# Chapter 6 resource masters

Chapter 6 Resource Masters Consumable Workbooks Many of the worksheets contained in the Chapter Resource Masters booklets are available as consumable workbooks. Study Guide and Intervention Workbook Skills Practice Workbook Practice Workbook 0-07-828029-X 0-07-828023-0 0-07-828024-9 ANSWERS FOR WORKBOOKS The answers for Chapter 6 of these workbooks can be found in the back of this Chapter Resource Masters booklet. Glencoe/McGraw-Hill Copyright © by The McGraw-Hill Companies, Inc. All rights reserved. Printed in the United States of America. Permission is granted to reproduce the material contained herein on the condition that such material be reproduced only for classroom use; be provided to students, teacher, and families without charge; and be used solely in conjunction with Glencoe's Algebra 2. Any other reproduction, for use or sale, is prohibited without prior written permission of the publisher. Send all inquiries to: The McGraw-Hill Companies 8787 Orion Place Columbus, OH 43240-4027 ISBN: 0-07-828009-5 1 2 3 4 5 6 7 8 9 10 066 11 10 09 08 07 06 05 04 03 02 Algebra 2 Chapter 6 Resource Masters Contents Vocabulary Builder . . . . . . . . . . . . . . . . vii Lesson 6-1 Study Guide and Intervention . . . . . . . . 313—314 Skills Practice . . . . . . . . . . . . . . . . . . . . . . . . 315 Practice . . . . . . . . . . . . . . . . . . . . . . . . . . . 316 Reading to Learn Mathematics . . . . . . . . . . 317 Enrichment . . . . . . . . . . . . . . . . . . . . . . . . . 318 Lesson 6-6 Study Guide and Intervention . . . . . . . . 343—344 Skills Practice . . . . . . . . . . . . . . . . . . . . . . . . 345
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Practice . . . . . . . . . . . . . . . . . . . . . . . . . . . 340 Reading to Learn Mathematics . . . . . . . . . . 341 Enrichment . . . . . . . . . . . . . . . . . . . . . . . . . 342 © Glencoe/McGraw-Hill iii Glencoe Algebra 2 Teacher's Guide to Using the Chapter 6 Resource Masters The Fast File Chapter Resource system allows you to conveniently file the resources you use most often. The Chapter 6 Resource Masters includes the core materials needed for Chapter 6. These materials include worksheets, extensions, and assessment options. The answers for these pages appear at the back of this booklet. All of the materials found in this booklet are included for viewing and printing in the Algebra 2 TeacherWorks CD-ROM. Pages vii—viii include a student study tool that presents up to twenty of the key vocabulary terms from the chapter. Students are to record definitions and/or examples for each term. You may suggest that students highlight or star the terms with which they are not familiar. Vocabulary Builder There is one master for each lesson. These problems more closely follow the structure of the Practice and Apply section of the Student Edition exercises. These exercises are of average difficulty. Practice WHEN TO USE These provide additional practice options or may be used as homework for second day teaching of the lesson. WHEN TO USE Give these pages to students before beginning Lesson 6-1. Encourage them to add these pages to their Algebra 2 Study Notebook. Remind them to add definitions and examples as they complete each lesson. Reading to Learn Mathematics Study Guide and Intervention Each lesson in Algebra 2 addresses two objectives. There is one Study Guide and Intervention master for each objective. WHEN TO USE Use these masters as One master is included for each lesson. The first section of each master asks questions about the opening paragraph of the lesson in the Student Edition. Additional questions ask students to interpret the context of and relationships among terms in the lesson. Finally, students are asked to summarize what they have learned using various representation techniques. reteaching activities for students who need additional reinforcement. These pages can also be used in conjunction with the Student Edition as an instructional tool for students who have been absent. There is one master for each lesson. These provide computational practice at a basic level. used with students who have weaker mathematics backgrounds or need additional reinforcement. WHEN TO USE This master can be used as a study tool when presenting the lesson or as an informal reading assessment after presenting the lesson. It is also a helpful tool for ELL (English Language