Estimation of optimal hedge ratios - strategies

Naïve or one-to-one hedge assumes that futures and cash prices move closely together. In this traditional view of hedging, the holding of both the initial spot asset and the futures contract used to offset the risk of the spot asset are of equal magnitude but in opposite direction. In this case the hedge ratio (h) is one-to-one (or unit) (-1) over the period of the hedge.

This approach fails to recognize that the correlation between spot and futures prices is less than perfect and also fails to consider the stochastic nature of futures and spot prices and resulting time variation in hedge ratios (Miffre, City University).

The beta hedge recognizes that the cash portfolio to be hedged may not match the portfolio underlying the futures contract. With the beta hedge strategy, his calculated as the negative of the beta of the cash portfolio.

Thus, for example, if the cash portfolio beta is 1. 5, the hedge ratio will be -1. 5, since the cash portfolio is expected to move by 1. 5 times the movement in the futures contract, where the cash portfolio is that which underlies the futures contract. The traditional strategy and the beta strategy yield the same value for h (Butterworth and Holmes 2001).

Minimum Variance Hedge Ratio (MVHR) was proposed by Johnson (1960) and Stein (1961). This approach takes into account the imperfect correlation between spot and futures markets and was developed by Ederington (1979). According to him, the objective of a hedge is to minimize the risk, where risk is measured by the variance of the portfolio return. The hedge ratio is identified as:

$h^* = -\sigma_{S, F} / \sigma^2_F$ (1)

Where, $\sigma_{S, F}$ is the variance of the futures contract and $\sigma_{S, F}$ is the covariance between the spot and futures position. The negative sign mean that the hedging of a long stock position requires a short position in the futures market. The relation between spot and futures can be represented as:

$S_t = \alpha + h^*F_t + e_t$ (2)

Eq. (2), which is expressed in levels, can also be written in price difference as:

$S_t - S_{t-1} = \alpha + h^*(F_t - F_{t-1}) + \varepsilon_t$ (3)

or in price returns as:

$S_t - S_{t-1} / S_{t-1} = \alpha + h^*(F_t - F_{t-1} / F_{t-1}) + \varepsilon_t$ (4)

Eq. (4) can be approximated by:

$\log S_t - \log S_{t-1} = \alpha + h^*(\log F_t - \log F_{t-1}) + \varepsilon_t$ (5)

Eq. (6) can be re-written as:

$RS_t = \alpha + h^*RF_t + \varepsilon_t$ (6)

Where, RS_t and RF_t are returns on spot and futures position at time t.

Equation (2) and (3) assume a linear relationship between the spot and futures while eq. (4)-(6) assumes that two prices follow a log-linear relation.

Relative to equation (2)-(3), the hedge ratio represents the ratio of the number of units of futures to the number of units of spot that must be hedged, whereas, relative to eq. (4), hedge ratio is the ratio of the value of futures to the value of spot. (Scarpa and Manera, 2006)

Eq. (2) can easily produce auto correlated and heteroskedastic residuals (Ederington, 1979; Myers and Thompson, 1989: cited in Scarpa and Manera, 2006). Due to this reason, some authors suggest the use of eq (3)-(6), so that the OLS classical assumption of no correlation in the error terms is not violated.

Empirically, optimal hedge ratio h* can be obtained by simple Ordinary Least Square (OLS) approach, where the coefficient estimates of the futures gives the hedge ratio. This is can only be done when there is no co-integration between spot and futures prices/values and conditional variance-covariance matrix is time invariant (Casillo, XXXX). Even though application of MVHR relies on unrealistic assumptions, it provides an unambiguous benchmark against which to assess hedging performance (Butterworth and Holmes, 2001).

Error Correction Model (ECM) approach for determining optimal hedge ratio takes in to account the important role played by the theory of co-integration between futures and spot market, which is ignored by MVHR (Casillo, XXXX). The theory of co-integration is developed by Engle and Granger (1981), who shows that if two series are co-integrated, there must exist an error correction representation that permits to include both the short-run dynamics and the long-run information.

