Good example of research paper on research on mathematical proofs

Law, Evidence

What Mathematical Proofs are?

In mathematics, a proof is called a chain of logical reasoning, showing that at some set of axioms and inference rules of a true statement. Depending on the context, may refer to the formal proof of (built by the special rules sequence of statements recorded in a formal language) or text in natural language, in which, if desired, you can restore a formal proof. Provable statements called theorems in mathematics (in mathematical texts usually means that someone found proof; exceptions to this custom is mainly composed of works on logic, which explores the notion of proof); if neither approval nor its negation is not yet proved, that such a statement is called a hypothesis. Sometimes in the process of proving Theorem allocated evidence is less complicated statements, called lemmas.

According to Pólya, G. (1954), " formal proof by a special branch of mathematics is a theory of evidence. Formal proofs are almost never used, because human perception they are very complicated and often occupy a lot of space". Usually proof takes the form of text, in which the author, based on axioms and theorems proved earlier by logical means shows the truth of a statement. Unlike other sciences, mathematics empirical inadmissible evidence: all assertions proved extremely logical ways. In mathematics, the important role played by mathematical intuition and analogies between different objects and theorems; nevertheless, all these tools are used by scientists only when searching for evidence themselves evidence cannot be based on such vehicles. Evidence, written in natural language, may not be very detailed, based on the fact that prepared the reader he can recover detail. Strictness of proof is guaranteed that it can be represented as a record in a formal language (this happens when the computer checking evidence).

Erroneous evidence is text that contains logical errors, that is, one which cannot be restored by a formal proof. In the history of mathematics have been cases where eminent scientists published false " evidence", but usually their colleagues or themselves pretty quickly found the error. (One of the most often misunderstood proves the theorem - Fermat's Last Theorem. Until now, there are people who do not know that it has been proved and offering new false " evidence.") Can only be erroneous recognition of the " evidence" in a natural or formal language proof; formal proof cannot be wrong by definition.

In mathematics, there are unsolved problems, the solution of which scientists would love to find. Some of them can be found in the article " Hypothesis." For evidence of particularly interesting and important mathematical statements society prescribed premium.

Why Mathematical Proofs are Important?

The importance for a formal proof of the allegations is one of the main characteristics of mathematics as a deductive discipline, respectively, the notion of proof plays a central role in the subject of mathematics, and the availability of evidence and determine the status of the correctness of any of the mathematical results (Fallis, Don, 2002).

Throughout the history of mathematics understanding of the methods and techniques of admissible evidence varied considerably, mainly in the direction of greater formalization and more restrictions. A key milestone in the formalization of proof issue was the creation of mathematical logic in the XIX century and the formalization of its main means of proof techniques. In XX century was built the proof theory that studies proof as a mathematical object. With the advent of the second half of the XX century, an emphasis on the computers using methods of mathematical proof for the verification and synthesis of programs, and even structural correspondence was found between computer programs and mathematical proofs (compliance Curry - Howard), on the basis of which we have created tools for automatic proof. Basic techniques used in constructing proofs: direct proof, mathematical induction and its generalization, proof by contradiction, contrapositive, construction, brute force, establishing bijections, double counting; applications as mathematical proofs are also involved methods that do not give a formal proof, but to ensure the practical applicability of the result - the probability, statistics, approximate. Depending on the branch of mathematics or mathematical formalism used by schools, not all methods can be made unconditionally; in particular, the structural evidence suggests serious limitations.

What Proofs Consist of and Their Purpose

There are many different types of mathematical proofs. According to Franklin (2011), the most common are the following:

- direct proof
- induction
- contrary

- contrapositive
- construction
- running out of options
- bijection

Direct proof involves the use only of direct deductive considered true statements (axioms, previously proved lemmas and theorems), without the use of judgment with the denial of any allegations. For example, direct evidence deemed acceptable the following figures (in natural deduction notation):

,,

In some cases, indirect evidence, using arguments with the denial, particularly with respect to finite objects can be simply reduced to a straight with no loss of generality, but on the allegations of infinite set is not always the case, and with increasing values of constructive proofs in mathematics XX century considered important to find direct evidence for the allegations deemed proven, but indirect methods.

Inductive method to move from private to general statements, the most interesting when applied to an infinite set of objects, but its formulation and application differ greatly depending on the application (Franklin, 2011). Inductive method is the simplest mathematical induction, inference concerning the natural numbers, the idea that in approving a law for all natural numbers based on the facts of its implementation for the unit and follow the truth for each subsequent number in natural deduction notation:

Mathematical induction in a natural way can be used for any countable sets of objects, is considered a reliable and legitimate both in classical and in intuitionistic and constructive systems evidence.

