Lecture week

Science, Mathematics

Find thestressin the same elastic plate under the combined loading. Solution: Known the stress functions in the two different loading cases. Thus the stress functions can be added directly as per the " superposition principle". 4. 7 Solution Approaches and Skills Introduction After define the B. C. , one should solve for three groups of unknowns: Displacement: u, v, w Strain Stress It is however impossible to solve for these unknowns altogether. We often have to solve one or two groups first. As such we have four different methods: displacement teeth, strain method, stress method and mixed method.

Fig. 4. 8 Flowchart of displacement method (replace stress and strain by displacement). Displacement Method Unknowns: u, v, w Procedure: Other two sets of the unknown variables must be eliminated from the equations. Thus we replace strain and stress in displacements, which can be done as follows: We derive (refer to Tutorial Question #3, Week 5) where Lovelace operator: and After obtain u, v, w, one can calculate strain by using strain-displacement equation and then calculate the stress by using Hooker's law.

Note that the solution must satisfy the boundary conditions. Stress Method Unknowns: Procedure: Solve for stress component first and then strains and displacements. Strain Method 4. 8 Problem 1: Solution to Cylinder under Internal and External Pressure Introduction It is convenient to use cylindrical coordinate system for many engineering problem which involves in circular geometry (e. G. Fig. 4. 8). Cylindrical coordinate system Similar to Cartesian coordinate system, cylindrical system consists of 3 independent coordinates: (r, 0, z) as shown in Fig. 4. 9.

Equilibrium equations in AD cylindrical system (can be derived by replacing coordinate): Strain-Displacement relations: Normal: Hooker's Law in AD: Displacement Method Step 1: Check the Boundary conditions: At: ћ Shear: Step 2 Analysis: The deformation is asymmetric and under plane strain. So the deformation is independent of coordinate z and 0. Thus the circumferential and axial displacement v and w vanish, and displacements can be expressed as: Step 3 Strain - Displacement relation: Step 4 Apply Hooker's law: Step 5: Equilibrium Equations The second and third equations are satisfied automatically.

The first equation is: Substitution of Hooker's law into the above equation of Thus Step 6: Solve for this linear and static ordinary differential equation Thus its solution an be assumed as (Displacement Method) (in which CLC and co are constants to be determined by using B. C.) Step 7: plug this trial function (solution) into the Strain - Displacement equations Similarly, we can have: where . Now the question is how to determine A and B. Equations. Step 8: Apply B. C. O determine the constants which leads to: and From A and B we can calculate CLC and co: Step 9: Calculate all the functions Displacements: Strains: Stresses: Plane Stress Problem: Replacing E and 0 by and , we can further obtain the solution to the corresponding plane stress problems. Plant stress Fig. 4. 0 Pressurized cylinder with plane strain and plane stress Displacement: Remarks: are independent on material properties. The cylinder made of any materials will have the same stress values and thus if strength is the major concern, one should select the highest strength material.

However, the displacement and strains are dependent on material properties. If the stiffness is the main concern, a higher E modulus material should be chosen. When , one have Since , the radial stress (always negative) and (always positive). Thus: . As all shear stresses are zero, thus the principal stresses are: 4. 9 Saint-Vents Principle In the cantilever beam problem, some observed some difference of stress contours as shown in Fig. 4. 11.

Saint Vents observed that in pure bending of a beam conforms a rigorous solution only when the external forces applied at the ends of beams are distributed over the end is the same as internal stress distribution, I. E. Linear distribution. Saint Vents Principle: If the force acting on a small portion of the surface of an elastic body are replaced by another statically equivalent system of forces acting on the same portion of the surface, such redistribution of loading produces substantial change in stress locally tit a linear dimensions of the surface on which the force are changed".

Two key assumptions: (1) very small loading area compared with the whole dimension. The affected area will be much smaller than the unaffected area Unaffected>> Affected. E. G in the tensile bar as shown in Fig 4. 12, L>> a, in which the affected area will take roughly: Aziza. (2) Force replaced must be statically equivalent. The replacement must not change either the resultant force or resultant couple. For example the slender bar is stretched in different ways as below, where one can approximately define the effected and unaffected areas.

Tensile test In the tensile test, the way of holding a specimen has no effect on the stress and deformation in the middle region of the specimen. In test code requires a sufficient length of the specimen to avoid the end effect on the testing result. It is an application of Saint-Pennant's principle. Four-point bending The better positioning of strain gauge should be in a far field as shown below to get more stable and reliable testing result. Cantilever beam in FEE The end force can be applied in different way, which only affects a small area as shown.