ECM approach augments the standard OLS regression used in MVHR by incorporating error correction term (residual) and lagged variables to capture deviation from the long run equilibrium relationship and short-run dynamics respectively (XXXXect). The presence of the efficient market hypothesis and the absence of arbitrage opportunity imply that spot and futures are co-integrated and an error correction representation must exist (Casillo, XXXX) of the following form:

i= 1

j= 1

$\Delta S_t = \alpha e_{t-1} + \beta \Delta F_t + \Sigma \gamma_i \Delta F_{t-i} + \Sigma \delta_j \Delta S_{t-j} + u_t$ (7)

Where, β is the optimal hedge ratio and $e_{t-1} = S_{t-1} - \alpha F_{t-1}$

All the above mentioned approaches employ constant variance and covariance to measure hedge ratio, which have some problems. The return series of many financial securities exhibit non-constant variance, besides having a skewed distribution. This has been demonstrated by Engle 1982, Lamoureux and Lastrapes 1990, Glosten, Jagannathan and Runkle 1993, Sentana 1995, Lee and Brorsen 1997 and Lee Chen and Rui 2001 (Rose, et al., 2005).

Non-constant variance, linked to unexpected events is considered to be uncertainty or risk, and this uncertainty is particularly important to investors who wish to minimize risks. In order to cope with these problems, Engle (1982) introduced the Autoregressive Conditional Heteroskedasticity (ARCH) model to estimate conditional variance. It takes into account changing variance over time, by imposing an autoregressive structure on the conditional variance. Bollerslev, Engle and Wooldridge (1988) expanded the univariate GARCH described above to a multivariate dimension to simultaneously measure the conditional variance and covariance of more than one time series. Thus, the multivariate GARCH model is applied to calculate a dynamic hedge ratio that varies over time based upon the variance-covariance between time series. (Rose, et al., 2005)

Finally, other researchers have proposed more complex techniques and some special case of the above techniques for the estimation of the OHR. Among these we mention the random coefficient autoregressive offered by Bera et al. (1997), the Fractional Cointegrated Error Correction model by Lien and Tse (1999), the Exponentially Weighted Moving Average Estimator by Harris and Shen (2002), and the asymmetric GARCH by Brooks et al. (2002). (Casillo, XXXX)

Despite the existence of massive literature on all the above approaches, no unanimous conclusion has been reached regarding the superiority of a particular methodology for determining the optimal hedge ratio. However, it would be wise to suggest that the choice of a strategy for deriving optimal hedge ratio should be based on the subjective assessment to be made in relation to investor preferences (Butterworth and Holmes, 2001).

Development of Research:

Figlewski (1984) conducted the first analysis of hedging effectiveness of stock index futures in US. He examined the hedging effectiveness for Standard and Poor’s 500 stock index futures against the underlying portfolio of five major stock indexes for the period June 1, 1982 to September 20, 1983. All five indexes represented diversified portfolio, however they were different in character from one another. Standard and Poor's 500 index and New York Stock Exchange (NYSE) Composite included only the largest capitalization stocks. The American Stock Exchange composite (AMEX) and the National Association of Securities Dealers Automated Quotation System (NASDAQ) index of over-the-counter stocks contained only small companies which somewhat move independently of the Standard and Poor's index. Finally, the Dow Jones portfolio contained only 30 stocks of very large firms. Return series for the analysis included dividend payments as risk associated with dividends on the portfolio is presumably one of many sources that give rise to basis risk in a hedges position. However, it was found that their inclusion did not alter the results. Consequently, and given the relatively stable and predictable nature of dividends, subsequent studies have excluded dividends. Figlewski used beta hedge and minimum variance hedge strategies and showed that the latter can be estimated by Ordinary Least Square (OLS) approach using historical data. He found that for all indexes hedge performance using minimum variance hedge ratio (MVHR) was better than beta hedge ratio was used. MVHR resulted in lower risk and higher return. When MHHR was uses, risk was reduced by 70%-80% for large capitalization portfolios. However, hedging performance was considerably reduced for smaller stocks portfolios. Also, hedging performance was better for once week and four week hedges when compared with overnight hedges.

