Free cauchy sequences essay example

Sociology, Community

\n[toc title="Table of Contents"]\n

\n \t

\n[/toc]\n \n

There are several mathematical formulae that mathematicians and scientists have come up with over time. These mathematics formulae solve or address several issues or problems in the real world or mathematical and scientific field. One of the well-known mathematicians is Augustin Louis Cauchy. He is credited for coming up with the Cauchy sequence (the Cauchy sequence is named after him). This paper explores the Cauchy sequence, explaining its nature, form, application, and other related aspects of the sequence.
What is Cauchy sequence? This is a sequence whose elements become arbitrarily close to each other in the course of progression of the sequences (Lang, Serge, 1983). The sequence has the property of converging without having any limit. However, convergent series do not have the Cauchy property. A sequence (an) has the Cauchy property if;
$\forall \varepsilon > 0 \quad \exists N \in \mathbb{N} \quad \forall n \geq N \quad \forall k \geq N \quad a_n - a_k < \varepsilon$. Therefore, if a sequence is convergent, $an \to l$ as $n \to \infty$; then an, has the Cauchy property. In particular when given any small positive distances, all the elements save the finite ones are less compared to the distances from one another. Cauchy's utility lies in the fact that a complete metric space i. e. one that the sequence has been known to converge to a particular limit, and its convergence then, is determined by the term of the sequence itself. This differs on the definition

of convergence that uses the terms and its values. In most cases, the Cauchy sequences is exploited as an algorithm in theoretical and applied situations where iteration is used to show the Cauchy sequence. The iterates, therefore shall fulfill the logical conditions as a termination. There are generalizations of the Cauchy sequence describing the abstract nature of the sequence in the form of Cauchy nets and filters. The following diagram shows an illustration of the Cauchy sequence

The Cauchy sequence xn in blue against, n. Therefore, from the graph it can be shown that if the space with the sequences is complete, and then, the final destination of the sequence exists, i. e. the limit of xn exists.

If the sequence does not have the Cauchy property, the elements of the sequence are not arbitral in the progression. Therefore, the sequence fails to close towards each other. This is illustrated in the graph.

It is clear in the two graphs that the Cauchy property in the first one is tending make the elements move or become close to one another in the progression making a possible limit to exist. The second show the wide gap that propagation produces in the propagation making convergence impossible. There are many variations of the Cauchy sequences depending on the numbers, space, and other variations. It thus applies in specific ways depending on the mentioned factors.

Cauchy Sequences for real numbers

A sequence of real numbers as x1 , x2, x3, x4, is known as a Cauchy sequence if; for each of the positive real number ε, there exists a positive integer N in such a way that for all natural numbers in the sequence m, i. e. n> N, then, with the vertical bars denoting absolute values. The Cauchy sequences of rational or complex numbers are defined on a similar manner. In such a case, the Xm-xn has to be made infinitesimal for every pair of m and n, found in the sequences. An infinitesimal value is defined as a value that is too small to measure or see them. The word infinitesimal originally referred to the infinite-th item in a sequences introduced by Gottfried Wilhelm Leibniz (Katz, Mikhail; Sherry, and David, 2012).

Cauchy sequences in metric space

A metric space in mathematics is a set with distances between the members of the set defined. It is the distances considered together that are called the metric of a set. Thus, in a metric space X, the absolute values as indicated earlier in the real numbers, i. e. is substituted with the distance d(xm, xn). In this case, d: X multiplied by X tending towards R. the R has some properties found between xm, and xn.

For example, a sequences x1 , x2, x3, x4, as defined earlier is Cauchy if; for each of the positive real number ε there is an existing a positive integer N in such a way that there exists positive integers in the sequence m, i. e. n> N, the distance This indicates that the terms of the sequence get closer to each other suggesting that the sequences ought to have a limit in X. However, such a limit does not always exist within the X. It is worth noting that the main difference between the Cauchy sequence from real numbers and that of the metric space in two ways. In the real Cauchy sequences for real numbers use the positive integers without distances whereas, in the metric space, distances are used. Both use real positive integers. When the Cauchy sequence converges to an element X at a metric space X, then, it is described as being complete. The following show the property of completeness of the Cauchy sequence. For example, the real numbers are complete for the metric if it is induced by usual absolute values. One of the constructions of real numbers (the different fields that define real number systems as an ordered field) entails rational numbers in the Cauchy sequences. A metric space X also may have discrete metric; two points at fixed distance k from each other, the elements of X remain constant beyond the fixed points and eventually converge in a repetitive term.