Learner) students. There is one extension master for each lesson. These activities may extend the concepts in the lesson, offer an historical or multicultural look at the concepts, or widen students' perspectives on the mathematics they are learning. These are not written exclusively for honors students, but are accessible for use with all levels of students. Skills Practice Enrichment WHEN TO USE These masters can be WHEN TO USE These may be used as extra credit, short-term projects, or as activities for days when class periods are shortened. Glencoe Algebra 2 © Glencoe/McGraw-Hill iv Assessment Options The assessment masters in the Chapter 6 Resource Masters offer a wide range of assessment tools for intermediate and final assessment. The following lists describe each assessment master and its intended use. Intermediate Assessment - Four free-response quizzes are included to offer assessment at appropriate intervals in the chapter. - A Mid-Chapter Test provides an option to assess the first half of the chapter. It is composed of both multiple-choice and free-response questions. Chapter Assessment CHAPTER TESTS - Form 1 contains multiple-choice questions and is intended for use with basic level students. - Forms 2A and 2B contain multiple-choice questions aimed at the average level student. These tests are similar in format to offer comparable testing situations. - Forms 2C and 2D are composed of freeresponse questions aimed at the average level student. These tests are similar in format to offer comparable testing situations. Grids with axes are provided for questions assessing graphing skills. - Form 3 is an advanced level test with free-response questions. Grids without axes are provided for questions assessing graphing skills. All of the above tests include a freeresponse Bonus question. - The Open-Ended Assessment includes performance assessment tasks that are suitable for all students. A scoring rubric is included for evaluation guidelines. Sample answers are provided for assessment. - A Vocabulary Test, suitable for all students, includes a list of the vocabulary words in the chapter and ten questions assessing students' knowledge of those terms. This can also be used in conjunction with one of the chapter tests or as a review worksheet. Continuing Assessment - The Cumulative Review provides students an opportunity to reinforce and retain skills as they proceed through their study of Algebra 2. It can also be used as a test. This master includes free-response questions. - The Standardized Test Practice offers continuing review of algebra concepts in various formats, which may appear on the standardized tests that they may encounter. This practice includes multiplechoice, grid-in, and quantitativecomparison questions. Bubble-in and grid-in answer sections are provided on the master. Answers - Page A1 is an answer sheet for the Standardized Test Practice questions that appear in the Student Edition on pages 342—343. This improves students' familiarity with the answer formats they may encounter in test taking. - The answers for the lesson-by-lesson masters are provided as reduced pages with answers appearing in red. - Full-size answer keys are provided for the assessment masters in this booklet. © Glencoe/McGraw-Hill v Glencoe Algebra 2 NAME ______________________________________________ DATE ____________ PERIOD _____ 6 Reading to Learn Mathematics Vocabulary Builder Vocabulary Builder This is an alphabetical list of the key vocabulary terms you will learn in Chapter 6. As you study the chapter, complete each term's definition or description. Remember to add the page number where you found the term. Add these pages to your Algebra Study Notebook to review vocabulary at the end of the chapter. Vocabulary Term Found on Page Definition/Description/Example axis of symmetry completing the square constant term discriminant dihs·KRIH·muh·nuhnt linear term maximum value minimum value parabola puh·RA·buh·luh quadratic equation kwah·DRA·tihk Quadratic Formula ! " # " $ ! " # " $ ! " " # " " $ (continued on the next page) Glencoe/McGraw-Hill © vii Glencoe Algebra 2 NAME ______________________________________________ DATE ____________ PERIOD _____ 6 Reading to Learn Mathematics Vocabulary Builder Vocabulary Term Found on Page (continued) Definition/Description/Example quadratic function quadratic inequality quadratic term roots Square Root Property vertex vertex form Zero Product Property zeros © Glencoe/McGraw-Hill viii Glencoe Algebra 2 NAME ______________________________________________ DATE ____________ PERIOD _____ 6-1 Study Guide and Intervention Graphing Quadratic Functions Graph Quadratic Functions Quadratic Function Graph of a Quadratic Function A function defined by an equation of the form f (x) b x-coordinate of vertex: 2a ax 2 bx c, where a 0 b ; 2a A parabola with these characteristics: y intercept: c ; axis of symmetry: x ( 3) x or 3 . The x-coordinate of the vertex is 3 . 