

Section ii newtonian mechanics

[Science](#), [Physics](#)



SECTION II NEWTONIAN MECHANICS - PAGE 9 - Chapter 2: Kinematics

Rectilinear Motion Non-linear Motion a. Define displacement, speed, velocity and acceleration. Distance: Displacement: Speed: Velocity: Total length covered irrespective of the direction of motion. Distance moved in a certain direction Distance travelled per unit time. is defined as the rate of change of displacement, or, displacement per unit time {NOT: displacement over time, nor, displacement per second, nor, rate of change of displacement per unit time} is defined as the rate of change of velocity. Acceleration: b. Use graphical methods to represent distance travelled, displacement, speed, velocity and acceleration. Self-explanatory c. Find displacement from the area under a velocity-time graph. The area under a velocity-time graph is the change in displacement. d. Use the slope of a displacement-time graph to find velocity. The gradient of a displacement-time graph is the {instantaneous} velocity. e. Use the slope of a velocity-time graph to find acceleration. The gradient of a velocity-time graph is the acceleration. f. g. Derive, from the definitions of velocity and acceleration, equations that represent uniformly accelerated motion in a straight line. Solve problems using equations which represent uniformly accelerated motion in a straight line, including the motion of bodies falling in a uniform gravitational field without acceleration. 1. 2. 3. 4. $v = u + at$ $s = \frac{1}{2}(u + v)t$ $v^2 = u^2 + 2as$ $s = ut + \frac{1}{2}at^2$: derived from definition of acceleration: $a = (v - u) / t$ derived from the area under the v-t graph derived from equations (1) and (2) derived from equations (1) and (2) These equations apply only if the motion takes place along a straight line and the acceleration is constant; {hence, for eg., air resistance must be negligible.} h. Describe qualitatively the motion of

bodies falling in a uniform gravitational field with air resistance. Consider a body moving in a uniform gravitational field under 2 different conditions: A WITHOUT AIR RESISTANCE $\ddot{y} = -g$, Highest point $v = 0$, $t = \frac{v_0}{g}$, $W = +ve$, $\ddot{y} = -g$ Moving up $\ddot{y} = -g$ Moving down Assuming negligible air resistance, whether the body is moving up, or at the highest point or moving down, the weight of the body, W , is the only force acting on it, causing it to experience a constant acceleration. Thus, the gradient of the v - t graph is constant throughout its rise and fall. The body is said to undergo free - PAGE 10 - fall. B v WITH AIR RESISTANCE $\ddot{y} = -g$, Highest point $\ddot{y} = -g$, gradient = 9.81 m s^{-2} Terminal velocity $\ddot{y} = 0$ Moving up $\ddot{y} = -g$ Moving down $\ddot{y} = -g$ If air resistance is NOT negligible and if it is projected upwards with the same initial velocity, as the body moves upwards, both air resistance and weight act downwards. Thus its speed will decrease at a rate -2 greater than 9.81 m s^{-2} . This causes the time taken to reach its maximum height reached to be lower than in the case with no air resistance. The max height reached is also reduced. At the highest point, the body is momentarily at rest; air resistance becomes zero and hence the only force -2 acting on it is the weight. The acceleration is thus 9.81 m s^{-2} at this point. As a body falls, air resistance opposes its weight. The downward acceleration is thus less than 9.81 m s^{-2} . As air resistance increases with speed (Topic 5), it eventually equals its weight (but in opposite direction). From then there will be no resultant force acting on the body and it will fall with a constant speed, called the terminal velocity. i. Describe and explain motion due to a uniform velocity in one direction and uniform acceleration in a perpendicular direction. Equations that are used to describe the horizontal and vertical motion x direction (horizontal — axis) $s_x = u_x t$ (displacement)

$$1 \ 2 \ s \ x = u_x \ t + 2 a_x \ t \ y \ \text{direction (vertical — axis)} \ 1 \ 2 \ s \ y = u_y \ t + 2 \ a \ y \ t$$

