

Vector analysis essay sample

[Environment](#), [Water](#)



1. 1. 1 Show how to find A and B, given $A+B$ and $A-B$.

1. 1. 2 The vector A whose magnitude is 1.732 units makes equal angles with the coordinate axes. Find A_x , A_y , and A_z . 1. 1. 3 Calculate the components of a unit vector that lies in the xy-plane and makes equal angles with the positive directions of the x- and y-axes. 1. 1. 4 The velocity of sailboat A relative to sailboat B, v_{rel} , is defined by the equation $v_{rel} = v_A - v_B$, where v_A is the velocity of A and v_B is the velocity of B. Determine the velocity of A relative to B if $v_A = 30$ km/hr east
 $v_B = 40$ km/hr north.

ANS. $v_{rel} = 50$ km/hr, 53.1° south of east.

1. 1. 5 A sailboat sails for 1 hr at 4 km/hr (relative to the water) on a steady compass heading of 40° east of north. The sailboat is simultaneously carried along by a current. At the end of the hour the boat is 6.12 km from its starting point. The line from its starting point to its location lies 60° east of north. Find the x (easterly) and y (northerly) components of the water's velocity. ANS. $v_{east} = 2.73$ km/hr, $v_{north} \approx 0$ km/hr.

1. 1. 6 A vector equation can be reduced to the form $A = B$. From this show that the one vector equation is equivalent to three scalar equations.

Assuming the validity of Newton's second law, $F = ma$, as a vector equation, this means that a_x depends only on F_x and is independent of F_y and F_z . 1. 1.

7 The vertices A, B, and C of a triangle are given by the points $(-1, 0, 2)$, $(0, 1, 0)$, and $(1, -1, 0)$, respectively. Find point D so that the figure ABCD forms a plane parallelogram. ANS. $(0, -2, 2)$ or $(2, 0, -2)$. 1. 1. 8 A triangle is

defined by the vertices of three vectors A, B and C that extend from the origin. In terms of A, B, and C show that the vector sum of the successive

sides of the triangle ($AB + BC + CA$) is zero, where the side AB is from A to B , etc. 1. 1. 9 A sphere of radius a is centered at a point r_1 . (a) Write out the algebraic equation for the sphere.

(b) Write out a vector equation for the sphere.

ANS. (a) $(x - x_1)^2 + (y - y_1)^2 + (z - z_1)^2 = a^2$.

(b) $r = r_1 + a$, with $r_1 = \text{center}$.

(a takes on all directions but has a fixed magnitude a .)

1. 1. 10 A corner reflector is formed by three mutually perpendicular reflecting surfaces. Show that a ray of light incident upon the corner reflector (striking all three surfaces) is reflected back along a line parallel to the line of incidence.

Hint. Consider the effect of a reflection on the components of a vector describing the direction of the light ray.

1. 1. 11 Hubble's law. Hubble found that distant galaxies are receding with a velocity proportional to their distance from where we are on Earth. For the i th galaxy, $v_i = H_0 r_i$, with us at the origin. Show that this recession of the galaxies from us does not imply that we are at the center of the universe. Specifically, take the galaxy at r_1 as a new origin and show that Hubble's law is still obeyed.

1. 1. 12 Find the diagonal vectors of a unit cube with one corner at the origin and its three sides lying along Cartesian coordinates axes. Show that there are four diagonals with length $\sqrt{3}$. Representing these as vectors, what are their components? Show that the diagonals of the cube's faces have length $\sqrt{2}$ and determine their components.