# The miller and modigliani capital structure irrelevance theorem finance essay



Contrary to Modigliani and Miller (1958, MM hereafter), Capital Structure is not irrelevant when we consider a firm with a dividend payout policy. This article extends the MM capital structure theorem by relaxing the full payout assumption and introducing retention policy. The theoretical contribution shows that it is possible to verify the theorem when we suppose an investor who exchanges his initial holding for another portfolio composed of consumption and investment. The empirical analysis of this new approach is based on a data set of the USA Electric Utilities and Oil companies for the period 1990-1998. The results show that the relationships between leverage and firm value are significantly affected by the firm's payout ratio.

#### 1. Introduction

Miller and Modigliani's (1958) irrelevance theorem is one of the important and puzzling issues in modern corporate finance theory [1], which has challenged the traditional view[2], that an optimum leverage exists. The main source of the puzzle stems from the fact that financial research don't seem to explain the firm financing behaviour as we attempt to reconcile the MM theory with the evidence(Myers 1984, Gordon1994, Rajan and Zingales1995). The MM theorem(proposition I) has shown that under a perfect market hypothesis the market value of any firm is independent of its capital structure (Stulz2006). This fundamental proposition explicitly indicates that the aptitude of investors to engage in personal or "homemade" leverage is sufficient to ensure that corporate leverage in itself cannot modify the total market value of the firm [3]. In other words, the theorem provides conditions under which arbitrage by individuals keeps the value of the firm depend only on cash flow generated by the investment

policy. Literature about the validity of the MM-proposition is discussed about whether investors can really accomplish the required conditions of the arbitrage method without changing the overall value of the company. In this context, many authors have shown the inadequacy of the theorem when variables that deal with the real world are introduced.

Following the seminal paper of MM (1958), most theories have been put forward in corporate finance to reconcile the shortcomings of the irrelevance theorem with variables that explain the firm's choice of capital structure. According to the previous debate, criticism against this theorem can be grouped in two types of arguments: on the one hand, there are papers which deal with the limitations of the arbitrage conditions; on the other hand, there are studies which analyze the effect of market imperfections on the firm's choice of capital structure. Despite the importance of these interventions, we note that all of the limitations deal with the explicit assumptions used by MM, but none deals with the critiques of the MM's implicit assumptions. More recently, DeAngelo and DeAngelo (2006, DD hereafter) have challenged MM's irrelevance dividend policy. Dealing with this alternative of earnings as fully distributed, these authors have showed the irrelevance of the MM dividend irrelevance theorem when MM's assumptions are relaxed to allow retention. As DeAngelo and DeAngelo(2006, page 294) wrote "When MM's assumptions are modified to allow retention with the NPV of Investment policy fixed, a firm can reduce its value by paying out less than the full present value of FCF, and so Payout policy matters and Investment policy is not the sole determinant of value ". According to DD(2006), the MM's irrelevance theorem forces firms to choose only among dividend policies that

distribute the full present value of free cash flow(FCF) to shareholders.

Distributions below the totality of earnings are ruled out by the implicit hypothesis.

Dealing with this alternative of fully-distributed earnings, MM(1958) used the same hypothesis in the development of the irrelevance of capital structure... As pointed by the authors ".... as will become clear later, as long as management is presumed to be acting in the best interests of the stockholders, retained earnings can be regarded as equivalent to a fully subscribed, pre-emptive issue of common stock. Hence, for present purposes, the division of the stream between cash dividends and retained earnings in any period is a mere detail." MM, 1958 p266. However, MM(1958) failed to recognize that proposition I implies that firms distribute all their cash flow to shareholders without paying any attention to their retention policy. This paper constitutes a new extended proof of the MM theorem by not considering the hypothesis of earnings as fully distributed. We will show that it is possible to verify the theorem when we suppose an investor who exchanges his initial holding for a mix of consumption and investment. The rest of the paper is organized as follows: in the next section, we demonstrate the irrelevance of the MM's capital structure irrelevance when earnings are not fully distributed. We propose the possibility of extending of the MM theorem. Furthermore, we show that the two firms are not forced to distribute their full earnings; and the irrelevance is hold in the presence of the mix of investment and consumption. Section III describes the data set, introduces the methodology, examines the hypothesis of the variables and investigates whether the empirical Modigliani-Miller capital

structure irrelevance is influenced by dividend payout ratio. Section IV provides some concluding remarks.