2(1) 2 2 3 Next make a table of values for x near . 2 x 0 1 3 2 a 1, b 3, and c 5, so the y-intercept is 5. The equation of the axis of symmetry is x2 02 12 3 2 2 3x 3(0) 3(1) 3 3 2 5 5 5 5 5 5 f(x) 5 3 11 4 (x, f(x)) (0, 5) (1, 3) 3 11 , 2 4 f (x ) 2 3 22 32 3(2) 3(3) 3 5 (2, 3) (3, 5) O x Exercises For Exercises 1—3, complete parts a—c for each quadratic function. a. Find the y-intercept, the equation of the axis of symmetry, and the x-coordinate of the vertex. b. Make a table of values that includes the vertex. c. Use this information to graph the function. 1. f(x) x2 6x 8 2. f(x) x2 2x 2 3. f(x) 2x2 4x 3 8, x x f (x) 3 1 3, 2 0 12 8 4 (x ) 3 1 3 4 0 2, x x f (x) 1 3 4 —8 —4 O —4 1, 0 2 f (x ) 1 2 2 1 1 3, x x f (x) 1, 1 1 1 f (x ) 12 0 3 2 3 3 9 4 8x 8 4 —4 O 4 8 —8 —4 O —4 4 x —8 x © Glencoe/McGraw-Hill 313 Glencoe Algebra 2 Lesson 6-1 Find the y-intercept, the equation of the axis of symmetry, and the x-coordinate of the vertex for the graph of f(x) x2 3x 5. Use this information to graph the function. Example NAME ______________________________________________ DATE ____________ PERIOD _____ 6-1 Study Guide and Intervention Graphing Quadratic Functions (continued) Maximum and Minimum Values Maximum or Minimum Value of a Quadratic Function The y-coordinate of the vertex of a quadratic function is the maximum or minimum value of the function. The graph of f(x ) ax 2 bx c, where a 0, opens up and has a minimum when a 0. The graph opens down and has a maximum when a 0. Determine whether each function has a maximum or minimum value. Then find the maximum or minimum value of each function. a. f(x) 3x 2 6x 7 For this function, a 3 and b 6. Since a 0, the graph opens up, and the function has a minimum value. The minimum value is the y-coordinate of the vertex. The x-coordinate of the b 6 vertex is 1. 2a 2(3) Example b. f(x) 100 2x x 2 For this function, a 1 and b 2. Since a 0, the graph opens down, and the function has a maximum value. The maximum value is the y-coordinate of the vertex. The x-coordinate of the vertex b 2 is 1. 2a 2( 1) Evaluate the function at x 1 to find the minimum value. f(1) 3(1)2 6(1) 7 4, so the minimum value of the function is 4. Evaluate the function at x 1 to find the maximum value. f( 1) 100 2( 1) ( 1)2 101, so the minimum value of the function is 101. Exercises Determine whether each function has a maximum or minimum value. Then find the maximum or minimum value of each function. 1. f(x) 2x2 x 10 2. f(x) x2 4x 7 3. f(x) 3x2 3x 1 min., 9 4. f(x) 7 4x x2 min., 5. f(x) x2 11 7x 11 min., 6. f(x) 1 x2 6x 4 16 max., 20 7. f(x) x2 5x 2 min., 8. f(x) 20 5 6x x2 max., 5 9. f(x) 4x2 x 3 min., 10. f(x) 17 x2 4x 10 max., 29 11. f(x) x2 10x 5 min., 2 15 12. f(x) 6x2 12x 21 max., 14 13. f(x) 25x2 100x 350 min., 14. f(x) 20 0. 5x2 0. 3x 1. 4 max., 27 15. f(x) x2 2 x 4 8 min., 250 © min., 1. 445 314 max., 7 31 Glencoe Algebra 2 Glencoe/McGraw-Hill NAME ___________________________________________ DATE ___________ PERIOD _____ 6-1 Skills Practice Graphing Quadratic Functions For each quadratic function, find the y-intercept, the equation of the axis of symmetry, and the x-coordinate of the vertex. 1. f(x) 3x2 2. f(x) x2 1 3. f(x) x2 6x 15 0; x 4. f(x) 0; 0 2x2 11 1; x 5. f(x) 0; 0 x2 10x 5 15; x 6. f(x) 2x2 3; 3 8x 7 11; x 0; 0 5; x 5; 5 7; x 2; 2 Complete parts a—c for each quadratic function. a. Find the y-intercept, the equation of the axis of symmetry, and the x-coordinate of the vertex. b. Make a table of values that includes the vertex. c. Use this information to graph the function. 7. f(x) 2x2 8. f(x) x2 4x 2 0 4 4 6 9. f(x) x2 0 6x 2 0 3 8 4 6 8 0; x x f (x) 0; 0 2 8 1 0 2 0 f (x ) 4; x 1 2 2 8 x 2; 2 2 0 4 16 8; x x f (x) 8 f (x ) 3; 3 1 0 f (x) 16 4 f (x ) 16 O x 12 8 4 O —2 O 2 4 6x x Determine whether each function has a maximum or a minimum value. Then find the maximum or minimum value of each function.

