The relationship between a series of straight, non-parallel, infinite lines on a ...

Business

Aim: In this investigation I will explore the relationship between a series of straight, non-parallel, infinite lines on a plane surface and analyze the number of lines, maximum number of crossover points and open and closed regions.

I will investigate patterns that emerge from the collected data (relating to number of lines, the maximum number of crossover points and the maximum number of open and closed regions obtained). Method: I will use diagrams to illustrate my investigation, and use mathematical notation in the form of tables to describe the sequences that appear and apply what I have learnt about sequences to determine formulas or ‘ rules’ to predict the results for more lines. In the course work hand out we were given, we were presented with a diagram which had four lines, five cross-over points and a total of ten regions. For the purposes of my investigation I will start with 1 line and tabulate my findings (with regard to number of lines, the maximum number of crossover points and the maximum number of open and closed regions). I will redraw the diagrams adding one more line every time until I have a diagram with six lines.

I should then have enough information to be able to predict the results for a diagram with 7 lines with the use of the formulae, which I will find to summarise the rules for the sequences. Diagram 1: 1 Line (open regions are depicted with numbers)In this diagram there are: Number of Lines (n)Cross-Over PointsOpen RegionsClosed RegionsTotal Regions10202Diagram 2A: 2 Lines (open regions are depicted with numbers)In Diagram 2A the lines do not cross over each other, therefore no cross-over points are formed, and there are only 3 open regions. This diagram does not represent the maximum number of crossover points and the maximum number of open and closed regions. Diagram 2B: (open regions are depicted with numbers, crossover point is high-lighted with a red circle)Diagram 2 B represents the most regions and crossover points possible for two straight lines. Therefore I can deduce that in order to create the maximum number of crossover points and the maximum number of open and closed regions all the lines in the diagram must cross over every other line.

In this diagram there are: Number of Lines (n)Cross-Over PointsOpen RegionsClosed RegionsTotal Regions1020221404Diagram 3A: Diagram 3BIn Diagram 3A the lines do not cross over each other, therefore no cross-over points are formed, and there are only 4 open regions. Diagram 3A does not represent the maximum number of crossover points and the maximum number of open and closed regions. I will investigate this further by drawing the lines indifferent positions. In Diagram 3B there are more open regions than in 3A, but only one cross over point. Diagram 3B does not represent the maximum number of crossover points and the maximum number of open and closed regions, therefore, I continue the investigation. Diagram 3C: (all 3 lines cross each other, regions are depicted with numbers, crossover points are high-lighted with red circles)Diagram 3 C represents the most regions and crossover points possible for three straight lines.

Therefore I can deduce that in order to create the maximum number of crossover points and the maximum number of open and closed regions all the lines in the diagram must cross over every other line in the diagram. In other words, no two lines in the diagram can ever be parallel. I will apply this rule in the rest of the investigation. In Diagram 3C there are: Number of Lines (n)Cross-Over PointsOpen RegionsClosed RegionsTotal Regions102022140433617*Course work Diagram:*In the course work hand out we were given, we were presented with a diagram which had four lines, five cross-over points and a total of ten regions. The diagram we were presented with had two parallel lines and therefore all the lines in the diagram did not cross each other. It is interesting to note that by applying the rule discovered in Diagrams 2 and 3, when every line in the diagram crosses over every other line, there are in fact a maximum of six crossover points and a total of 11 regions.

Diagram 4: (all 4 lines cross each other, regions are depicted with numbers, crossover points are high-lighted with red circles)In this diagram there are: Number of Lines (n)Cross-Over PointsOpen RegionsClosed RegionsTotal Regions102022140433617468311Diagram 5: (all 5 lines cross each other, regions are depicted with numbers, crossover points are high-lighted with red circles)In this diagram there are: Number of Lines (n)Cross-Over PointsOpen RegionsClosed RegionsTotal Regions1020221404336174683115101061 6Diagram 6: (all 6 lines cross each other, regions are depicted with numbers, crossover points are high-lighted with red circles)In this diagram there are: Number of Lines (n)Cross-Over PointsOpen RegionsClosed RegionsTotal Regions10202214043361746831151010616615121022I will start this part of my investigation by looking for a recursive pattern (the way each term relates to the one before it) and use what I have learnt about sequences to create formulas to predict the results for a diagram with 7 (or ‘ n’) lines.