(Note: If projectile ends at same level as the start, then $s_y = 0$) $-2 \ u$ (initial

velocity) $u_x \ u_y \ v$ (final velocity) (Note: At max height, $v_x = 0$) $v_x = u_x + a_x t$

$v_y = u_y + a t$ $v_y = u_y + 2 \ a \ s_y \ 2 \ 2 \ a$ (acceleration) (Note: Exists when a force

in x direction present) a_x (Note: If object is falling, then $a_y = -g$) $a_y \ t$ (time) t

$t \ v_y$ Parabolic Motion: $\tan \theta_{\pm} = v_x / s_y$ θ_{\pm} : direction of tangential velocity

{NOT: $\tan \theta_{\pm} = s_x / s_y$ } x - PAGE 11 - Chapter 3: Dynamics Newton's laws of

motion Linear momentum and its conservation a. State each of Newton's

laws of motion. Newton's First Law Every body continues in a state of rest

or uniform motion in a straight line unless a net (external) force acts on it.

Newton's Second Law The rate of change of momentum of a body is

directly proportional to the net force acting on the body, and the momentum

change takes place in the direction of the net force. Newton's Third Law

When object X exerts a force on object Y, object Y exerts a force of the same

type that is equal in magnitude and opposite in direction on object X. The

two forces ALWAYS act on different objects and they form an action-reaction

pair. b. Show an understanding that mass is the property of a body which

resists change in motion. Mass: is a measure of the amount of matter in a

body, & is the property of a body which resists change in motion. c. Describe

and use the concept of weight as the effect of a gravitational field on a mass.

Weight: is the force of gravitational attraction (exerted by the Earth) on a

body. d. Define linear momentum and impulse. Linear momentum of a body

is defined as the product of its mass and velocity ie $p = m \ v$ Impulse of a

force I is defined as the product of the force and the time Δt , t during which

it acts ie $I = F \times \Delta t$, t {for force which is const over the duration Δt , t } For a

variable force, the impulse = Area under the F-t graph { if $\int F dt$; may need to "count squares"} Impulse is equal in magnitude to the change in momentum of the body acted on by the force. Hence the change in momentum of the body is equal in mag to the area under a (net) force-time graph. {Incorrect to define impulse as change in momentum} e. Define force as rate of change of momentum. Force is defined as the rate of change of momentum, ie $F = \frac{d(mv)}{dt} = ma$ or $F = m \frac{dv}{dt}$ -2 The {one} Newton is defined as the force needed to accelerate a mass of 1 kg by 1 m s⁻². f. Recall and solve problems using the relationship $F = ma$ appreciating that force and acceleration are always in the same direction. Self-explanatory g. State the principle of conservation of momentum. Principle of Conservation of Linear Momentum: When objects of a system interact, their total momentum before and after interaction are equal if no net (external) force acts on the system. or, ie The total momentum of an isolated system is constant $m_1 u_1 + m_2 u_2 = m_1 v_1 + m_2 v_2$ if net $F = 0$ {for all collisions } NB: Total momentum DURING the interaction/collision is also conserved. h. Apply the principle of conservation of momentum to solve problems including elastic and inelastic - PAGE 12 - interactions between two bodies in one dimension. (Knowledge of coefficient of restitution is not required.) (Perfectly) elastic collision: Inelastic collision: Perfectly inelastic collision: Both momentum & kinetic energy of the system are conserved. Only momentum is conserved, total kinetic energy is not conserved. Only momentum is conserved, and the particles stick together after collision. (i. e. move with the same velocity.) i. Recognise that, for a perfectly elastic collision between two bodies, the relative speed of approach is equal to the relative speed of separation. For all elastic

collisions, $u_1 - u_2 = v_2 - v_1$ ie. relative speed of approach = relative speed of separation or, $\frac{1}{2} m_1 u_1^2 + \frac{1}{2} m_2 u_2^2 = \frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2$ j. Show an understanding that, whilst the momentum of a system is always conserved in interactions between bodies, some change in kinetic energy usually takes place. In inelastic collisions, total energy is conserved but Kinetic Energy may be converted into other forms of energy such as sound and heat energy.