## 2. How do we reconcile MM's capital structure irrelevant theorem with the firm's payout choice?

2. 1 The failure of the MM theorem when earnings are not fully distributed.

As indicated by Rubinstein (2003), "the law of the conservation of investment value" of MM(1958) was anticipated by many studies (Fisher (1930), Williams[5] (1938), Durand (1952); Morton (1954) for examples) but none of these authors have used arbitrage mechanism to prove the invariance of the cost of capital under changes in leverage. The MM's theorem demonstrates that under certain hypothesis of market conditions, the value of the firm is independent of its debt-equity ratio and is given by capitalizing the expected return generated by its assets. This model can be expressed as:

for any firm j in class k (1)

Where V stands for the market value of the firm, S for the market value of its common shares, D for the market value of its debts, X for its expected earnings before interest on its assets, for the capitalization rate "appropriate to its class".

The analysis of the MM's arbitrage steps shows the implicit hypothesis of full payout ratio which plays a crucial role in the model. The MM's capital structure irrelevance theorem constrains firms to distribute all of their earnings. In particular, we note that the validity of the proof developed by

MM is based on this implicit assumption. MM(1958) consider (see MM(1958) pages 269-270) the return of the investor Y as a fraction of the net income available (X-rD for levered firm and X for unlevered firm) for the stockholders.

(2)

Where: is the return of the investor before arbitrage process, L is levered firm and U is Unlevred firm and is fraction of the total outstanding shares owned by the investor. Obviously, MM(1958) confuse artificially return of the investor(dividend return) and net income which should be distributed between dividend and retention. MM(1958 page 266) assert that "the division of the stream between cash dividends and retained earnings in any period is a mere detail".

When we derive the MM capital structure theorem for firms that are not distributing all their earnings as dividends, it follows a non-adequacy of the arbitrage operations, a non-proof of the irrelevance model. Table I shows the two cases used by MM(1958) when we introduce a level of payout different from 100%. Therefore, when we use the same arbitrage as MM(1958), we must then admit that the two firms distribute all the available income to verify the leverage irrelevance proposition. As will be shown later, this assumption can modify the validity of the MM theorem. To justify this thesis, we suppose the same steps of the MM first proposition but with a slight difference: here we suppose that firms are not constrained to distribute all of their earnings. This means that we introduce in the arbitrage reasoning the payout ratio (PR) as a new variable. Table I below shows that MM theorem is

not verified. The difference between returns (before and after arbitrage operations) is not the same as showed by MM (1958).

## Table I. The irrelevance of the MM capital structure irrelevance when payout ratio is different from 100%

First possibility : VL > VU

Second possibility: VU > VL

First stage : the initial return of the investor YL

### **Second Stage:**

Arbitrage process

- Sold his initial worth of the firm L
- Borrows an additional amount dL with the same interest rate r
- Acquired new shares of the firm u

sold his initial worth of the firm U

Acquired new shares of the firm L

Acquired new bonds b of the firm L

Third stage: the return of the investor YU

#### Final stage:

Difference of earnings

â^†Y= YU -YL

#### **Interpretations**

It is not possible to verify the MM results when we introduce the hypothesis of payout ratio different from 100%, the difference of returns will depend on the all components of the equation. When we pose PRL= PRU= 1, it is easy to obtain the same difference of returns as MM(1958):

or

Notes: Using the MM formulation, we consider two firms L and U, for which the expected return is the same XL = XU = X. Company U is financed entirely by stock SU and company L by stock SL and debt D. The market value of each firm is then VU = SU and VL = SL + D, We denote PRL and PRU the payout ratios of the levered and unlevered firms (MM 1958 suppose PRL = PRU = 100% all expected return is distributed). SL = SL, SU = SU denote the value of shares owned respectively by an investor in the levered and unlevered firm with a fraction

2. 2 The possibility of extension; The two firms are not obliged to distribute all their income: the mix of investment and consumption solution.

- X. Company U is financed entirely by stock SU and company L by stock SL and debt D. The market value of each firm is then VU = SU and VL = SL + D.
- st Case 1: we suppose the value of the levered firm VL , to be greater than that of the Unlevered firm VU ( ).