10. f(x) 6x2 11. f(x) 8x2 12. f(x) x2 2x min.; 0 13. f(x) x2 2x 15 max.; 0 14. f(x) x2 4x 1 min.; 15. f(x) x2 1 2x 3 min.; 14 16. f(x) 2x2 4x 3 max.; 3 17. f(x) 3x2 12x 3 min.; 18. f(x) 4 2x2 4x 1 max.; 1 min.; 9 315 min.; 1 © Glencoe/McGraw-Hill Glencoe Algebra 2 Lesson 6-1 NAME ____________________________________________ DATE ___________ PERIOD _____ 6-1 Practice (Average) Graphing Quadratic Functions Complete parts a—c for each quadratic function. a. Find the y-intercept, the equation of the axis of symmetry, and the x-coordinate of the vertex. b. Make a table of values that includes the vertex. c. Use this information to graph the function. 1. f(x) x2 0 8x 15 4 6 8 2. f(x) x2 6 4 4x 12 3. f(x) 2x2 2x 1 2 5 15; x x 4; 4 2 1 3 15 12; x x 2; 2 2 0 2 1; x x f (x) 5 0. 5; 0. 5 1 0 0. 5 1 1 0. 5 1 f (x) 15 3 16 12 8 4 O 2 4 f (x) 0 12 16 12 0 f (x ) 16 12 8 4 f (x ) 6 8x —6 —4 —2 O 2x Determine whether each function has a maximum or a minimum value. Then find the maximum or minimum value of each function. 4. f(x) x2 2x 8 5. f(x) x2 6x 14 6. v(x) x2 14x 57 min.; 7. f(x) 9 2x2 4x 6 min.; 5 8. f(x) x2 4x 1 max.; 9. f(x) 8 2 2 x 3 8x 24 min.; 8 max.; 3 max.; 0 10. GRAVITATION From 4 feet above a swimming pool, Susan throws a ball upward with a velocity of 32 feet per second. The height h(t) of the ball t seconds after Susan throws it is given by h(t) 16t2 32t 4. Find the maximum height reached by the ball and the time that this height is reached. 20 ft; 1 s 11. HEALTH CLUBS Last year, the SportsTime Athletic Club charged $20 to participate in an aerobics class. Seventy people attended the classes. The club wants to increase the class price this year. They expect to lose one customer for each $1 increase in the price. a. What price should the club charge to maximize the income from the aerobics classes? $45 b. What is the maximum income the SportsTime Athletic Club can expect to make? $2025 © Glencoe/McGraw-Hill 316

Glencoe Algebra 2 NAME ______________________________________________ DATE ____________ PERIOD _____ 6-1 Reading to Learn Mathematics Graphing Quadratic Functions How can income from a rock concert be maximized? Read the introduction to Lesson 6-1 at the top of page 286 in your textbook. - Based on the graph in your textbook, for what ticket price is the income the greatest? $40 - Use the graph to estimate the maximum income. about $72, 000 Pre-Activity Reading the Lesson 5x is the linear 4 . term, and 3 is the 4 ax2 x 3x2, a b. For the quadratic function f(x) c 2. Consider the quadratic function f(x) a. The graph of this function is a b. The y-intercept is 3 , b 1 , and bx c, where a . 0. parabola . c x c. The axis of symmetry is the line d. If a 0, then the graph opens b 2a . and the function has a upward minimum e. If a value. 