Some definitions: If (n) is the number of lines (where n can be any natural number) thenthe number of Open Regions is given by: OR(n)the number of Cross -Over Points is given by: COP(n)the number of Closed Regions is given by: CR(n)the number of Total Regions is given by: TR (n)Step 1To investigate the first pattern I will list the number of Open Regions: OR(n)By inspection, OR(1) = 2OR(2) = 4OR(3) = 6OR(4) = 8OR(5) = 10OR(6) = 12This sequence can also be written as: Terms (n)123456Value: Open Regions24681012First Difference+2+2+2+2+2I found that the values in the ‘ First Difference’ line are the same or ‘ constant’ number + 2. The terms are even numbers or multiples of 2. For each new (non-parallel) line that is added, there will be two more Open Regions. I can summarise this rule for working out even numbers with a formula like this: Thus, OR(n) = 2(n) by inspectionI can check my formula by using data I have already collected and know to be correct.

OR (2) using the formula 2(n) let n = 22(2) = 4 CORRECT! OR (3) using the formula 2(n) let n = 32(3) = 6 CORRECT! OR (5) using the formula 2(n) let n = 52(5) = 10 CORRECT! I predict: OR (7) using the formula 2(n) let n = 72(7) = 14To test the prediction I made I draw a diagram with 7 lines. Counting the open regions confirms the result of 14This diagram, where lines (n) = 7 and every line crosses every other line, (no lines are parallel) proves that the formula I used to predict the amount of Open Regions2(n) is correct because there are 14 Open Regions in this diagram, and that is what I predicted. Step 2: I will list the number of cross-over points to investigate the pattern in the second sequence. The number of Cross -Over Points is given by: COP(n)(n) is the number of lines (where n can be any natural number) thenBy inspection, COP(1) = 0COP(2) = 1COP(3) = 3COP(4) = 6COP(5) = 10COP(6) = 15This sequence can be written as: Terms (n)123456Value Cross-Over Points01361015First Difference+1+2+3+4+5Second Difference+1+1+1+1Since the values of the first difference are not the same, I find the second difference. I found that the values in the ‘ Second Difference’ line are the same or ‘ constant’. From what I have learnt in class, I know that the polynomial for this sequence of values is quadratic (an expression is quadratic if the highest power of the variable is 2).

It is of the form2an + bn + c = 0Investigation (i)To solve for the sequence I will find (b) where:(n) can be defined as the number of lines or terms(a) can be defined as 1/2 the second difference(c) can be defined as the zero term where the n term is equal to ‘ 0’let n = 1a = 1/2c = -1(1/2) x (1)2 + b x (1) – 1 = 01/2 + b – 1 = 0b = 1/2Substitute (b) as found above into the formula to check its accuracy for the first term in the sequence COP(1). b = 1/2n = 1a = 1/2c = -1(1/2) x (1)2 + (1/2) x (1) – 1 = COP(1)(1/2) + (1/2) -1 = COP(1)0 = COP(1) – which is correctInvestigation (ii)To solve for the sequence I will find (b) where:(n) is the number of lines or terms(a)is 1/2 the second difference(c)is the zero term where the n term is equal to ‘ 0’let n = 2a = 1/2c = -1(1/2) x (2)2 + b x (2) – 1 = 0(1/2) x (4) + b x (2) – 1 = 02b = -1b = -1/2Substitute (b) into the formula to check its accuracy for the second term in the sequence COP(2).(1/2) x (2)2 + (-1/2) x (2) – 1 = COP(2)(1/2) x (4) + (-1/2) x (2) -1 = COP(2)2 + (-1) -1 = COP(2)0 = COP(2) – which is not correctIt would seem that the formula: an2 + bn + c = 0 does not work for the sequence COP(n). However I have noticed another possibility if I were to rearrange the formula using the values for (a), (b) and (c) that I found in Investigation (i)As before let: b = 1/2 (as found in Investigation (i))a = 1/2 (can be defined as 1/2 the second difference)c = -1 (can be defined as the zero term where the n term is equal to ‘ 0’)COP (n) = 1/2 n2 + 1/2 n -1= n2 + n – 1= 1/2n (n+1) – 1= n(n+1)/2 – 1COP (2) is the second term of the sequence let n = 2COP (2) = 2 (2 +2)/2 – 1= 8/2 – 1= 4 – 1= 3COP (2) is not 3. Therefore this formula does not work.

