The fencing problem

Science, Mathematics

A farmer has exactly 1000 metres of fencing; with it she wishes to fence off a plot of land. She is not concerned about the shape of the plot, but it must have a perimeter of 1000m. So it could be or anything else with a perimeter (or circumference) of 1000m What she does wish to do is fence off the plot of land which contains the maximum area. Investigate the shape, or shapes of the plot of land which have a maximum area.

Throughout this investigation I will check that the perimeter is 1000 meters by finding the total of all the outer sides. Also I will use refining as a way of finding the maximum area. When I talk about using the maximum area of the previous table the maximum area of each table will be highlighted.

Rectangles

The first shape I will test will be a rectangle. Having been told that the perimeter must be 1000 meters I will find the areas of three rectangles, each with different lengths of sides, making sure that the perimeter is kept the same.

Having carried out the above calculations I will create a spreadsheet with formulae to carry out more calculations. The headings will consist of Length, Width, Perimeter and Area. Under length there will be a variable number (less than 500 and greater than 0). The first formula will be put under the width heading. The width will be calculated by taking the length away from 500. This will guarantee the perimeter to be 1000m.

The formula will be = 500-B2 where B2 is the cell in which the length is. To double check that the perimeter is 1000m under the perimeter heading there will be another formula. This will be =(B2+C2)*2 where B2 is the length and C2 is the perimeter. It will be multiplied by 2 because the answer in the brackets would be just the total of two sides and not all four. Finally under the area heading there will be a formula. This will be = B2*C2 where B2 is the length and C2 is the width. This formula is the same as the one used previously to calculate the area of a rectangle. The formulas and headings will be entered in as shown in the table below.

Having entered the correct information I will be able to calculate the areas of many different sizes of rectangles with a perimeter of 1000m. I can do this in Microsoft Excel by dragging the formula boxes down, thus duplicating them but allowing them to refer to different lengths.

(Please see tables and graphs [Fencing Problem for Rectangles])

To start with I used my spreadsheet to find the area of a few rectangles within the range of 1m and 499m. I then plotted a graph showing length against area. It showed a perfect curve. I decided that the line of symmetry of this curve would help to find the length that would give me the maximum area. I found the line of symmetry to be along the 250m mark on the x axis of the graph.

Hypothesis

I predict that the length of a rectangle that will give me the maximum area will be 250m. I have decided this having found the line of symmetry on the graph.

Poof (Please see tables and graphs [Fencing Problem for Rectangles])

To prove my hypothesis I refined my search around the maximum area of the first table and then the second table, followed by the third table and so on. Eventually I found that, even to 1 decimal place above or below 250m that, the maximum area was given by rectangle of sides 250m by 250m. This shows that a square gives the maximum area for a rectangle.

Isosceles Triangles

The second shape that I will test will be an isosceles triangle. Having carried out tests for a rectangle I am going to see whether the maximum area will be bigger, smaller or the same as that of a rectangle. I am also going to find out whether the number of sides affects the results and whether there are any similarities in results to a triangle. This will help me find the shape that gives the maximum area.

As previously for rectangles I will test some different sized isosceles triangles that have an area of 1000m.

The formula for the area of a triangle is BASE x HEIGHT divided by 2 or bh/2. I cannot find the area without knowing what the height of the triangle is. To find the height of the triangle I must use Pythagoras. This states that for a right-angled triangle a2+b2= c2 or the square hypotenuse is equal to the sum of square of the other two sides. Therefore to find the height I must split the triangle in half and then use half of the base to help me find the height. The square height will therefore be equal to the square of the hypotenuse minus the square of half the base. In the below examples

After completing the above tests I will create a spreadsheet with formulae to carry out more calculations. The headings will consist of Base, 1 equal side, Perimeter, Height and Area. Under the base heading there will be a variable number between 1 and 500. The first formula will be used to calculate the length of one equal side of the isosceles triangle. The formula will be =(1000-B2)/2 where B2 is the base. It will be divided by 2 because 1000-B2 would give the sum of the two equal sides together. As previously , for the rectangles, there will be a formula to check that the perimeter is 1000m.

This will be the base plus, one equal side multiplied by two or = B2+(C2*2). The main formula in this spreadsheet will be the one used to find the height. In a spreadsheet there are codes that represent calculations carried out. These are put at the front of the formula and the substitute for square root is SQRT. So my formula will be the square root of 1 equal side squared, minus half the base squared. However before entering my formula I found out that using the power sign (^) doesn't give accurate results and in order to square numbers I must multiply the number by itself instead of using such a sign. Therefore the formula entered into the spreadsheet will be = SQRT((C2*C2)-((B2/2)*(B2/2)))

Finally under the area heading there will be a formula. This will be =(B2*E2)/2 where B2 is the base and E2 is the height. This formula is the same as the one used previously to calculate the area of a triangle. The formulas and headings will be entered in as shown in the table below.

Having entered the correct information I will be able to calculate the areas of many different sizes of isosceles triangles with a perimeter of 1000m. I can do this in Microsoft Excel by dragging the formula boxes down, thus duplicating them but allowing them to refer to a different base.

(Please see tables and graphs [Fencing Problem for Isosceles Triangles})

As before I entered a range bases between 1m and 499m. I then plotted a graph of base against area and found that unlike the results for a rectangle there wasn't a perfect curve in order to find the line of symmetry, to aid my search. However I could tell that the maximum area would be given by a triangle with a base between 300m and 400m

Hypothesis

I predict that the maximum area will be given by a triangle with equal sides. I have decided this because the maximum area for a rectangle was given by a square and that my graph shows that the base must be between 300m and 400m. For a triangle with equal sides and a perimeter of 1000m the base would be 333. 33... meters.