- PAGE 13 - Chapter 4: Forces Types of force Equilibrium of force Centre of gravity Turning effects of forces a. Recall and apply Hooke's Law to new situations or to solve related problems. Within the limit of proportionality, the extension produced in a material is directly proportional to the force/load applied ie $F = kx$ Force constant $k = \text{force per unit extension (F/x)}$ {N08P3Q6b(ii)} b. Deduce the elastic potential energy in a deformed material from the area under a force-extension graph. Elastic potential energy/strain energy = Area under the F-x graph {May need to "count the squares"} For a material that obeys Hooke's law, Elastic Potential Energy, $E = \frac{1}{2} F x = \frac{1}{2} k x^2$ c. 2 Describe the forces on mass, charge and current in gravitational, electric and magnetic fields, as appropriate.

Forces on Masses in Gravitational Fields - A region of space in which a mass experiences an (attractive) force due to the presence of another mass.

Forces on Charge in Electric Fields - A region of space where a charge experiences an (attractive or repulsive) force due to the presence of another charge. - Refer to Chapter 15 Forces on Current in Magnetic Fields d. Solve problems using $p = \rho g h$. Hydrostatic Pressure $p = \rho g h$ {or, pressure difference between 2 points separated by a vertical distance of h } e. f. Show an understanding of the origin of the upthrust acting on a body in a fluid.

State that an upthrust is provided by the fluid displaced by a submerged or floating object. Upthrust: An upward force exerted by a fluid on a submerged or floating object; arises because of the difference in pressure between the upper and lower surfaces of the object. g. h. Calculate the upthrust in terms of the weight of the displaced fluid. Recall and apply the principle that, for an object floating in equilibrium, the upthrust is equal to the weight of the new object to new situations or to solve related problems. Archimedes's

Principle: ie Upthrust = weight of the fluid displaced by submerged object.

Upthrust = $V_{\text{submerged}} \rho_{\text{fluid}} g$. Show a qualitative understanding of frictional forces and viscous forces including air resistance. (No treatment of the coefficients of friction and viscosity is required.)

Frictional Forces: $\frac{1}{2} C_D \rho v^2 A$. The contact force between two surfaces = (friction + normal reaction). The component along the surface of the contact force is called friction. Friction between 2 surfaces always opposes relative motion {or attempted motion}, and its value varies up to a maximum value {called the static friction} Viscous Forces: - PAGE 14 -

j. A force that opposes the motion of an object in a fluid; Only exists when there is (relative) motion. Magnitude of viscous force increases with the speed of the object Use a vector triangle to represent forces in equilibrium.

See Chapter 1j, 1k k. Show an understanding that the weight of a body may be taken as acting at a single point known as its centre of gravity. Centre of Gravity of an object is defined as that pt through which the entire weight of the object may be considered to act. l. Show an understanding that a couple is a pair of forces which tends to produce rotation only. A couple is a pair of forces which tends to produce rotation only. m. Define and apply the

moment of a force and the torque of a couple. **Moment of a Force:** The product of the force and the perpendicular distance of its line of action to the pivot. **The produce of one of the forces of the couple and the perpendicular distance between the lines of action of the forces.** (WARNING: NOT an action-reaction pair as they act on the same body.) **Torque of a Couple:** n. Show an understanding that, when there is no resultant force and no resultant torque, a system is in equilibrium. **Conditions for Equilibrium (of an extended object):**

1. The resultant force acting on it in any direction equals zero
2. The resultant moment about any point is zero.

If a mass is acted upon by 3 forces only and remains in equilibrium, then

1. The lines of action of the 3 forces must pass through a common point.
2. When a vector diagram of the three forces is drawn, the forces will form a closed triangle (vector triangle), with the 3 vectors pointing in the same orientation around the triangle.

o. Apply the principle of moments to new situations or to solve related problems.