Figlewski (1885) studies hedging effectiveness of three US index futures (S&P500, NYSE Composite and Value Line Composite Index (VLCI)) in hedging five US indices (S&P500, NYSE Composite, AMEX Composite, NASDAQ and DJIA). Data was collected for 1982. He analyzed the hedging effectiveness for the holding period ranging from one day to three weeks using the standard deviation of the hedged position, divided by the standard deviation of the un-hedged position, as a measure of assessing hedging effectiveness. Hedge ratios were derived using beta strategy and MVHR. Assuming constant dividends, the weekly returns of each of the five indices were regressed on the on the returns of the indices underlying the three futures. Daily data was used to compute ex post risk-minimizing hedge ratios. In nearly every case, risk-minimizing hedge ratio outperformed the other in terms of hedging effectiveness, for both types of hedge ratio it was found that the hedges under a week were not very effective. It was also found that hedging was more effective for the S&P500, NYSE Composite and the DJIA than for NASDAQ and AMEX Composite. In other words, once again, portfolios of small stocks were hedged less effectively than were those comprising large stocks.

Junkas and Lee (1985) used daily spot and futures closing prices for the period 1982 to 1983 for three US indices: S&P500, NYSE Composite and VLCI. They investigated the effectiveness of various hedging strategies, including the MVHR and the one-to-one hedge ratio. This was done for each index using data for a month to compute the hedge ratio used during that same month in hedging the spot value of the corresponding index. MVHRs were computed by regressing changes in the spot price on changes in the futures price. The average MYHR was 0. 50, whike the average effectiveness, as measured by variance of un-hedged position minus variance of hedged position divided by variance of un-hedged position (HE), was 0. 72 for the S&P500 and the NYSE Composite, and 0. 52 for the VLCI. The effectiveness of the one-to-one hedge ratio was poor, leading to an increase in risk for the VLCI and the NYSE Composite, and an effectiveness measure of 0. 23 for the S&P500. In other words, MVHR was found to be most effective in reducing the risk of a cash portfolio comprising the index underlying the futures contract. There was little evidence of a relationship between contract maturity and effectiveness.

Peters (1986) examined the use of S&P500 futures to hedge three share portfolios; the NYSE Composite, the DJIA and the S&P500 itself. MVHR and beta hedge strategy was applied to the data for the period 1984 to 1985. For each of the portfolio, MVHR gave a hedged position with a lower risk that did beta.

Graham and Jennings (1987) were first to examine hedging effectiveness for cash portfolios not matching an index. They classifies US companies into nine categories according to their betas and dividend yield. For each beta-dividend yield category, ten equally weighted portfolios of ten shares each were constructed. Weekly returns were computed for each portfolio for 1982-83. They then investigated the performance of S&P500 futures in hedging these portfolios for periods of one, two and four weeks. Three alternative hedge ratios were uses: one to one, bets and MVHR. The MVHR produced hedged positions with returns that were about 75% higher than for the other two hedge ratios. The measure of hedging effectiveness HE ranged from 0. 16 to 0. 33. For the one and the two week hedges, the MVHR hedge was more effective, that is, had a higher HE value.

Morris (1989) investigated the performance of S&P500 futures in hedging the risk of a portfolio of the largest firms in the NYSE. The data was monthly from 1982 to 1987. The MVHR was estimated using data for the entire period, and gave a HE value of 0. 91.

Lindhal (1992) investigated hedge duration and hedge expiration effects for the MMI and S&P 500 future contract. Results showed that MVHR increased towards unity with an increase in the hedging duration. For S&P 500 hedge ratios were found to be 0. 927, 0. 965 and 0. 970 for one, two and four week hedge duration, respectively. It was concluded that hedge ratio and hedging effectiveness increase as duration increase. Lindhal’s examination of the hedge expiration effect is based on the fact that future prices converge towards spot prices as expiration approaches. According to him MVHR can be expected to converge towards the naïve hedge ratio if future prices also exhibit less volatility when approaching expiration. It was concluded that there was no obvious pattern in terms of risk reduction in relation to time to expiration.

Unlike previous studies which only investigate ex post hedging effectiveness, Holmes (1995) became the first individual in UK to examine the hedging effectiveness of FTSE-100 stock index futures contract using Ex Ante Minimum Variance Hedge Ratio strategy. The cash portfolio being hedged mirrored FTSE-100 stock index. Data for spot and future series was collected for the period July 1984 to June 1992 for hedging duration of one and two weeks. The results also demonstrated the superiority on MVHR over beta hedges and showed that ex ante hedge strategy resulted in risk reduction of over 80%. Greater risk reduction was also shown to be achieved by estimating hedge ratios over longer periods.

