Confirm equations (1) and (2) and show that the equivalent mass of the spring is ...

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- Analysis

actual mass.

Fig. 1-1 Schematic Diagram : Spring-Mass-Pulley system

In this derivation, the following variables are defined and used, according to Fig. 1-1:

k : Spring stiffness

Ms : Mass of the Spring

M : Mass of the Body

Ms : Mass of the Pulley, which includes Mass of the spring

g : Gravity Acceleration

T : Tensile Force in the Rope

1. 1 The natural frequency of free vibration of the assembled system :

Equilibrium at the body:

(1)

Equilibrium at the Pulley:

(2)

Meanwhile, the body moves twice as fast as the pulley, which means:

(3)

Substituting (3) into (1) :

(4)

Substituting (4) into (2) :

(5)

At this stage, the natural frequency of this system is assumed as follows:

: Angular Velocity of the Vibration

Then the motion of the pulley is described as follows:

(6)

General solution becomes,

(7)

When mass of the pulley is included in mass of the spring, or when mass of the pulley is

neglected,

(8)

1. 2 When a body is simply suspended from a spring:

Eq (2) becomes,

(9)

And eq (3) becomes,

(10)

Substituting (10) into (1) :

(11)

Substituting (11) into (9) :

(12)

Then the motion of the pulley is described as follows:

(13)

General solution becomes,

(14)

In this case, because there is no pulleys,

(15)

1. 3 Equivalent Concentrated Mass of Spring

When one end of a spring is stationary and the other end moves, the equivalent mass of

the spring concentrated at the moving end is one-third of the actual spring mass.

Show that the effective mass of a spring is 1/3 its actual mass

Body with mass (M) is suspended by a spring with stiffness (k), mass (m), length (L), and

y-coordinate is 0(zero) at the fixed end, the y= L at the free end.

Assuming that the velocity of the mass is v0 when it passes the neutral position of its natural vibration, then the vibration of any point inside the spring can be described as follows:

(16)

Meanwhile, any infinitesimal small volume along the spring can be regarded as a concentrated

mass as follows:

(17)

And the kinetic energy of any infinitesimal small volume is described as follows:

(18)

Integrating this quantity through x= 0 to L:

(19)

Finally the total kinetic energy of mass-spring system including the kinetic energy of the spring

becomes as follows:

(20)

Eq (20) implies that the mass-spring system has an additional mass, a third of actual spring mass

at its free end of the spring.

- Discussion

The discussion should include an estimation of the uncertainties in the measurements and

the final results and details of any methods used to minimize these

uncertainties.

2. 1 Experimental results

As a result of the experiment, the equivalent stiffness and natural frequency of the

pulley-mass-spring system are shown in Table 2-1 through Table 2-3, and

Fig. 2-1 through

Fig. 2-6:

According to those figures, the following trends can be seen:

- Standard deviation of the system's stiffness is in inverse proportion to the

summation of the

mass, i. e. equivalent concentrated mass of the spring and the pulley.

- Standard deviation of the system's natural frequency is directly

proportional to the summation of the mass, i. e. equivalent concentrated mass of the spring and the pulley.

The mass of pulley 2 is much lighter than pulley 1, which implies that the moment of inertia is larger in pulley 1 than pulley 2. Because of the large moment of inertia, the pulley 1plays a roll of flywheel, and stores rotational energy and resists changes in rotational speed. Therefore, in the case of pulley 1, the suspended mass would not move so sensitively, which stabilizes the

dynamic behavior of whole system.

In addition, because of the heavier mass of pulley 1, a higher frictional force is created at the bearing between pulley 1 and bracket than pulley 2, which would lower the natural frequency of the system.

Meanwhile in the case of pulley 2, because of the lighter mass, the rotational motion is not so stable as that of pulley 1, which implies that the motion of the suspended body is not correctly reflect the motion of the pulley 2.

Therefore, the deviation of the system's stiffness is larger in pulley 2 than pulley 1.

However, as can be seen from the comparison of both pulleys, the sensitivity of natural frequency against the mass is not so sensitive as that of stiffness, which implies that the stability of stiffness is more important than that of frequency.

Therefore, in accordance with the experimental results, the heavier pulley is more favorable than the lighter one.

You should also discuss the assumptions used in the derivation of equation (1) and their validity.

In the equation (1), the following factors are neglected:

- moment of inertia of pulley
- Frictional force at the bearing between the pulley and the bracket

Moment of inertia would stabilizes the motion, but would lower the natural frequency.

Frictional force would also lower the natural frequency of the system.

However, as far as the precision of the natural frequency is concerned, the heavier pulley might be better to use.

- Could any further assumptions have been made in the derivation which would not

have reduced the accuracy of the final result?

Because the elongation of the light cord is neglected, the dynamic behavior of the cord is not reflected in the system assumptions.

The velocity of the strain wave inside the cord is much faster than the natural frequency of the system, this factor can be neglected, but when combined with the slippage between the cord and the pulley, it might be some influence on its dynamic behavior.