A more complex issue is the possible extension of the inductive method to uncountable population. Within the framework of the naive set theory created the method of transfinite induction, which allows extending the inductive inference rule for any well-ordered sets scheme, similar to mathematical induction.

Proof by contradiction uses logical reductio ad absurdum reception and constructed as follows: in order to prove it is assumed that it is wrong, and then come to a deductive chain deliberately false statement, which according to the law of double negation of the truth of the conclusion of the original approval, notation natural deduction (Franklin, 2011):

.

Contrapositive proof uses the law of contraposition and is as follows: to prove the fact that the approval of A implies B is required to show that the negation of the negation of B follows A, in the symbolism of natural deduction:

.

For statements such as the existence theorems, which is formulated as a result of the presence of an object, for example, there exists a number satisfying any conditions, the most typical type of evidence - direct determination of the desired object using the methods of the corresponding formal system context or the corresponding section. Many classical existence theorems are proved by contradiction: reduction to the absurd assumption that the non-existence of the object with the specified properties, but such evidence is considered unhelpful and, accordingly, in intuitionistic and constructive mathematics for such allegations are only proof construction.

In some cases, to prove enumerates all possible options together with respect to which the above assertions (coarse search) or all possible options are broken down into a finite number of classes that represent special cases, and with respect to each of which the proof is carried out separately. As a rule, the proof method of exhaustion options, consists of two phases:

- establishing all possible special cases, and prove that no other special cases,
- proof of each particular case.

The number of options can be quite high, for example, to prove the four color conjecture bust took almost two thousand different options using a computer. The appearance of this type of evidence at the end of the XX century in connection with the development of computer technology, have raised the question of their status in mathematical science because of possible problems with verifiability.

Proof method of establishing the bijection is used to establish the allegations of the amount or structure of all or comparability conjunction with any other aggregate and consists in constructing a one - to-one correspondence between the studied set A and the set with the known properties of B. In other words, the proof of the allegations of a certain set of reduced to the proof of the construction of the bijection, possibly with additional constraints, with the collection for which this is known.

The simplest examples of bijective proof is a proof combinatorial allegations including combinations or sets the number of elements, more complex examples - establishing isomorphisms, homeomorphisms, diffeomorphisms bimorphism, whereby on the studied object or set of A transferred property already known object B, invariant with respect to the or special kind of bijections.

What Difficulties do Students Have With Proofs?

Efficiency of the process of teaching mathematics in our time is determined by many factors. The skill of the teacher's ability to manage the process of the formation of students' knowledge, developing their ability to think largely depends on whether the student creatively studied material. Its task is primarily to educate actively thinking person (D. Solow, 2004).

Acquiring knowledge of mathematics skills, students must learn how to conduct reasoned evidence to master such complex categories as definition, classification, analysis and synthesis skills to get inductive and deductive reasoning.

Often have to deal with such cases, where the student learns the course material, without thinking, fills his hand in the use of specific algorithms and has the laziness of mind that prevents him to consider the difficulties encountered.

Severely hampered the study of mathematics lack of habit to follow closely the chain of inferences to interpret them critically, the lack of adequate notice for completeness output units reasoning. Sometimes students do not only cope with bad Retrieving these links, but do not see the need in the logical proof.

In the proposed work was an attempt to develop a common methodology training students how to construct logically unmistakable evidence schemes, as well as give them the skills to work in a meticulous search of the justification for any more or less important step in the proof.

According to Velleman (2006), “ education evidence theory is often not effective enough. In mathematics it is clear that many children find it difficult in solving problems on the proof in justifying mistakes solution”. One of the reasons is poor lighting in school textbooks of different ways to prove that leads to memorization and formal Learning without critical reflection. Among other reasons, attention is drawn to the fact that the evidence of the data in the textbook, held only by synthesis. Advantages of this method are well known.

In learning theory the problem of analytical and synthetic method is often seen, excluding the time factor: today " teach" first on one task or theorem, tomorrow the second on a completely different problem. Testing will not be empty, and the synthesis will be meaningful if we these two methods will be considered as proof of a single process.

Works Cited

Pólya, G. (1954), Mathematics and Plausible Reasoning, Princeton University Press.

Fallis, Don (2002), " What Do Mathematicians Want? Probabilistic Proofs and the Epistemic Goals of Mathematicians", Logique et Analyse 45: 373–388.

Franklin, J.; Daoud, A. (2011), Proof in Mathematics: An Introduction, Kew Books, ISBN 0-646-54509-4.

Solow, D. (2004), How to Read and Do Proofs: An Introduction to

Mathematical Thought Processes, Wiley, ISBN 0-471-68058-3.

Velleman, D. (2006), How to Prove It: A Structured Approach, Cambridge University Press, ISBN 0-521-67599-5.