History of algebra essay sample

Science, Mathematics

Algebra may divided into “ classical algebra” (equation solving or “ find the unknown number” problems) and “ abstract algebra”, also called “ modern algebra” (the study of groups, rings, and fields). Classical algebra has been developed over a period of 4000 years. Abstract algebra has only appeared in the last 200 years. The development of algebra is outlined in these notes under the following headings: Egyptian algebra, Babylonian algebra, Greek geometric algebra, Diophantine algebra, Hindu algebra, Arabic algebra, European algebra since 1500, and modern algebra. Since algebra grows out of arithmetic, recognition of new numbers – irrationals, zero, negative numbers, and complex numbers – is an important part of its history. The development of algebraic notation progressed through three stages: the rhetorical (or verbal) stage, the syncopated stage (in which abbreviated words were used), and the symbolic stage with which we are all familiar. The materials presented here are adapted from many sources including Burton, Kline’s Mathematical Development From Ancient to Modern Times, Boyer’s A History of Mathematics , and the essay on “ The History of Algebra” by Baumgart in Historical Topics for the Mathematics Classroom – the 31st yearbook of the N. C. T. M.

Egyptian Algebra

Much of our knowledge of ancient Egyptian mathematics, including algebra, is based on the Rhind papyrus. This was written about 1650 B. C. and is thought to represent the state of Egyptian mathematics of about 1850 B. C. They could solve problems equivalent to a linear equation in one unknown. Their method was what is now called the “ method of false position.” Their algebra was rhetorical, that is, it used no symbols. Problems were stated and solved verbally. The Cairo Papyrus of about 300 B. C. indicates that by this time the Egyptians could solve some problems equivalent to a system of two second degree equations in two unknowns. Egyptian algebra was undoubtedly retarded by their cumbersome method of handling fractions.

Babylonian Algebra

The mathematics of the Old Babylonian Period (1800 – 1600 B. C.) was more advanced that that of Egypt. Their " excellent sexagesimal [numeration system]. . . led to a highly developed algebra" [Kline]. They had a general procedure equivalent to solving quadratic equations, although they recognized only one root and that had to be positive. In effect, they had the quadratic formula. They also dealt with the equivalent of systems of two equations in two unknowns. They considered some problems involving more than two unknowns and a few equivalent to solving equations of higher degree. There was some use of symbols, but not much. Like the Egyptians, their algebra was essentially rhetorical. The procedures used to solve problems were taught through examples and no reasons or explanations were given. Also like the Egyptians they recognized only positive rational numbers, although they did find approximate solutions to problems which had no exact rational solution.

Greek Geometrical Algebra

The Greeks of the classical period, who did not recognize the existence of irrational numbers, avoided the problem thus created by representing quantities as geometrical magnitudes. Various algebraic identities and constructions equivalent to the solution of quadratic equations were expressed and proven in geometric form. In content there was little beyond what the Babylonians had done, and because of its form geometrical algebra was of little practical value. This approach retarded progress in algebra for several centuries. The significant achievement was in applying deductive reasoning and describing general procedures.

Diophantine Algebra

The later Greek mathematician, Diophantus (fl. 250 A. D.), represents the end result of a movement among Greeks (Archimedes, Apollonius, Ptolemy, Heron, Nichomachus) away from geometrical algebra to a treatment which did not depend upon geometry either for motivation or to bolster its logic. He introduced the syncopated style of writing equations, although, as we will mention below, the rhetorical style remained in common use for many more centuries to come. Diophantus' claim to fame rests on his Arithmetica, in which he gives a treatment of indeterminate equations – usually two or more equations in several variables that have an infinite number of rational solutions. Such equations are known today as " Diophantine equations". He had no general methods. Each of the 189 problems in the Arithmetica is solved by a different method. He accepted only positive rational roots and ignored all others. When a quadratic equation had two positive rational roots he gave only one as the solution. There was no deductive structure to his work.

Hindu Algebra

The successors of the Greeks in the history of mathematics were the Hindus of India. The Hindu civilization dates back to at least 2000 B. C. Their record in mathematics dates from about 800 B. C., but became significant only after influenced by Greek achievements. Most Hindu mathematics was motivated by astronomy and astrology. A base ten, positional notation system was standard by 600 A. D. They treated zero as a number and discussed operations involving this number. The Hindus introduced negative numbers to represent debts. The first known use is by Brahmagupta about 628. Bhaskara (b. 1114) recognized that a positive number has two square roots. The Hindus also developed correct procedures for operating with irrational numbers. They made progress in algebra as well as arithmetic. They developed some symbolism which, though not extensive, was enough to classify Hindu algebra as almost symbolic and certainly more so than the syncopated algebra of Diophantus. Only the steps in the solutions of problems were stated; no reasons or proofs accompanied them. The Hindus recognized that quadratic equations have two roots, and included negative as well as irrational roots. They could not, however, solve all quadratics since they did not recognize square roots of negative numbers as numbers. In indeterminate equations the Hindus advanced beyond Diophantus. Aryabhata (b. 476) obtained whole number solutions to $ax \pm by = c$ by a method equivalent to the modern method. They also considered indeterminate quadratic equations.

