = \frac{4}{3}\pi \times 1728 \therefore R = 12$  Hence, the radius of the  
 resulting sphere is 12 cm. Q. 3. A 20 m deep well with diameter 7 m is dug  
 and the earth from digging is