Investigation (iii)I will investigate a new formula for the sequence of numbers. If I start the sequence with 1. Add 2 to get 3. Add 3 to get 6. Add 4 to get 10.

Continue the pattern to get: 1, 3, 6, 10, 15, 21, 28, 36, 45, …I notice that these numbers are triangular numbers. The terms of this sequence or ‘ triangular numbers’ increase in a way that can be represented by triangular patterns of dots like this: COP(2) COP(3) COP(4) COP(5) COP(6)1 3 6 10 15The formula for producing them is: n(n + 1)/2I will apply this formula to the sequence COP (n)[where (n) is the number of lines in the diagram]COP(2) let n = 22 (2+1)= 2= 62= 3COP (3) let n = 33 (3+1)= 2= 122= 6I can check that the formula is true for triangular numbers by looking at the dot pattern diagrams above.

But the sequence for Cross-over Points is: COP(1) = 0COP(2) = 1COP(3) = 3COP(4) = 6COP(5) = 10COP(6) = 15COP (2) is not equal to 3 and COP (3) is not equal to 6. Therefore this formula does not work. Investigation (iiii)The same problem keeps emerging for all the formulas for Cross-over Points I have tried so far. However I can see that the sequence for COP (n) and Triangular numbers are very similar. The only difference is this: The first term of the sequence COP (1) is: 0.

The first term of a sequence of triangular numbers is: 1. When I looked at Diagrams 1- 6 again it became clear that when no lines are parallel, each new line intersected exactly once with each previous line. Thus, when the nth line is added, it makes (n-1) new intersections or cross-over points. I decided to incorporate this concept into the formula I have for discovering the nth term in a sequence of triangular numbers. So instead ofn (n+1) n (n-1)2 I'll try 2 [where (n) is the number of lines in the diagram]To test this formula I will substitute some known terms into the equation.

COP(2) let n = 22 (2-1)= 2= 22= 1 CORRECT! COP (3) let n = 33 (3-1)= 2= 62= 3 CORRECT! COP (6) let n = 66 (6-1)= 2= 302= 15 CORRECT! I predict: COP(7) using the formula n (n-1)2[where (n) is the number of lines in the diagram]let n = 7= 7 (7-1)2= 422= 21To test the prediction I made, I draw a diagram with 7 lines. This diagram, where lines (n) = 7 and every line crosses every other line, (no lines are parallel) proves that the formula I used to predict the amount of cross-over points in the sequence COP (7) is correct because there are 21 cross-over points in this diagram, and that is what I predicted. Step 3: I will list the number of Closed Regions to investigate the pattern in the third sequence. The number of Closed Regions is given by: CR(n)(n) is the number of lines in the diagram (where n can be any natural number) thenBy inspection, CR(1) = 0CR(2) = 0CR(3) = 1CR(4) = 3CR(5) = 6CR(6) = 10This sequence can be written as: Terms (n)123456Value Closed Regions0013610First Difference+0+1+2+3+4Second Difference+0+1+1+1It seems to me that this series is identical to the sequence COP(n), except that the start point is moved back by one step. Thus, if COP(n), = n (n-1)2when I looked at Diagrams 1- 6 it becomes obvious that when no lines are parallel, each new line intersected exactly once with each previous line. Thus, when the nth line is added, it makes (n-1) new intersections or cross-over points and (n-2) closed regions.