We denote respectively, PRL and PRU the payout ratios of the levered and unlevered firms (MM 1958 page 269) suppose PRL = PRU = 100% all expected return is distributed).

- First stage (initial return): consider an investor who owns sL dollars' worth of the stock in the company L representing a fraction of the total outstanding shares SL, where sL= SL. His return YL can be written as:

(3)

The return from this portfolio, denoted by YL, will be a fraction of the income distributed for the stockholders of company L, which equals the multiplication of the payout ratio PRL by the difference between to total return X and the interest charge r DL. Where, r is the interest rate which the firm pays on its debt D.

- Second Stage (Arbitrage process): now suppose that an individual investor who adjusts his own personal leverage in order to increase his profits. He makes the following operations:
- (a ) Sold his worth sL of the company L and he divided it as follows: (i) he partially invested an amount IU = PRL. sL (which equals: IU = PRLSL) in

acquiring shares (ii) he consumes the remainder CL= (1-PRL)SL. where sL= IU + CL.

- (b) Borrowed an additional amount .
- (c) Acquired an amount of the shares of the company U. He could so by using the amount IU from the sales of his initial holding and the amount d from borrowing.
- Third Stage (the new return): the income of the investor ((i) who holds sU dollars' worth of the shares of the company U (ii) and who must pay interest of personal debt d would be:

(4)

- Last Stage: Arbitrage profit: Comparing (4) with (3) we obtain:

(5)

Thus, under this approach we can distinguish two situations:

First situation: If PRU = PRL = 1 then we find the same result as obtained by MM (1958 page 270).

(6)

Second situation: We can also verify the same result of MM(1958 page 270) without the hypothesis of PRU = PRL = 1, we can simply assume PRU = 1, while the payout ratio of the levered firm PRL is likely to vary between 0% and 100%, we get then:

(7)

(8)

From equation (7), we conclude that as long we must verify, so that it pays shareholders of corporation L to sell their investments, by this means decreasing SL and hence VL, and replace them with a mix of consumption and portfolio investment, which contains shares of the unlevered firm and personal debt, thereby growing SU and thus VU. This arbitrage process will be finished when equilibrium restores the stated equalities between the values of the two firms.

- \* Case 2: we suppose the value of the unlevered firm VU , to be larger than that of the Levered one VL ( ).
- First stage: The return of the investor who holds sU dollars of shares of company U representing a fraction of the total outstanding stock SU . Where

The return from this portfolio denoted by YU will be a fraction of the income distributed to shareholders of the unlevered firm U.

- Second stage: suppose that the investor exchanges his initial holding in U by another portfolio in the levered firm L. The arbitrage process with consumption behaviour will take the following form: the investor sold his worth of company U: and divided it as follows:
- (i) He invested partially of the shares of the company L
- (ii) He invested also of bonds of the company L

(iii) The remainder will be consumed.

From IL and IB, we can write respectively:

- Third stage: The return of the investor (i) who holds IL dollars worth of the shares of the company L (ii) and who holds IB dollars worth of bonds of the company L.

(9)

- Last stage: Arbitrage profit: comparing YL (from 9) with YU (from 8) we obtain:

(10)

In order to get a profitable arbitrage opportunity for the investor, we must consider a positive difference of returns. Analysing equation (10), we can easily formulate two possibility of payout ratio:

In the first, if we suppose a full earning model for the two firms (PRL = PRU = 1), therefore we will obtain the same results as showed by MM(1958) (page 270). According to this situation, equation (10) can be written as:

(11)

In the second, the MM's results can also be obtained if we just assume a full earnings for levered firm PRL= 1 while the payout ratio of the unlevered firm PRU is likely to vary between 0% and 100% implying that the firm can use a payout policy, which is not restricted to full earnings. Such a representation is written as:

(12)

In this context, it is also important to show that as we must obtain , hence it

pays the shareholders of company U to sell their holdings and substitute

them with a mix of consumption and portfolio investment, which contains

shares and bonds. If, all investors in firm U will accomplish the three stages

below, decrease the value of the unlevered firm U and increase the price of

the levered firm L. This switching process will be over when equilibrium

restores the stated equalities between the values of the two firms.