Arabic Algebra

In the 7th and 8th centuries the Arabs, united by Mohammed, conquered the land from India, across northern Africa, to Spain. In the following centuries (through the 14th) they pursued the arts and sciences and were responsible for most of the scientific advances made in the west. Although the language was Arabic many of the scholars were Greeks, Christians, Persians, or Jews. Their most valuable contribution was the preservation of Greek learning through the middle ages, and it is through their translations that much of what we know today about the Greeks became available. In addition they made original contributions of their own. They took over and improved the Hindu number symbols and the idea of positional notation. These numerals (the Hindu-Arabic system of numeration) and the algorithms for operating with them were transmitted to Europe around 1200 and are in use throughout the world today. Like the Hindus, the Arabs worked freely with irrationals. However they took a backward step in rejecting negative numbers in spite of having learned of them from the Hindus. In algebra the Arabs contributed first of all the name. The word “ algebra” come from the title of a text book in the subject, Hisab al-jabr w’al muqabala, written about 830 by the astronomer/mathematician Mohammed ibn-Musa al-Khowarizmi. This title is sometimes translated as “ Restoring and Simplification” or as “ Transposition and Cancellation.” Our word “ algorithm” in a corruption of al-Khowarizmi’s name. The algebra of the Arabs was entirely rhetorical.

They could solve quadratic equations, recognizing two solutions, possibly irrational, but usually rejected negative solutions. The poet/mathematician Omar Khayyam (1050 – 1130) made significant contributions to the solution of cubic equations by geometric methods involving the intersection of conics. Like Diophantus and the Hindus, the Arabs also worked with indeterminate equations.

European Algebra after 1500

At the beginning of this period, zero had been accepted as a number and irrationals were used freely although people still worried about whether they were really numbers. Negative numbers were known but were not fully accepted. Complex numbers were as yet unimagined. Full acceptance of all components of our familiar number system did not come until the 19th century. Algebra in 1500 was still largely rhetorical. Renaissance mathematics was to be characterized by the rise of algebra. In the 16th century there were great advances in technique, notably the solution of the cubic and quartic equations – achievements called by Boyer " perhaps the greatest contribution to algebra since the Babylonians learned to solve quadratic equations almost four millennia earlier." Publication of these results in 1545 in the Ars Magna by Cardano (who did not discover them) is often taken to mark the beginning of the modern period in mathematics. Cardano was the best algebraist of his age, but his algebra was still rhetorical. Subsequent efforts to solve polynomial equations of degrees higher than four by methods similar to those used for the quadratic, cubic, and quartic are comparable to the efforts of the ancient Greeks to solve the three classical construction problems: they led to much good mathematics but only to a negative outcome.

There were also at this time many important improvements in symbolism which made possible a science of algebra as opposed to the collection of isolated techniques (" bag of tricks") that had been the content of algebra up to this point. The landmark advance in symbolism was made by Viète (French, 1540-1603) who used letters to represent known constants (parameters). This advance freed algebra from the consideration of particular equations and thus allowed a great increase in generality and opened the possibility for studying the relationship between the coefficients of an equation an the roots of the equation (“ theory of equations”). Viète’s algebra was still syncopated rather than completely symbolic. Symbolic algebra reached full maturity with the publication of Descartes’ La Géométrie in 1637. This work also gave the world the wonderfully fruitful marriage of algebra and geometry that we know today as analytic geometry (developed independently by Fermat and Descartes). “ By the end of the 17th century the deliberate use of symbolism – as opposed to incidental and accidental use – and the awareness of the power and generality it confers [had] entered mathematics.” [Kline] But logical foundations for algebra comparable to those provided in geometry by Euclid were nonexistent.

Abstract Algebra

In the 19th century British mathematicians took the lead in the study of algebra. Attention turned to many “ algebras” – that is, various sorts of mathematical objects (vectors, matrices, transformations, etc.) and various operations which could be carried out upon these objects. Thus the scope of algebra was expanded to the study of algebraic form and structure and was no longer limited to ordinary systems of numbers. The most significant breakthrough is perhaps the development of non-commutative algebras. These are algebras in which the operation of multiplication is not required to be commutative. (The first example of such an algebra were Hamilton’s quaternions – 1843.) Peacock (British, 1791-1858) was the founder of axiomatic thinking in arithmetic and algebra. For this reason he is sometimes called the “ Euclid of Algebra.” DeMorgan (British, 1806-1871) extended Peacock’s work to consider operations defined on abstract symbols. Hamilton (Irish, 1805-1865) demonstrated that complex numbers could be expressed as a formal algebra with operations defined on ordered pairs of real numbers ((a, b) + (c, d) = (a+b, c+d) ; (a, b)(c, d) = (ac-bd, ad+bc)).

Gibbs (American, 1839-1903) developed an algebra of vectors in three-dimensional space. Cayley (British, 1821-1895) developed an algebra of matrices (this is a non-commutative algebra). The concept of a group (a set of operations with a single operation which satisfies three axioms) grew out of the work of several mathematicians. Perhaps the most important steps were by Galois (French, 1811-1832). By the use of this concept Galois was able to give a definitive answer to the broad question of which polynomial equations are solvable by algebraic operations. His work also led to the final, negative resolution of the three famous construction problems of antiquity – all were shown to be impossible under the restrictions imposed. The concept of a field was first made explicit by Dedekind in 1879. Peano (Italian, 1858-1932) created an axiomatic treatment of the natural numbers in 1889. It was shown that all other numbers can be constructed in a formal way from the natural numbers. (“ God created the natural numbers. Everything else is the work of man.” – Kronecker) Abstract algebra is a branch of mathematics in which researchers have been very active in the twentieth century.