

3 work, power and energy



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3 Work, Power and Energy At the end of this section you should be able to: a. b. c. d. e. f. describe potential energy as energy due to position and derive potential energy as mgh describe kinetic energy as energy due to motion and derive kinetic energy as $\frac{mv^2}{2}$ state conservation of energy laws and solve problems where energy is conserved define power as rate of energy transfer define couple, torque and calculate work done by variable force or torque solve problems where energy is lost due to friction

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3.6.3 Work, Power and

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 Hanna & Hiller 180 - 210 Fundamentals of Mechanics — Kinetics: Section 3 -
 Work, Powers and Energy 1 3. 1 Work done by a constant force When the
 point at which a force acts moves, the force is said to have done work. When
 the force is constant, the work done is defined as the product of the force
 and distance moved. work done = force \times distance moved in direction of
 force Consider the example in Figure 3. 1, a force F acting at the angle θ ,
 moves a body from point A to point B. $F \sin \theta$, $s \cos \theta$ A θ , s B Figure 3. 1: Notation
 for work done by a force The distance moved in the direction of the force is
 given by Distance in direction of force = $s \cos \theta$, So the work done by the
 force F is Work done = $F s \cos \theta$, Equation 3. 1 If the body moves in the same
 direction as the force the angle is 0° so Work done = Fs When the angle is
 90° then the work done is zero. The SI units for work are Joules J (with force,

F, in Newton's N and distance, s, in metres m). Worked Example 3. 1 How much work is done when a force of 5 kN moves its point of application 600mm in the direction of the force. Solution the force. work done = $5 \times 10^3 \times 600 \times 10^{-3} = 3000 \text{ J} = 3 \text{ kJ}$ Worked Example 3. 2 () () Find the work done in raising 100 kg of water through a vertical distance of 3m. Solution The force is the weight of the water, so Fundamentals of Mechanics — Kinetics: Section 3 - Work, Powers and Energy 2 work done = $(100 \times 9.81) \times 3 = 2943 \text{ J}$ 3. 2 Work done by a variable force Forces in practice will often vary. In these cases Equation 3. 1 cannot be used. Consider the case where the force varies as in Figure 3. 2 For the thin strip with width ds - shown shaded in Figure 3. 2 - the force can be considered constant at F. The work done over the distance ds is then work done = $F ds$ This is the area of the shaded strip. The total work done for distance s is the sum of the areas of all such strips. This is the same as the area under the Force/distance curve. force F 0 ds s distance Figure 3. 2: Work done by a variable force So for a variable force work done = area under force/distance curve Equation 3. 2 Clearly this also works for a constant force - the curve is then a horizontal line. In general you must use some special integration technique to obtain the area under a curve. Three common techniques are the trapezoidal, mid-ordinate and Simpson's rule. They are not detailed here but may be found in many mathematical text books. 3. 3 Energy A body which has the capacity to do work is said to possess energy. For example, water in a reservoir is said to possess energy as it could be used to drive a turbine lower down the valley. There are many forms of energy e. g. electrical, chemical heat, nuclear, mechanical etc. The SI units are the same as those for work, Joules J. In this module only purely mechanical energy will be considered. This may be of