From these demonstrations (case 1 and case 2) we can conclude that we are

not compelled to suppose that the two firms distribute all of their returns. In

other words we can make arbitrage process merely by considering that the

overpriced firm (levered firm L in the first case and unlevered firm U in the

second case) has a payout ratio PR which is not restricted to be 100% of the

earnings. The table below summarizes the theoretical findings.

Table II: the MM's arbitrage and the payout hypothesis

#### **Conditions**

#### Conclusions

MM's arbitrage conditions without dividend payout

MM's(1958) irrelevance theorem

MM's arbitrage conditions with a payout ratio

Failure of the MM's proof

MM's arbitrage conditions with a payout ratio and consumption hypothesis

Proof of the MM's irrelevance theorem(Extension)

#### 3. The Empirical Analysis

The previous part of this paper provides a new extension of the relationship between firm value and capital structure when the firm has a payout policy. In this section, we attempt some possible empirical tests. The central issue is, whether or not the leverage ratio affects firm value when earnings are not fully distributed?.

Modigliani and Miller (1958) have taken two samples of 43 electric utilities during 1947-1948 and 42 oil companies during 1953. The data are provided respectively by two studies conducted by Allen (1954) and Smith (1955); and they estimated the weighted average cost of capital (wacc) according to the financial leverage of the firm. The regression form of the model was:

(13)

Where wacc is the weight cost of capital approximated by X /V , here X is the expected return net of taxes, V is the market value of all securities and the financial leverage of the firm measured by the ratio D/V, where D is the market value of Bonds and preferred stock. The results of the tests (as shown MM(1958page 282) are favourable to Modigliani and Miller (1958)'s hypothesis. The values of the correlations coefficients are small and not statistically significant. Weston (1963) criticizes Modigliani-Miller empirical result. In particular, he assumes that the lack of effect of capital structure on the overall value of the firm is due to deficiency of the approach to take

account of other factors that may be influencing the firm's cost of capital.

Contrary to MM, the author shows in the empirical tests that leverage is

correlated negatively with firm value in the presence of the hypothesis of
earnings growth.

#### 3. 1 Data and Methodology

In order to conduct an empirical analysis similar to MM's, we have collected data on the same sectors from the same country as done by Modigliani and Miller 1958. The data we use are annual standardized financial information of US firms observed in the period 1990-1998. Our sample is formed by two sub samples: from the Electric sector we use 256 companies, and from the oil sector we take 223 companies. These data were obtained from the Worldscope Database (SIC Code 13 and 49). Contrary to Weston(1963), we consider the hypothesis of risk-class can be verified in the oil industry and the electric sector (as supposed by MM 1958).

According to MM(1958), a linear model was constructed to explain the relationship between leverage and the firm value. The variables used in our regressions are constructed (see table III) as the same way as presented by these authors. The corresponding models used by MM(1958) are: For Model 1: see MM(1958) page284 (note 38), for model 2, see MM(1958) page282; For Model 3, see MM(1958) page284 (note 39); For. With regard to the basic capital structure irrelevance theorem to be estimated; we propose three regression models as follows:

Model 1: (14)

Model 2: (15)

Model 3: (16)

Where wacc is the weighted average cost of capital; Leverage 1: first measure of leverage; ML1: modified leverage 1; Value: the ratio of the firm value; , ER: earnings ratio; DR debt ratio.

The purpose of model 1 is to test the effect of leverage (as measured by Debt ratio DR) on firm value, while the Model 2 and model 3 test the effect of leverage (measured by Leverage1) on the cost of capital (measured by WACC). The variable ML1(modified leverage 1) is included in model3 to test the U-shaped hypothesis that the coefficient e of this variable should be significant and positive to confirm the traditional view, and not significantly different from zero to confirm the irrelevance theorem.. Note also that according to our approach the correlation between these variables should be different from zero.

To test the validity of the MM's proposition when earnings are not fully distributed, we alternatively estimate all the above regressions in the absence (model MM58 and the model MM58supp) and the presence of the payout ratio. We validate this last alternative in two steps: In the first step, we test the models for all firms (model MMExt). In the second step, we test the models for subsamples: First Quartile sample (Firm's Payout ratio is less than 25%), Second Quartile sample (firm's payout ratio is between 25% and 50%), Third Quartile sample (firm's payout ratio is between 50% and 75%), and Fourth Quartile sample (firm's payout ratio is more than 75%). The tableIII below reports the different measures of variables and their predicted effects.

