Deriving keplers laws of planetary motion

Business

Deriving Kepler's Laws Tanner Morrison November 16, 2012 Abstract Johannes Kepler, a world renowned mathematician and astronomer, formulated three of today's most in? uential laws of physics. These laws describe planetary motion around the sun.

Deriving these laws (excluding Kepler's First Law) will stress the concept of planetary motion, as well as provide a clear understanding of how these laws became relevant. 1 Kepler's First Law Kepler's First Law states: The orbit of every planet is an ellipse with the Sun at one of the two foci. 2 Kepler's Second Law

Kepler's Second Law states: A line joining a planet and the Sun sweeps out equal areas during equal time intervals. In more simpler terms, the rate at which the area is swept by the planet is constant (dA = constant). dt 2. 1 Derivation Of Kepler's Second Law To start this derivation, we will need to know how to ? nd the area that is swept out by the planet.

This area is equal to ? r A= rdrd? = 0 r2 ? 2 (1) 0 The position can be de? ned by the planetary motion. r = r cos ?? + r sin ?? i j (2) The velocity can then be found by taking the derivative of the position. r = (? r sin ? d? dr d? dr + cos ?)? + (r cos ? i sin ?)? j dt d? dt d? (3) As noted during the derivation of Kepler's First Law, h is a constant, due to the fact that r ? r is a constant. h = r ? r = constant To ? nd the constant vector h evaluate the determinate that is given by the cross product of r ? r . ? ? ? ? ? i j k h=? r cos ? r sin ? 0? dr d? dr d? ? r sin ? dt + d? cos ? r cos ? dt + d? sin ? 0 Once the determinate is evaluated it can be simpli? ed to h = r2 1 d? ? k dt (4) The magnitude of this vector being (the same). | h| = r2 d? dt (5) by the de? nition of h this value is a constant.

Recall that the area swept out by the planet can be described as. r A= rdrd? = 0 r2 ? 2 0 The area swept through a little change in time (dt) is then equal to r2 d? dA = dt 2 dt Notice dA dt (6) looks alot like h = r2 d? dt h dA = dt 2 Showing that a constant. 3 dA dt is constant. Showing that the area swept out by the planet is Kepler's Third Law Kepler's Third Law states: The square of the orbital period of a planet is directly proportional to the cube of the semi-major axis of its orbit. This derivation will show that 4 ? 2 a 2 b2 T2 = h2 3. 1 Deriving Kepler's Third Law From the derivation of Kepler's Second Law we know that h dA = dt 2 By using integration we can ? d the area swept out during a certain time interval (T), the period.

The fundamental theorem of calculus states that the integral of the derivative is equal to the integrand, T T dA = 0 h 2 dt 0 2 by simplifying we get the area of the planetary motion h T 2 A= (7) recall that A = ? ab, inputting this into our area equation we get ? ab = h T 2 Solving for the period (T), we get 2? ab h T= By squaring this period we get, 4 ? 2 a 2 b2 h2 T2 = (8) 2 Recall the directrix of an ellipse is (d = h) and the eccentricity of an ellipse is c c (e = GM). Multiplying these together and simplifying we get ed = 2 e h2 = eGM GM (9) Also recall that the square of half of the major axis of an ellipse is a2 = and the square of half of the minor axis is b2 = v Consider v a2 = e2 d2 (1 ? e2) 2 e2 d 2 (1? e2) . = a= e2 d2 (1? e2)2 Solving for a ed 1 ? e2 2 b a b2 e2 d2 (1 ? e2) = = ed a (1 ? e2) ed (10) Equating equations (9) and (10) yields h2 b2 = GM a Simplifying this we get h2 = recalling T 2 = 4? 2 a2 b2 , h2 b2 GM a (11) inserting the new found h we get T2 = 4? 2 a2 b2 a 4? 2 a3 = h2 GM GM (12) Showing that the square of the period (T 2) is proportional to the cube of the semi-major axis (a3). 3