two kinds, potential and kinetic. Fundamentals of Mechanics — Kinetics: Section 3 - Work, Powers and Energy 3 3. 3. 1 Potential Energy There are different forms of potential energy two examples are: i) a pile driver raised ready to fall on to its target possesses gravitational potential energy while (ii) a coiled spring which is compressed possesses an internal potential energy. Only gravitational potential energy will be considered here. It may be described as energy due to position relative to a standard position (normally chosen to be the earth's surface.) The potential energy of a body may be defined as the amount of work it would do if it were to move from its current position to the standard position. 3. 3. 2 Formulae for gravitational potential energy A body is at rest on the earth's surface. It is then raised a vertical distance h above the surface. The work required to do this is the force required times the distance h . Since the force required is its weight, and weight, $W = mg$, then the work required is mgh . The body now possesses this amount of energy - stored as potential energy - it has the capacity to do this amount of work, and would do so if allowed to fall to earth. Potential energy is thus given by potential energy = mgh Equation 3. 3 where h is the height above the earth's surface. Worked example 3. 3 What is the potential energy of a 10kg mass: a) 100m above the surface of the earth b) at the bottom of a vertical mine shaft 1000m deep. Fundamentals of Mechanics — Kinetics: Section 3 - Work, Powers and Energy 4 Solution a) potential energy = $mgh = 10 \times 9.81 \times 100 = 9810 \text{ J} = 9.81 \text{ kJ}$ potential energy = $mgh = 10 \times 9.81 \times (1000) = 98100 \text{ J} = 98.1 \text{ kJ}$ b) 3. 3. 3 Kinetic energy Kinetic energy may be described as energy due to motion. The kinetic energy of a body may be defined as the amount of work it can do before being brought to rest. For example when a hammer is used to knock

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in a nail, work is done on the nail by the hammer and hence the hammer must have possessed energy. Only linear motion will be considered here. 3.

3. 4 Formulae for kinetic energy Let a body of mass m moving with speed v be brought to rest with uniform deceleration by a constant force F over a distance s . Using Equation 1. 4 $v^2 = u^2 + 2as$ $0 = u^2 + 2as$ $s = \frac{-u^2}{2a}$ And work done is given by $v^2 = u^2 + 2as$ work done = force \times distance = $Fs = F \frac{v^2 - u^2}{2a}$

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5 The force is $F = ma$ so $v^2 - u^2 = 2as$ work done = $ma \frac{v^2 - u^2}{2a} = \frac{1}{2} m(v^2 - u^2)$ Thus the kinetic energy is given by kinetic energy = $\frac{1}{2} mv^2$ Equation 3. 4 3. 3. 5 Kinetic energy and work done

When a body with mass m has its speed increased from u to v in a distance s by a constant force F which produces an

acceleration a , then from Equation 1. 3 we know $v^2 = u^2 + 2as$ $v^2 - u^2 = 2as$

$\frac{1}{2} m(v^2 - u^2) = mas$ multiplying this by m give an expression of the increase in

kinetic energy (the difference in kinetic energy at the end and the start) $\frac{1}{2} m(v^2 - u^2)$

$= \frac{1}{2} mv^2 - \frac{1}{2} mu^2 = mas$ Thus since $F = ma$ increase in kinetic energy = Fs

$Fs =$ work done but also we know So the relationship between kinetic energy

can be summed up as Work done by forces acting on a body = change of

kinetic energy in the body Equation 3. 5 This is sometimes known as the

work-energy theorem. Worked examp . 4 A car of mass 1000 kg travelling at

30m/s has its speed reduced to 10m/s by a constant breaking force over a

distance of 75m. Find: a) The cars initial kinetic energy b) The final kinetic

energy c) The breaking force Solution a) Fundamentals of Mechanics —

Kinetics: Section 3 - Work, Powers and Energy 6 Initial kinetic energy = $\frac{1}{2} mv^2$

$= \frac{1}{2} \times 1000 \times 30^2 = 450000 \text{ J} = 450 \text{ kJ}$ b) Final kinetic energy = $\frac{1}{2} mv^2$

$= \frac{1}{2} \times 1000 \times 10^2 = 50000 \text{ J} = 50 \text{ kJ}$ c) Change in kinetic energy = 400 kJ By