There are also instances when the Cauchy sequences are not complete. Rational numbers say Q are not complete for the usual distance when; there exists rational that converge at R to irrational numbers. The scenario here describes Cauchy sequences without limits at Q. if X is an irrational real number, the sequence xn that has its nth term truncated to n decimal places in the expansion of the X. This produces a Cauchy sequences of rational numbers whose limit is irrational i. e. with irrational limit x. This is possible since rational numbers exists.

There are other properties associated with the Cauchy sequences. For example, every sequence that is convergent e. g. p is a Cauchy sequence provided in a given real set of numbers, $\varepsilon > 0$ and beyond, certain fixed points exist, thus the sequence has fixed distances of $\varepsilon/2$ of p, therefore, each element of the sequences is within the distance ε of each. Cauchy sequences of the real or complex numbers is bounded. In the metric space, a Cauchy sequence that has convergent subsequences with a limit t is convergent. The completeness of the Cauchy's sequence is done using the proof of the Bolzano-Weierstrass theorem. In the summation of infinite series, the Cauchy sequences come in handy. It is also used to proof the completeness of the sequences.

There are various generalizations of the Cauchy sequences. These include the following. The topological vector spaces; this concept of the Cauchy sequence exists in the topological vector space X, for example, if using a local base B for X, about the origin, then Xk forms a Cauchy sequence when for each member, , there exists some number N such that when n, m> N, then xn-xm, forms part of V.

In topological groups, a vector space has its definition of a Cauchy sequence requiring there be a continuous subtracting operation then it would be appropriate in this context of the topological group; Xk in topological group G becomes a Cauchy sequences when each of the open neighborhood U of the identity in G, there shall then exist a number N in such a way that whenever m, n> M, it follows that,. Similarly, one can define the binary relation on Cauchy in G. there is a concept of Cauchy sequence found in a group G. This can be explained as follows. Let H= Hr be a sequence that decreases of a normal subgroups of G with finite index then the sequence Xn (the relative size of the H in a group G) the resulting sequence is a Cauchy sequence with respect to H if and only if, for any random value of r there is N such that,. This forms the topological group as described earlier, but for a particular topology G with H as the local base.

Cauchy series are given modulus in constructive mathematics. It is a must for the modulus of Cauchy convergence to be provided since it is very useful. For example, if x1 , x2, x3, x4, is Cauchy sequences in the set X. then, the modulus of the Cauchy convergence of the sequences is a function resulting from a function derived from a natural set of natural numbers in it. Any sequence that has the modulus of the Cauchy convergence forms a Cauchy sequence. It thus follows that Cauchy sequence that has modulus comes from the well-ordered property of natural numbers. It is not the case in constructive mathematics. I. e. in constructive mathematics, there is no the well-ordered property. It is also follow that, on the converse considering the depend choices principle is accepted in constructive mathematics. Therefore, the moduli for the Cauchy convergence are directly needed when constructive mathematicians wish not to use any form of choice (Bourbaki, Nicolas, 1972).

The modulus of the Cauchy is used in the simplification for definitions and theorems used in constructive mathematics and analysis. However, there is a more useful aspect of the Cauchy sequence. For example, the regular Cauchy sequence that form the Cauchy series with a particular modulus of convergence. Any one modulus of Cauchy sequences is equivalent-by consideration of the metric form completion, to a regular Cauchy sequence. The proof of this does not require the use of any form of axiom choices. The regular Cauchy series have been used by several mathematicians as Erret Bishop in his book called foundations of constructive analysis. Douglas Bridge also used the concept in mathematical constructivism.

Work cited

Katz, Mikhail; Sherry, David " Leibniz’s Infinitesimals: Their Fictionality, Their Modern Implementations, and Their Foes from Berkeley to Russell and Beyond", Erkenntnis. 2012.

Lang, Serge Algebra (Third ed.), Reading, Mass.: Addison-Wesley Pub. Co 1993.

Bourbaki, Nicolas Commutative Algebra (English translation ed.). Addison-Wesley 1972.