## Table III. Measures of variables and predicted signs

Variables	1	8
Symbol		
Measure		
MM Hypothesis		
Our Hypothesis		
Dependants variables		
Weighted average cost of capital		
WACC		
X/V		
Firm value ratio		
Value		
V/A		
The explanatory variables		
First measure of leverage		
Leverage 1		
D/V		
Zero effect		

# Significant effect Modified Leverage 1 measure ML1 D. D/V. S Zero effect Significant effect Earnings ratio ER X/A Debt ratio DR D/A Zero effect Significant effect Payout ratio Payout Div/NI

Not tested

Significant effect

Notes: the table reports the different measures of variables where V: firm value= market value of equity S +market value of debt D, X: Earnings before interest and Taxes (EBIT), A: is the value of the total assets, NI net income. ML1 modified leverage 1 measure =  $(D/V)^2/(1-D/V)$ . We measure the value of the Debt D by the amount of total liabilities.

#### 3. 2 Descriptive statistics

As indicated in Table IV, the descriptive statistics shows that the average value of cost of capital is 5. 92% for electric utilities and 4. 48% for oil companies[6]. On average, we have a leverage ratio of 51. 79%(37. 85%), this measure is 62% (50. 2%) when we use total assets as deflator. The average firm has a value ratio of 1, 38 for electric utilities which is much weaker than those of oil companies (1, 99). For these firms, earnings ratio ranges from 0% to 2. 7% for electric utilities (0% to 66% for oil companies). In terms of net income, the average value of payout is more important for electric utilities (45%) ranging from 0% to 99, 9%, than those of oil companies (16%). These results show that the division of the stream between cash dividend and retained earnings in any period is not a mere detail as supposed by Modigliani and Miller (1958 page 266). None of firms in the two samples and during the whole period (1990-1998) has distributed the totality of its income. For the normal distribution of the series around the mean (see table IV), all of the distributions of the variables are not symmetric since their skewness values are different from zero. This

conclusion is also verified by the values of the Kurtosis which are quite different from 3.

# Table IV. Descriptive Statistics of Variables (256 Electric Utilities and 223 Oil Companies)

Variables	<b>-</b>		
Sample			
Mean			
Minimum			
Maximum			
Std. Dev			
Skewness			
Kurtosis			
Obs			
WACC			
Elect			
0. 05924			
0. 00000			
0. 29090			

0. 03188
0. 292328
6. 376099
2304
Oil
0. 04481
0. 00000
0. 69582
0. 05448
4. 75993
42. 0526
2007
Leverage1
Elect
0. 51796
0. 01573
0. 99416

0. 17873	
-0. 46925	
3. 36365	
2304	
Oil	
0. 37857	
0. 0000	
0. 98237	
0. 21714	
0. 20952	
2. 36431	
2007	
Value	
Elect	
1. 38155	
0. 09087	
9. 77112	

0. 82268	raper Example
5. 51989	
45. 7871	
2304	
Oil	
1. 99172	
0. 14447	
138. 56	
5. 40308	
18. 7716	
397. 615	
2006	
ER	
Elect	
0. 07353	
0. 0000	
0. 027612	

0. 04158
0. 77790
7. 94274
2304
Oil
0. 06418
0. 0000
0. 664303
0. 06683
2. 104262
11. 546
2007
<b>DR</b> Elect
0. 62322
0. 02761
0. 995066

The miller and modigliani capital struct Paper Example
0. 14891
-0. 9991
4. 78983
2304
Oil
0. 50220
0. 0000
0. 9978
0. 22065
-0. 2593
2. 4847
2006
ML1 Elect
1. 34913
0. 000252
169. 346

6. 6480	3	•	·	
17. 3645				
344. 950				
2304				
Oil				
0. 61298				
0. 0000				
23. 2454				
1. 5346				
8. 6309				
103. 96				
2006				
Payout Elect				
0. 45169				
0. 00000				
0. 99980				