Principle of Moments: For a body to be in equilibrium, the sum of all the anticlockwise moments about any point must be equal to the sum of all the clockwise moments about that same point. - PAGE 15 - Chapter 5: Work, Energy and Power

Work Energy conversion and conservation **Potential energy and kinetic energy** **Power**

- a. Show an understanding of the concept of work in terms of the product of a force and displacement in the direction of the force.
- b. Calculate the work done in a number of situations including the work done by a gas which is expanding against a constant external pressure: $W = p\Delta V$.

Work Done by a force is defined as the product of the force and displacement (of its point of application) in the direction of the force ie $W = F \cdot s \cos \hat{I}$, Negative work is said to be done by F if x or its compo. is anti-parallel

to F If a variable force F produces a displacement in the direction of F , the work done is determined from the area under F - x graph. {May need to find area by “ counting the squares”. } c. Give examples of energy in different forms, its conversion and conservation, and apply the principle of energy conservation to simple examples. By Principle of Conservation of Energy, Work Done on a system = KE gain + GPE gain + Thermal Energy generated {ie Work done against friction} d. 1 Derive, from the equations of motion, the formula $E_k = \frac{1}{2}mv^2$. Consider a rigid object of mass m that is initially at rest. To accelerate it uniformly to a speed v , a constant net force F is exerted on it, parallel to its motion over a displacement s . Since F is constant, acceleration is constant, Therefore, using the equation: $v^2 = u^2 + 2as$, $1 \cdot v^2 = 2 \cdot (v - u) \cdot s$ Since kinetic energy is equal to the work done on the mass to bring it from rest to a speed v , The kinetic energy, $E_k = \text{Work done by the force } F = Fs = mas = \frac{1}{2}mv^2 = \frac{1}{2}m(v - u)^2$ e. 1 Recall and apply the formula $E_k = \frac{1}{2}mv^2$. 2 Self-explanatory f. Distinguish between gravitational potential energy, electric potential energy and elastic potential energy. Gravitational potential energy: this arises in a system of masses where there are attractive gravitational forces between them. The gravitational potential energy of an object is the energy it possesses by virtue of its position in a gravitational field. Elastic potential energy: this arises in a system of atoms where there are either attractive or repulsive 2 short-range inter-atomic forces between them. (From Topic 4, $E. P. E. = \frac{1}{2}kx^2$.) Electric potential energy: this arises in a system of charges where there are either attractive or repulsive - PAGE 16 - electric forces between them. g. Show an understanding of and use the relationship between force and potential

energy in a uniform field to solve problems. The potential energy, U , of a body in a force field {whether gravitational or electric field} is related to the force F it experiences by: $F = -\frac{dU}{dx}$. Derive, from the defining equation $W = Fs$ the formula $E_p = mgh$ for potential energy changes near the Earth's surface. Consider an object of mass m being lifted vertically by a force F , without acceleration, from a certain height h_1 to a height h_2 . Since the object moves up at a constant speed, F is equal to mg . The change in potential energy of the mass = Work done by the force $F = Fs = Fh = mgh$. Recall and use the formula $E_p = mgh$ for potential energy changes near the Earth's surface. Self-explanatory j. Show an appreciation for the implications of energy losses in practical devices and use the concept of efficiency to solve problems. Efficiency: ie k. The ratio of (useful) output energy of a machine to the input energy. $\text{Efficiency} = \frac{\text{Useful Output Energy}}{\text{Input Energy}} \times 100\%$ Define power as work done per unit time and derive power as the product of force and velocity. Power {instantaneous} is defined as the work done per unit time. $P = \frac{W}{t}$ Total Work Done Total Time $W = Ft$ Since work done $W = F \times s$, $F \times s = P \times t = Fv$ for object moving at const speed: $F =$ Total resistive force {equilibrium condition} for object beginning to accelerate: $F =$ Total resistive force + ma