I can show that moving the start point will affect the formula in the following way: CR(n) = (n-1) (n-2)2, for the real positive integers above 1. I can expand the polynomial so that it becomes: CR(n) = (n2 -3n + 2)2To test this formula I will substitute some known terms into the equation. I can use this sequence to check that the answers I get when using the formula are correct. CR(1) = 0CR(2) = 0CR(3) = 1CR(4) = 3CR(5) = 6CR(6) = 10(i) CR(1) let n = 1CR(1) = (12 – 3(1) + 2)2CR(1) = 1 – 3+ 22CR(1)= 0 CORRECT!(ii) CR(2) let n = 2CR(2) = (22 – 3(2) + 2)2CR(2) = 2 – 6+ 22CR(2)= 0 CORRECT!(iii) CR(3) let n = 3CR(3) = (32 – 3(3) + 2)2CR(3) = 9 – 9+ 22CR(3)= 1 CORRECT!(iv) CR(4) let n = 4CR(4) = (42 – 3(4) + 2)2CR(4) = 16 – 12+ 22CR(4) = 62CR (4) = 3 CORRECT! CR(6) let n = 6CR(6) = (62 – 3(6) + 2)2CR(6) = 36 – 18+ 22CR(6) = 202CR (6) = 10 CORRECT! I predict: CR(7) using the formula CR(n) = (n2 -3n + 2)2[where (n) is the number of lines in the diagram]let n = 7CR(7) = (72 -3(7) + 2)2CR(7) = 49 -21 + 22CR(7) = 302CR(7) = 15To test the prediction I made, I draw a diagram with 7 lines. This diagram, where lines = (n) = 7 and every line crosses every other line, (no lines are parallel) proves that the formula I used to predict the amount of closed regions in the sequence CR (7) is correct because there are 15 closed regions in this diagram, and that is what I predicted.

Step 4: I will list the number of Total Regions to investigate the pattern in the fourth sequence. The number of Closed Regions is given by: TR(n)(n) is the number of lines in the diagram (where ‘ n’ can be any natural number) thenBy inspection, TR(1) = 2TR(2) = 4TR(3) = 7TR(4) = 11TR(5) = 16TR(6) = 22From looking at the information I gathered when looking at Diagram 6 I noticed the following: Number of Lines (n)Cross-Over PointsOpen RegionsClosed RegionsTotal Regions10202214043361746831151010616615121022Open Regions + Closed Regions = Total RegionsBased on this observation, I will investigate the following theory: Adding the two formulas I have found for OR(n) (Open Regions) and CR (n) (Closed Regions) together, should create an accurate formula to define the sequence TR(n)OR (n) is defined by the formula: 2(n)CR (n) is defined by the formula: (n2 -3n + 2)2Let TR(n) = 2(n) + (n2 -3n + 2)2To test this formula I will substitute some known terms into the equation. I can use the known sequence for TR (n) to check that the answers I get when using the formula are correct.(i)TR(1) let n = 1TR(1) = 2(1) + (12 -3(1) + 2)2TR(1) = 2 + (1-3 +2)2TR(1) = 2 CORRECT!(ii)TR(2) let n = 2TR(2) = 2(2) + (22 -3(2) + 2)2TR(2) = 4 + (4-6 +2)2TR(2) = 4 CORRECT!(iii)TR(3) let n = 3TR(3) = 2(3) + (32 -3(3) + 2)2TR(3) = 6 + (9-9 +2)2TR(3) = 7 CORRECT!(iv)TR(6) let n = 6TR(6) = 2(6) + (62 -3(6) + 2)2TR(6) = 12 + (36-18 +2)2TR(6) = 22 CORRECT! I predict: TR(7) using the formula TR(n) = 2(n) + (n2 -3n + 2)2[where (n) is the number of lines in the diagram]let n = 7TR(7) = 2(7)+ (72 -3(7) + 2)2TR(7) = 14 + 49 -21 + 22CR(7) = 14 + 302CR(7) = 14 + 15CR(7) = 29To test the prediction I made, I draw a diagram with 7 lines. This diagram, where lines = (n) = 7 and every line crosses every other line, (no lines are parallel) proves that the formula I used to predict the amount of total regions in the sequence TR (7) is correct because there are 29 total regions in this diagram, and that is what I predicted.