Holmes(1996) examined the ex post hedging effectiveness for the same data and return series used in the earlier study (1995) and showed that the standard OLS estimated MVHR provided the most effective hedge when compared to beta hedge strategy, error correction method and GARCH estimation. Results also suggested increase in hedging effectiveness with increase in hedging duration. This can be explained as variance of returns increases with an increase in the duration, resulting in the reduction of the proportion of the total risk accounted for by the basis risk.

Butterworth and Holmes (2001) provided an unprecedented insight in to the hedging effectiveness of investment trust companies (ITCs) using Mid250 and FTSE100 stock index futures contract , the former being introduced in February 1994 with an aim to provide better hedging for small capitalization stocks. Analysis is based on daily and weekly hedge durations for the cash and future return data of thirty-two ITCs and four indices for the period of February 1994 to December 1996. FTSE100 index futures and FTSE Mid250 index futures are used to hedge cash positions. Apart from well established OLS approach, consideration is also given to Least Trimmed Squares (LTS) approach for estimation which involves trimming of regression by excluding the outliers. Four hedging strategies including traditional hedge, beta hedge, minimum variance hedge and composite hedge were compared on the basis if within sample performance. Composite hedge ratio was generated by considering returns on synthetic index futures formed by weighted average of returns on FTSE100 and FTSE-Mid250 contracts. Results demonstrated that traditional and beta hedge performed worst. MVHR strategy for daily and weekly hedges using Mid250 contracts outperformed the same strategy using FTSE100 contacts in terms of risk reduction for ITCs. However the superiority of Mid250 over FTSE100 is significantly less for cash portfolios based on broad market indexes. The composite hedge strategy demonstrated only minor improvements over results of the Mid250 contract. The LTS approach suggested similar results as OLS.

Seelajaroen (2000) attempted to investigate the hedging effectiveness of All Ordinance Share Price Index (SPI) to reduce price risk of All Ordinary Index (AOI) portfolio in the Australian financial market. Hedging effectiveness was investigated for one, two and four week hedge duration. Hedge ratios were generated by using Working's model and the Minimum variance model and their effectiveness was determined by comparison with naïve strategy. Data for the analysis consisted if daily closing prices of the SPI and API for the period January 1992 to July 1998. Minimum variance model consisted of both ex post and ex ante approach. Results demonstrated superiority of both Working's model and Minimum variance model over naïve hedge strategy. Working's strategy was found to be more effective in long run, however, in short run the strategy is more sensitive to basis level used in the decision rule. Minimum variance strategy was also found to be highly effective, as even the standard use of the hedge ratio derived from past data was able to achieve risk reduction of almost 90%. Also, longer duration hedges were found to be more viable than short duration hedges and finally effects of time expiration on hedge ratio and effectiveness was found be ambiguous.

DATA & METHODOLOGY:

This paper examines the cross hedging effectiveness of five of the world's most actively traded Stock Index Futures to reduce the risk of KSE100 index. The 5 stock index futures include S&P500, NASDAQ100, FTSE100, HANG SENG and NIKKEI 225. All 5 stock index futures and KSE100 index are arithmetic weighted indexes, where the weights are market capitalization. Analysis is based on daily and weekly hedge durations by using spot and futures return data for the period commencing from 1 st January 2003 to 31 st July 2008. Due to problems of sample size hedge durations of more than one week are not considered. Each daily return series consists of 1457 observations, out of which last 157 (from 1 st January 2008 to 31 st July 2008) are used to calculate out of sample (ex ante) hedging performance. Each weekly series consists of 292 observations, out of which last 31 (from 1 st January 2008 to 31 st July 2008) are used to measure ex ante hedging performance. The return series for each index is calculated as a logarithmic value change:

$R_t = \log V_t - \log V_{t-1}$ (2)

Where, R_t is the daily or weekly return on either the spot or futures position and V_t is the value of the index at time t.

Value is the daily or weekly closing value of all 6 indexes. All data was obtained from Datastream.

Two hedging strategies are considered. First, is the MVHR, and the second, is an extension of the first strategy by applying the theory of co-integration, formally known as Error Correction Model.

MVHR is estimated by regressing spot returns (KSE 100 in this case) on futures returns using historical information:

$RS_{t} = \alpha + bRF_{t} + e_{t}$ (3)

Where, RS_t is the return on KSE100 index in time period t; RF_t is the return on the futures contract in the time period t; e_t is the error term and α and b are regression parameters.