{N07P1Q10, N88P1Q5} - PAGE 17 - Chapter 6: Motion in a Circle Kinematics of uniform circular motion Centripetal acceleration Centripetal force a_c . Express angular displacement in radians. Radian (rad) is the S. I. unit for angle, $\hat{\theta}$, and it can be related to degrees in the following way. In one complete revolution, an object rotates through 360° , or 2π rad. As the object moves through an angle $\hat{\theta}$, with respect to the centre of rotation, this

angle $\hat{\theta}$, is known as the angular displacement. b. Understand and use the concept of angular velocity. Angular velocity ($\dot{\theta}$) of the object is the rate of change of angular displacement with respect to time. $\dot{\theta} = \frac{d\theta}{dt}$ c. $\hat{\theta} = t \dot{\theta} = T$ (for one complete revolution) Recall and use $v = r\dot{\theta}$. Linear velocity, v , of an object is its instantaneous velocity at any point in its circular path. $v = \frac{\text{arc length}}{\text{time taken}} = r\dot{\theta}$, $t = \frac{r\dot{\theta}}{v}$ Note: (i) The direction of the linear velocity is at a tangent to the circle described at that point. Hence it is sometimes referred to as the tangential velocity. $\dot{\theta}$ is the same for every point in the rotating object, but the linear velocity v is greater for points further from the axis. (ii) d. Describe qualitatively motion in a curved path due to a perpendicular force, and understand the centripetal acceleration in the case of a uniform motion in a circle. A body moving in a circle at a constant speed changes velocity {since its direction changes}. Thus, it always experiences an acceleration, a force and a change in momentum. e. Recall and use centripetal acceleration $a = \frac{v^2}{r}$, $a = r\dot{\theta}^2$, $v = r\dot{\theta}$ Centripetal acceleration, f. a {in magnitude} Recall and use centripetal force $F = m\dot{\theta}^2 r$, $F = \frac{mv^2}{r}$ Centripetal force is the resultant of all the forces that act on a system in circular motion. {It is not a particular force; "centripetal" means "centre-seeking". Also, when asked to draw a diagram showing all the forces that act on a system in circular motion, it is wrong to include a force that is labelled as "centripetal force". } Centripetal force, $F = m\dot{\theta}^2 r = \frac{mv^2}{r}$ {in magnitude} r A person in a satellite orbiting the Earth experiences "weightlessness" although the gravitational field strength at that height is not zero because the person and the satellite would both have the same acceleration; hence the contact force between man & satellite/normal reaction on the

person is zero {Not because the field strength is negligible.} - PAGE 18 -

Chapter 7: Gravitation Gravitational Field Force between point masses Field of a point mass Field near to the surface of the Earth Gravitational Potential

a. Show an understanding of the concept of a gravitational field as an example of field of force and define gravitational field strength as force per unit mass. Gravitational field strength at a point is defined as the gravitational force per unit mass at that point. b. Recall and use Newton's law of gravitation in the form $F = \frac{GMm}{r^2}$ Newton's law of gravitation: The (mutual) gravitational force F between two point masses M and m separated by a distance r is given by $F = \frac{GMm}{r^2}$ where G : Universal gravitational constant r or, the gravitational force of between two point masses is proportional to the product of their masses & inversely proportional to the square of their separation. c. Derive, from Newton's law of gravitation and the definition of gravitational field strength, the GM equation $g = \frac{GM}{r^2}$ for the gravitational field strength of a point mass. r Gravitational field strength at a point is the gravitational force per unit mass at that point. It is a vector and its S. I. unit is N kg^{-1} . F By definition, $g = \frac{F}{m}$ By Newton Law of Gravitation, $F = \frac{GMm}{r^2}$ GM Combining, magnitude of $g = \frac{GM}{r^2}$ GM Therefore $g = \frac{GM}{r^2}$, M = Mass of object "creating" the field r d. GM for the gravitational field strength of a point mass to new situations or to solve related problems. Recall and apply the equation $g = \frac{GM}{r^2}$ Example 7D1 6 24 Assuming that the Earth is a uniform sphere of radius $6.4 \times 10^6 \text{ m}$ and mass $6.0 \times 10^{24} \text{ kg}$, find the gravitational field strength g at a point (a) on the surface, $GM = 11.24 \times 10^{16} \text{ m}^3 \text{ s}^{-2}$ $g = \frac{GM}{r^2} = \frac{(6.67 \times 10^{-11})(6.0 \times 10^{24})}{(6.4 \times 10^6)^2} = 9.77 \text{ m s}^{-2}$ (b) at height 0.50 times the radius of above the Earth's