Equation 3. 5 work done = change in kinetic energy so $Fs =$ change in kinetic

energy Breaking force $\tilde{A} - 75 = 400\,000$ Breaking force = 5333 N 3. 4

Conservation of energy The principle of conservation of energy state that the total energy of a system remains constant. Energy cannot be created or destroyed but may be converted from one form to another. Take the case of a crate on a slope. Initially it is at rest, all its energy is potential energy. As it accelerates, some of it potential energy is converted into kinetic energy and some used to overcome friction. This energy used to overcome friction is not lost but converted into heat. At the bottom of the slope the energy will be purely kinetic (assuming the datum for potential energy is the bottom of the slope.) If we consider a body falling freely in air, neglecting air resistance, then mechanical energy is conserved, as potential energy is lost and equal amount of kinetic energy is gained as speed increases. If the motion involves friction or collisions then the principle of conservation of energy is true, but conservation of mechanical energy is not applicable as some energy is converted to heat and perhaps sound. Worked Example 3. 5 A cyclist and his bicycle has a mass of 80 kg. After 100m he reaches the top of a hill, with slope 1 in 20 measured along the slope, at a speed of 2 m/s. He then free wheels the 100m to the bottom of the hill where his speed has increased 9m/s. How much energy has he lost on the hill? Solution 100m h Figure 3. 3:

Dimensions of the hill in worked example 3. 5 If the hill is 100m long then the height is: Fundamentals of Mechanics — Kinetics: Section 3 - Work, Powers and Energy 7 $h = 100$ So potential energy lost is $1 = 5m\ 20\ mgh = 80\ \tilde{A} - 9. 81\tilde{A} - 5 = 3924\ J\ 1\ 2\ 1\ 1\ mv\ \hat{\wedge}'\ \mu\ 2 = m\ v\ 2\ \hat{\wedge}'\ u\ 2\ 2\ 2\ 2 = 40(81\ \hat{\wedge}'\ 4) = 3080\ J$ Increase in kinetic energy is () By the principle of conservation of energy Initial energy = Final energy + loss of energy(due to friction etc.) loss of energy (due to friction etc.) = $3924\ \hat{\wedge}'\ 3080 = 844\ J$ 3. 5

Power Power is the rate at which work is done, or the rate at which energy is used transferred. Power = work done time taken Equation 3. 6 The SI unit for power is the watt W. A power of 1W means that work is being done at the rate of 1J/s. Larger units for power are the kilowatt kW (1kW = 1000 W = 10³ W) and the megawatt MW (1 MW = 1000000 W = 10⁶ W). If work is being done by a machine moving at speed v against a constant force, or resistance, F, then since work done is force times distance, work done per second is Fv, which is the same as power. Power = Fv Equation 3. 7 Worked Example 3. 6 A constant force of 2kN pulls a crate along a level floor a distance of 10 m in 50s. What is the power used? Solution Work done = force \times distance = 2000 \times 10 = 20 000 J Fundamentals of Mechanics — Kinetics: Section 3 - Work, Powers and Energy 8 Power = work done time taken 20 000 = 50 = 400W Alternatively we could have calculated the speed first v= distance time 10 = = 0. 2 m / s 50 and then calculated power Power = Force \times Speed = Fv = 2000 \times 0. 2 = 400W Worked Example 3. 7 A hoist operated by an electric motor has a mass of 500 kg. It raises a load of 300 kg vertically at a steady speed of 0. 2 m/s. Frictional resistance can be taken to be constant at 1200 N. What is the power required? Solution Total mass = m = 800 kg Weight = 800 \times 9. 81 = 7848 N Total force = 7848 + 1200 = 9048 N Fundamentals of Mechanics — Kinetics: Section 3 - Work, Powers and Energy 9 From Equation 3. 7 Power = force \times speed = 9048 \times 0. 2 = 1810W = 1. 81kW Worked example 3. 8 A car of mass 900 kg has an engine with power output of 42 kW. It can achieve a maximum speed of 120 km/h along the level. a) What is the resistance to motion? b) If the maximum power and the resistance remained the same what would be the maximum speed the car could achieve up an incline of 1 in 40 along the slope? Solution

