Good research paper on the number e

Environment, Nature

Introduction

One of the famous irrational number in mathematics is the number e which has a value: e= 2. 71828 18284 59045 23536 02874 71352 66249 77572 47093 69995

The number e was named by the Swiss-German mathematician, Leonhard Euler, in 17th century. It is frequently called Euler's number after his name; however, the letter e stands for the word exponential. The number e is the base of the natural logarithms that is created by John Napier. In the year 1737, Euler employed the symbol for pi to denote the ratio of the circumference to the diameter within a circle. Then, in 1777, he used i to represent the value of the square root of -1. He was the first mathematician to include five most important numbers in a single equation which is called the Euler equation or Euler identity (Sandifer 1).

eπi+ 1= 0

The value of e is very important in several fields especially in calculus. The objective of this paper is to provide a deeper understanding and appreciation regarding the constant number e. The paper aims to provide a background of the number e so to allow understanding base from its establishment.

This paper tackles the constant exponential number e. In order to provide a smooth flow of discussion, the paper is divided according to the history of the constant e, its definition, derivation, properties, application as well as its relation to another famous constant number such as pi.

History of constant number e

The constant e was first published in 1618 by John Napier under his works in logarithms. It was supposed that the table of appendix which contains the constant was created by William Oughtred. But the discovery of the constant e was credited to Jacob Bernoulli with his endeavor to find the value of the expression that is actually equivalent to e. The expression is defined as the limn→∞1+1nn. In 1731, Leonhard Euler created the notation e to be the base of natural logarithm with his letter to Christian Goldbach. He uses the symbol in his unpublished works in the years that followed. The first appearance of the number e is in the year 1736 when Euler published his Euler's Mechanic. In the following years, many constants have been used to denote the value of e but the constant e became the standard (Coolidge 371).

In the survey during 1988 done by David Wells, the equation ranked first as the most beautiful theorem. In 2004, it was the second greatest equation recognized by physics world. In 2007, it was the third in an MAA course of Euler’s greatest theorems.

Definition

The constant e has many definitions. The definition of e is the limit as n approaches infinity of (1+1n)n. This can be shown in the Figure 1.

As the computed value becomes bigger, the frequency of compounding increases. However, the growth rate is declining or slowing. The growth can be observed to approach a certain value and is getting nearer to the value of 2. 71828. This is the exponential constant number e. This value is very important especially in the fields of physical sciences and mathematics. Calculus cannot exist without it.

It can also be defined as the limit as n approaches zero of (1+n)1n. The number e is also the base of the function y= ax with the slope of the tangent curve equivalent to 1 at point (0, 1).

The number e is also equivalent to the sum of the infinite series: e= 11!+ 12!+13!+ 14!+15!+16! Using the function of the natural logarithm represented by the equation below, we can calculate the value of e. Using the Newton's formula, the sum can be computed to as many decimal places as desired (Gardner, 7).

lnt= 1t1xdx

This equation has a small value of area if t is small, and can have a large value of area when t is really large. The area of this equation is equal to 1 when the value of t is equivalent to 2. 7. Now we can denote that the area of the equation is equal to 1 under the conditions y= 1x and x= 1 to x= e. The value of e is the number when lne= 1. Using the inverse function, e= 1ln (Gardner 5).

Deriving the value of e

Consider solving the derivative of an exponential function defined as ax, where a a∈R+. The derivative can be stated as

ddxax= limΔx→0ax+Δx- axΔx

Using the laws of exponents, the expression can be simplified as

ddxax= limΔx→0axaΔx- 1Δx= ax{limΔx→0aΔx- 1Δx}

The equation shows that the derivative of ax has a proportional value to itself with a constant. We can use the quantity and denote it as M(a) which is a base function of an exponential expression. This M(a) is independent of x.

Simplifying, we have:

ddxax= axM(a)

Now use the constant e such that M(e) = 1. Substituting, we can have the equation

ddxex= exMe= ex , ddxex= 1; x= 0.

Using the chain rule, dwdx= 1ew. From the equations solved awhile ago, ew= x. Thus the derivative of lnx= 1/x. To get the derivative of ax

ddxax= ddx(elna)x= ddxeslna

Applying chain rule, ddxeslna= lnax eslna= lnax as

limn→∞1+1nn= e{limn→∞1+1nn}= e1= e

Using the equation, the computed value of e is approximately 2. 71. Thus, we have derived the value of the constant number e. Just to reiterate, the number e is defined as the number when the derivative of a number is raised to some power, the result would be identical to the original number raised to some power.

Properties of e

The constant number e is considered to be normal, which means that when it is expressed in exponential notation, the base will have a uniform distribution.

The constant number e can be applied in the number theory. The constant number e was demonstrated to be irrational by Euler through simple continued fraction expanded in an infinite manner. The Lindemann–Weierstrass theorem stated that e is transcendental. This means that e is not a solution set of any polynomial expression with non-constant with rational coefficient.

Other interesting property of the constant number e is that it possesses relationship with π and i. This is the Euler equation: eπi= -1 (Andreou and Lambright 378).

The relationship shows that the imaginary number i provide a relationship among pi and e using two operations. Calculating the logarithms using infinite series, on the other hand, started in 1670 by Gregoryy, Halley and Walis. These calculations give way to the development of present day calculations.

The constant number " e" is also called the " natural" exponential since it occurs unsurprisingly in mathematics as well as in physical sciences which has real life applications. It is comparable to pi which also occurs in nature of geometry.

Applications

There are real life applications of the constant number e. These applications include plane waves in the field of electrodynamics, Newton's Law of heating and cooling and Boltzmann factor in the field of thermodynamics (Andreou and Lambright 376).

In mathematics, Euler's formula is applied for complex numbers. Other mathematical applications of the constant number e are for solving Eigen value problems, Savings model, Threshold model and Logistic mode.

For electrical applications, e is also applied in the damping factor to characteristic roots, employed in solving oscillator problems like spring-mass problems or LRC circuits. The constant is also found in charging capacitors. Moreover, the constant e is also applied in decay problems such as radioactivity or growth problems in population concepts. The constant is also applied in probability and derangement (Andreou and Lambright 375).

Ramanujan constant

There exist an irrational constant that relates the number e to the value of pi. This constant is called Ramanujan constant and is expressed by the equation, R= eπ163. The constant is approximately equal to 262537412640768743. 99999999999925. The Ramanujan constant can be established by means of the modular function theory. The nine numbers of Heegner contribute to a profound quantity of theoretic property associated to a few wonderful characteristics of the j-function. Even though Ramanujan provided not so many rather stunning illustrations of approximately integers, he did not really state the exacting characteristics mentioned beforehand. Hermite has known this property of the number163 in the year 1859 previous to Ramanujan's investigation. The term " Ramanujan's constant" was created by Simon Plouffe (Ball et al. 387).

There is a small clearing up associated with the Ramanujan constant epÖ163. The constant is an almost-integer. The constant was named after Ramanujan because of an April fool’s joke done by Martin Gardner. He stated that Ramanujan conjectured that the constant is really an integer. But actually, the form epÖd with integer d being positive are transcendental. This statement was proven by Aleksandr Gelfond. At first, the naming of the constant was intended to be just an April fool’s joke, but in turned out that the name was suitable for Ramanujan since his works were in line with the value of the constant. The expression of the Ramanujan constant (epÖd) must be near to an integer for particular values of n. It is important when n is a big Heegner value (Ball et al. 387).

Works Cited

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