- 0.35978
- -0. 15569
- 1.40417
- 2304

Oil

- 0.16381
- 0.0000
- 0.9991
- 0.27721
- 1.50967
- 3.90646

2006

## 3. 3 The effect of Leverage on the firm value (model 1)

The MM(1958)'s theorem is confronted with our hypothesis in order to know the crucial effect of payout ratio on the sensitivity of firm value to leverage. If our prediction is true, we should find a significant coefficient of leverage ratio, otherwise the MM's view should be confirmed. As indicated in table V, estimates result shows that coefficients of earning ratio (ER) and debt ratio (DR) are significantly different from zero, which fails to support the MM's

view. Since our results, as presented below, demonstrate that the coefficient of debt ratio is significantly negative and contrary to the traditional view. We prefer to give more explanations of this relationship based on the presence of the payout policy. The latter has a negative influence on the two samples (see Model MMExt, table V) which is in the opposite direction as obtained by the cost of capital regressions (see tableVI). There are two main explanations for this result:

According to Brigham and Gordon(1968), the relationship between stock price and leverage depends on the association between R (return on assets and investment) and i (the rate of interest which the firm pays on its debt), not on the level of Leverage L. This can be written as:

(16)

Where E is the book value of the common equity per share, k is the rate at which dividend is discounted. It is evident, when R is less than i, the leverage effect on stock price P will be negative. Furthermore, the negative influence of the dividend ratio on the firm value confirms the leverage impact when the return on investment is less than the cost of debt. This means that firms experiencing lower rate of investment tend to use funds from internal and external resources to display higher payout ratio.

The leverage measure is not the same: in Wacc regression, this variable is measured by debt on firm value (D/V), while in firm value regression (Value), the debt ratio is measured by debt on total Assets (D/A). The fact that both variables are divided by different deflators may be affected by a random

disturbances of the market value of the firm. This bias correlation is not observed in the firm value regression.

According to Modigliani and Miller (1958), the constant term in the previous regression should give more information on the value of the unlevered firm. As shown in table IV below, the estimated coefficient of this variable is not only significantly different from zero, but is quite positive and greatly relative to the coefficient of the debt ratio. This conclusion is confirmed for the two samples with large values for the oil companies.

# Table IV. Directs Pooled Least-Squares Estimates of the effects of leverage on the firm value

Coefficients of

Regressions

Sample

**Constant** 

ER

DR

**Payout** 

AdR<sup>2</sup>

Obs

MM 58

Elect

1. 893a	
-0. 158a	
-0. 805a	
_	
0. 025	
2304	
Oil	
2. 464a	
-6. 730a	
-0. 668	
<del>_</del>	
0. 048	
2007	
MM Ext	
Elect	
1. 963a	
-0. 131a	
-0. 466a	

-0. 625a		•	
0. 095			
2304			
Oil			
2. 465a			
-6. 703a			
-0. 642			
-0. 086			
0. 048			
2007			
First Quartile			
Elect			
1. 969a			
-0. 133b			
-0. 412c			
_			
0. 005			

801	
Oil	
2. 342a	
-7. 490a	
-0. 286	
_	
0. 052	
1440	
Second Quartile	
Elect	
1. 465a	
2. 650a	
-0. 554a	
_	
0. 187	
216	
Oil	

1. 659a

The fillier and modigital capital struct Taper Example
-0. 197
-0. 501a
0. 033
279
Third Quartile
Elect
1. 206a
1. 823a
-0. 249a
<del>_</del>
0. 096
738
Oil
1. 224a
3. 229a
-0. 055

_ 0. 113
207
Fourth Quartile
Elect
1. 080a
1. 809a
-0. 105
_ 0. 102
549
Oil
7. 197a
0. 983
-9. 064a
_ 0. 676
72

Notes: a, b and c indicate significance at the 1%, 5%, and 10% levels respectively.

## 3. 4 The effect of leverage on the cost of capital (model 2 and Model 3)

According to Modigliani and Miller's proposition I: " the average cost of capital Wacc (Xt/V) should tend to have the same value independently of the degree of leverage" MM (1958, page281). In other words, the leverage's coefficient parameter in the Wacc regression should be insignificant and statistically equal to zero. The results of the MM model tests are shown in table V (models: MM58 and MM58supp). According to this table, the MM hypothesis is only verified in the oil sample, while leverage in the electric utilities has a negative and significant effect (coefficient is equal -0, 1162) on the cost