surface. $GM = 112462g = r^2 = (6.67 \times 10^{-11})(6.0 \times 10^{24}) / (1.5 \times 10^6)^2 = 4.34 \text{ m s}^{-2}$

Example 7D2 -2 The acceleration due to gravity at the Earth's surface is 9.80 m s^{-2} . Calculate the acceleration due to gravity on a planet which has the same density but twice the radius of Earth. - PAGE 19

$g_P = g_E \frac{M_P}{M_E} \left(\frac{r_E}{r_P}\right)^2$
 $M_P = \rho \frac{4}{3}\pi r_P^3 = 2^3 \rho \frac{4}{3}\pi r_E^3 = 8M_E$
 $r_P = 2r_E$
 Hence $g_P = 2 \times 9.81 = 19.6 \text{ m s}^{-2}$. e. Show an appreciation

that on the surface of the Earth g is approximately constant and is called the acceleration of free fall. Assuming that Earth is a uniform sphere of mass M .

The magnitude of the gravitational force from Earth on a particle of mass m , located outside Earth a distance r from the centre of the Earth is $F = \frac{GMm}{r^2}$.

When a particle is released, it will fall towards the centre of the Earth, as a result of the gravitational force with an acceleration a . i. e. , $F = ma$

$\frac{GMm}{r^2} = ma$ Hence $a = g$ Thus gravitational field strength g is also numerically equal to the acceleration of free fall. Example 7E1 A ship is at rest on the

Earth's equator. Assuming the earth to be a perfect sphere of radius R and the acceleration due to gravity at the poles is g_0 , express its apparent

weight, N , of a body of mass m in terms of m , g_0 , R and T (the period of the earth's rotation about its axis, which is one day). Ans: At the North Pole,

the gravitational attraction is $F = \frac{GMm}{R^2} = mg_0$ At the equator, Normal Reaction Force on ship by Earth = Gravitational attraction - centripetal

force $N = mg_0 - mR\omega^2 = mg_0 - mR \left(\frac{2\pi}{T}\right)^2$ f. Define potential at a point as the work done in bringing unit mass from infinity to the point.

Gravitational potential at a point is defined as the work done (by an external agent) in bringing a unit mass from infinity to that point (without changing its

kinetic energy). i. GM Solve problems by using the equation $\frac{GM}{r} = \text{potential}$ for the

potential in the field of a point mass. $r \phi = W_{GM} = m r g$. Why gravitational potential values are always negative? As the gravitational force on the mass is attractive, the work done by an ext agent in bringing unit mass from infinity to any point in the field will be negative work {as the force exerted by the ext agent is opposite in direction to the displacement to ensure that $\phi, KE = 0$ } Hence by the definition of negative work, all values of ϕ are negative. - PAGE 20 - Relation between g and ϕ : $g = - d\phi/dr = -$ gradient of ϕ - r graph dr {Analogy: $E = -dV/dx$ } Gravitational potential energy U of a mass m at a point in the gravitational field of another mass M , is the work done in bringing that mass m {NOT: unit mass, or a mass} from infinity to that point. $U = m\phi =$ Change in GPE, $GMm/r \phi, U = m g h$ only if g is constant over the distance h ; {if h