$W \sin \hat{i}$, $N = 1200$ $N = 40$ $1 \hat{i}$, $W = 900g$ Figure 3. 4: Forces on the car on a slope in Worked Example 3. 8 $W \cos \hat{i}$, First get the information into the correct units: $120 \text{ km / h} = 120 \frac{1000}{3600} = 33.33 \text{ m / s}$ a) Calculate the resistance $\text{Power} = \text{Force} \times \text{speed} = \text{Resistance} \times \text{speed}$ $42000 = \text{Resistance} \times 33.33$ $42000 \text{ Resistance} = = 1260 \text{ N}$ 33. 33 Fundamentals of Mechanics — Kinetics: Section 3 - Work, Powers and Energy 10 b) Total force down the incline = frictional force + component of weight down incline = $1260 + mg \sin \hat{i}$, = $1260 + 900 \times 9.81 = 1260 + 221 = 1481 \text{ N}$ $\text{Power} = \text{force} \times \text{speed}$ $\text{Speed} = \frac{\text{Power}}{\text{force}}$ $\frac{42000}{1481} = 28.4 \text{ m / s}$ $\frac{28.4 \times 3600}{1000} = 102 \text{ km / h}$ 1 40 Or in km/h $\text{Speed} = 28.4$ 3. 6 Moment, couple and torque The moment of a force F about a point is its turning effect about the point. It is quantified as the product of the force and the perpendicular distance from the point to the line of action of the force. $F d$ O Figure 3. 4: Moment of a force In Figure 3. 5 the moment of F about point O is $\text{moment} = Fd$ Equation 3. 8 A couple is a pair of equal and parallel but opposite forces as shown in Figure 3. 6: Fundamentals of Mechanics — Kinetics: Section 3 - Work, Powers and Energy 11 F p F Figure 3. 6: A couple The moment of a couple about any point in its plane is the product of one force and the perpendicular distance between them: $\text{Moment of couple} = Fp$ Equation 3. 9 Example of a couple include turning on/off a tap, or winding a clock. The SI units for a moment or a couple are Newton metres, Nm. In engineering the moment of a force or couple is known as torque. A spanner tightening a nut is said to exert a torque on the nut, similarly a belt turning a pulley exerts a torque on the pulley. 3. 6. 1 Work done by a constant torque Let a force F turn a light rod OA with length r through an angle of \hat{i} , to position OB , as shown in Figure 3. 7. F B F s r A \hat{i} , O Figure 3. 7: Work done by a constant torque The tor TQ exerted

about O is force times perpendicular distance from O. $r TQ = Fr$ Equation 3.

10 Now work done by F is s is the arc of the circle, when $\hat{\theta}$ is measure in radians work done = Fs $s = r\hat{\theta}$ Fundamentals of Mechanics — Kinetics:

Section 3 - Work, Powers and Energy 12 work done = $Fr\hat{\theta}$ work done = $TQ\hat{\theta}$,

Equation 3. 11 The work done by a constant torque TQ is thus the product of the torque and the angle through which it turns (where the angle is

measured in radians.) As the SI units for work is Joules, TQ must be in Nm 3.

6. 2 Power transmitted by a constant torque Power is rate of doing work. It

the rod in Figure 3. 7 rotates at n revolutions per second, then in one second

the angle turned through is $\hat{\theta} = 2\pi n$ radians, and the work done per second

will be, by Equation 3. 11 work done per second = power = $TQ 2\pi n$ as

angular speed is $\omega = 2\pi n$ power = $2\pi n TQ$ power = ωTQ Equation 3. 12

then The units of power are Watts, W , with n in rev/s , ω in rad/s and TQ in

Nm . Worked Example 3. 9 A spanner that is used to tighten a nut is 300mm

long. The force exerted on the end of a spanner is 100 N. a) What is the

torque exerted on the nut? b) What is the work done when the nut turns

through 30° ? Fundamentals of Mechanics — Kinetics: Section 3 - Work,

Powers and Energy 13 Solution a) Calculate the torque by Equation 3. 10 TQ

= $Fr = 100 \times 300 \times 10^{-3} = 30 Nm$ b) Calculate the work done by

Equation 3. 11 () work done = $TQ\hat{\theta} = 30 \times \frac{\pi}{6} = 15.7 J = 30 \times \frac{\pi}{6}$ Worked Example

3. 10 An electric motor is rated at 400 W. If its efficiency is 80%, find the

maximum torque which it can exert when running at 285 Solution Calculate

the speed in rev/s using Equation 3. 12 power = $2\pi n TQ$ $n = \frac{power}{2\pi TQ}$