Value of b is obtained after running the above regression in e-views, which is the hedge ratio h^* shown earlier in equation 1. This hedge ratio is used in further calculation for determining risk reduction. Effectiveness of minimum variance hedge is determined by examining the percentage of risk reduced by the hedge (Ederington, 1979; Yang, 2001). Consequently, hedging effectiveness is measured by the ratio of the variance of the un-hedged position minus the variance of the hedged position, divided by the variance of the un-hedged position (Floros, Vougas 2006).

$Var(u) = \sigma^2_s$ (4)

$Var(h) = \sigma^2_s + h^2 \sigma^2_F – 2h\sigma_{S, F}$ (5)

Hedging Effectiveness (HE) = (Var(u) – Var(h)) / Var(u) (6)

Where, Var(u) is the variance on un-hedged position (KSE100); Var(h) is the variance on the hedged position; σ_S & σ_F are standard deviation on spot (KSE100) and futures returns respectively; h is the value of hedge ratio (b in equation 3); and $\sigma_{S, F}$ is the covariance between spot and future returns.

Error Correction Model (ECM) approach requires testing for co-integration. The return series are checked for co-integration by following a simple two step approach suggested by Engle and Granger. Consider two time series X_t and Y_t , both of which are integrated of order one (i. e. I(1)). Usually, any linear combination of X_t and Y_t will be I(1). However, if there exists a linear combination (Y_t – Ï•X_t) which is I(0), then according to Engle and Granger, X_t and Y_t are co-integrated, with the co-integrating parameter Ï•.

Generally, if X_t is I(d) and Y_t is I(d) but their linear combination (Y_t – Ï•X_t) is I(d-b), where b> 0 then X_t and Y_t are said to be co-integrated. Co-integration conjoins the long-run relationship between integrated financial variables to a statistical model of those variables (XYZ, 200N).

In order to test for co-integration, it is essential to check that each series is I(1). Therefore, the first step, is to determine the order of integration of each series. Order of integration is determined by testing for unit root by using Augmented Dickey Fuller (ADF) test. A variable X_t is I(1), if it requires differencing once to make it stationary. The null of unit root is rejected when probability is less than the critical level of 5%. Then the following OLS regression is estimated:

$$RS_t = \alpha + bRF_t + e_t$$

Where, variables are same as equation 3.

Empirical existence of co-integration is tested by constructing test statistics from the residuals of the above equation. If two series are co-integrated then e_t will be I(0). This is found by testing the residuals for unit root by using ADF test. The null of unit root is rejected if probability is less than 5%.

Once it is established that the series are co-integrated, their dynamic structure can be exploited for further investigation in step two. Engle and Granger show that co-integration implies and is implied by the existence of an error correction representation of the series involved. Error correction model (ECM) abstracts the short- and long-run information in modeling the data(XYZ, 200N). The relevant ECM to be estimated for generation of the optimal hedge ratio is given by:

j= 1

i= 1

$RS_t = \alpha e_{t-1} + \beta RF_t + \Sigma \gamma_i RF_{t-i} + \Sigma \delta_j RS_{t-j} + u_t$ (7)

Where, e_{t-1} is the error correction term and n and m are large enough to make u_t white noise; β is the hedge ratio. The appropriate values of n and m are chosen by the Akaike information criterion (AIC) (Akaike1974).

In short, returns on KSE100 are regressed on futures returns and residuals are collected by using OLS. ECM with appropriate lags is estimated by the OLS in the second stage.

Next phase is to determine the superiority of the two models MVHR and ECM, which were used to obtain the hedge ratios b and β respectively. This is achieved by conducting Wald Test of Coefficient on model (7). If anyone of the lags in model 7 turn out to be significant, then optimal hedge ratio obtained through model (7) will be superior then hedge ratio obtained through model (3). Hence, signaling the superiority of ECM over MVHR. The significance is tested by a hypothesis, where:

$H_o = C(1) = C(2) \ldots = C(i) = 0$

$H_1 = C(1) = C(2) \ldots = C(i) \neq 0$

The null is rejected if the probability of Chi-square statistic is less than the critical value of 5%.

Lastly, the superior hedge ratio will be used to determine ex ante performance. The hedging effectiveness of the superior hedge ratio will be based on the measure of risk reduction achieved through equation (6).