Poof (Please see tables [Fencing Problem for Isosceles Triangles})

To prove my hypothesis I refined my search around the maximum area of the first table and then the second table, followed by the third table and so on. Eventually I found that, to 2 decimal places, the maximum area was given by a triangle of equal sides which is 333. 33m to every side. This shows that an equilateral triangle gives the maximum area for a triangle and this proves my hypothesis right.

Regular Polygons

Having tested isosceles triangles and rectangles I found that regular sided shapes give the maximum area. I know this because the maximum area of an isosceles triangle is given when the sides are each 333. 33m. The maximum area given by a rectangle is give by a square with 250m sides. I have also that as you increase the number of sides the area increases because the maximum area for a rectangle is 62500m2, and the maximum area for an isosceles triangle is 48112. 52243m2. As a result of these findings I am going to test regular sided polygons.

Having split the pentagon into isosceles triangles and then into right angled triangles I can now find the area. I know that the base of the triangle is 100m however I do not know the height. Before finding the height I must work out what the internal angle is. To find this I will divide 360 by the number of right-angled triangles (in this case 10). I can now tell the following about the triangle:

I can now use Trigonometry to find the height of the triangle.

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I know what the opposite is and the angle, and I want to know what the adjacent is. I will therefore use the formula TAN= Opposite/Adjacent. Therefore Adjacent= Opposite/TAN. So the height in metres will be:

- Height = 100/TAN36
- Height = 137. 638192m
- Area of 1 Isosceles Triangle = (200*137. 638192)/2

- Area of 1 Isosceles Triangle = 13763. 819205m2
- Area of Pentagon = 13763. 819205*5
- Area of Pentagon = 68819. 09602 m2

After completing the above tests I will create a spreadsheet with formulae to carry out more calculations. The headings will consist of Number of Sides, 1 Equal Side, Perimeter, Internal Angle of 1 Triangle, Half Angle, Height (of internal isosceles triangle), Area of 1 Triangle and Total Area. Under the first heading (Number of Sides) there will be a variable, whole, number between 3 and as higher number as desired (e. g. 30). Under the second heading there will be a formula to calculate the length of one equal side. The formula will be = 1000/A3 where A3 is the number of sides. As in all the other tests there will be a formula to check that the perimeter is 1000m. This will tell me if I have made an error in any of the previous cells.

So far so good, however before I continue I must point out that a computer spreadsheet doesn't work in degrees to measure angles. It measures in radians where a complete rotation is 2?. Also ? is represented by PI() in a spreadsheet. So instead of using 360 in my formula under the Internal Angle of 1 Triangle heading I will use 2*PI()/A3 where A3 is the number of sides. Under the Half Angle heading there will be a formula that will be = D3/2 where D3 is the internal angle of one triangle. This gives the internal angle of 1 right-angled triangle.

My main formula will go under the Height heading and it will use Tan which is substituted by TAN in a spreadsheet. It will be =(B3/2)/TAN(E3) where B3 is 1 equal side and E3 is the angle inside a right-angled triangle. The area of one isosceles triangle will be calculated using the formula =(B3*F3)/2 where B3 is one equal side and F3 is the height. Finally the total area will be calculated by multiplying the area of one isosceles triangle by the number of sides. The formula entered will be = G3*A3 where G3 is the area of one triangle and A3 is the number of sides. The formulas and headings will be entered in as shown in the table below.

Having entered the correct information I will be able to calculate the areas of many regular polygons with different numbers of sides and with a perimeter of 1000m. I can do this in Microsoft Excel by dragging the formula boxes down, thus duplicating them but allowing them to refer to a different number of sides.

Hypothesis

I predict that as you increase the number of sides the area increases because the maximum area for a rectangle is 62500m2, and the maximum area for an isosceles triangle is 48112. 52243m2.

Proof (Please see graph and table [Fencing Problem for Regular Polygons])

Used my spreadsheet to calculate the areas of polygons with sides ranging from 3 to 30. The polygons with 3 and 4 sides were used to test that my formula worked correctly. I plotted a graph showing the number of sides against the area and found that, as predicted, as the number of sides increased so too did the area.

Circle

After my findings from carrying out tests on regular polygons I have decided to test circle. I have decided this because as the number of sides of a regular polygon increase so too does the area and a circle is an infinitely sided regular polygon.

Hypothesis

I predict that a circle will give the largest area because of my tests on regular polygons. I also predict that the maximum area given will be pretty close to that of a regular polygon with 30 sides (79286. 37045m2) because of the curve on the graph plotted for the regular polygon section.

To find the area of a circle I will be required to use the formulae 2? r and ? r2. The circumference must be 1000m and before finding the area I need to find the radius.

To complete this in a spreadsheet under the circumference heading I would enter 1000. Under the radius heading I would use the formula =(C2/2)/PI() where C2 is the circumference. Finally under the Area heading I would enter the formula = PI()*(D2*D2) where D2 is the radius. The headings and formulas will be entered as shown in the table below.

The table above clearly proves my hypothesis correct. The working out also proves my hypothesis correct.

Conclusion

Having completed the spreadsheet table I can conclude that a circle gives the maximum area and that the result was close to that given by a 30 sided regular polygon. A circle provides the maximum area possible for fencing of length 1000m. The maximum area possible is: - 79577. 47155m2