Discussion question

Science, Mathematics



Discussion question Discussion question How does Boolean algebra capture the essential properties of logic operations and set operations? Boolean algebra deals with the set of operations of union, intersection, complement and the logic operators NOT, OR, AND. Similar to any lattice, a Boolean algebra (A, and, or) produces a partially ordered set (A, \leq) through the definition a \leq b precisely when a = a land b.

This is also equal to b = a or b). A Boolean algebra can also be defined as a distributive lattice with greatest element 1 and least element 0. Every element x within this has a complement x such that

land x = 0 and x OR x = 1. In this case, and and OR are used to indicate the infimum (meet) and supremum (join) used for the two elements.

Complements in the above element are uniquely defined if the exist.

The algebraic and the order theory perspective can be used interchangeably for importing concepts and results from universal algebra and the order theory. A conjunction, disjunction, negation or ordering relation is all naturally available in practical examples so that it is basic to make use of this relationship. General approaches from duality can also be applied in order theory to Boolean algebras. The order dual of every Boolean algebra obtained by interchanging AND and OR is also a Boolean algebra. By interchanging 0 with 1, the law that applies to Boolean algebras can be changed into another valid dual law.

Each Boolean algebra (A, AND, OR) produces a ring (A,,*) that defines a + b = (a AND b) or (b AND a) and a * b = a AND b. the 0 element of this ring matches the 0 of the Boolean algebra while the multiplicative identity element is the 1 of the Boolean algebra. a *a = a for all the rings in A. such

rings are called Boolean rings.

On the other hand, given a Boolean ring A, this can be turned into a Boolean algebra by the definition x OR y = x + y + xy and x AND y = xy. We can conclude that every Boolean algebra rises from a Boolean ring and vice versa. Goodstein (2007).

References

Goodstein, R. L. (2007). Boolean algebra. Mineola, N. Y: Dover Publications.