0, then the graph opens downward and the function has a maximum value. (—2, 4) 3. Refer to the graph at the right as you complete the following sentences. a. The curve is called a b. The line x f (x ) parabola . O (0, —1) 2 is called the axis of symmetry . x c. The point ( 2, 4) is called the vertex 1), 1 is . d. Because the graph contains the point (0, the y-intercept . Helping You Remember 4. How can you remember the way to use the x2 term of a quadratic function to tell whether the function has a maximum or a minimum value? Sample answer: Remember that the graph of f(x) x 2 (with a 0) is a U-shaped curve that opens up and has a minimum. The graph of g(x) x 2 (with a 0) is just the opposite. It opens down and has a maximum. 317 Glencoe Algebra 2 © Glencoe/McGraw-Hill Lesson 6-1 1. a. For the quadratic function f(x) 2x2 5x 3, 2x2 is the quadratic constant term. term, NAME ______________________________________________ DATE ____________ PERIOD _____ 6-1 Enrichment Finding the Axis of Symmetry of a Parabola As you know, if f(x) ax2 bx c is a quadratic function, the values of x b b2 2a 4ac b . 2a that make f(x) equal to zero are and b b2 2a f(x) 4ac . The average of these two number values is value when x The function f(x) has its maximum or minimum b . Since the axis of symmetry 2a O b x = — —— 2a f(x) = ax 2 + bx + c of the graph of f (x) passes through the point where the maximum or minimum occurs, the axis of symmetry has the equation x b . 2a x b b (— —— , f (— —— (( 2a 2a Example Use x x 10 2(5) b . 2a Find the vertex and axis of symmetry for f(x) 5x 2 10x 7. 1 The x-coordinate of the vertex is 5x2 7 10x 12 7. 1. Substitute x 1 in f(x) 2 f( 1) 5( 1) 10( 1) The vertex is ( 1, 12). The axis of symmetry is x b , or x 2a 1. Find the vertex and axis of symmetry for the graph of each function using x 1. f(x) x2 b . 2a 4x 8 2. g(x) 4x2 8x 3 3. y x2 8x 3 4. f(x) 2x2 6x 5 5. A(x) x2 12x 36 6. k(x) 2x2 2x 6 © Glencoe/McGraw-Hill 318 Glencoe Algebra 2 NAME ______________________________________________ DATE ____________ PERIOD _____ 6-2 Study Guide and Intervention Solving Quadratic Equations by Graphing Solve Quadratic Equations Quadratic Equation Roots of a Quadratic Equation A quadratic equation has the form ax 2 bx c 0, where a 0. solution(s) of the equation, or the zero(s) of the related quadratic function The zeros of a quadratic function are the x-intercepts of its graph. Therefore, finding the x-intercepts is one way of solving the related quadratic equation. Example Solve x2 x 6 x2 0 by graphing. x 6. 1 , and the equation of the 2 1 . 2 O Graph the related function f(x) f (x ) x b The x-coordinate of the vertex is 2a 1 axis of symmetry is x . 2 Make a table of values using x-values around x f(x) 1 6 1 2 0 6 1 4 2 0 6 1 4 From the table and the graph, we can see that the zeros of the function are 2 and 3. Exercises Solve each equation by graphing. 1. x2 2x 8 O 0 2, f (x ) x 4 2. x2 4x O 5 f (x ) 0 5, 1 3. x2 5x 4 f (x ) 0 1, 4 x O x 4. x2 10x f (x ) 21 0 5. x2 4x 6 0 f (x ) 6. 4x2 4x 1 f (x ) 0 O x O x

O x 3, 7 © no real solutions 319 1 Glencoe Algebra 2 Glencoe/McGraw-Hill Lesson 6-2 NAME _____________________________________________ DATE ____________ PERIOD _____ 6-2 Study Guide and Intervention (continued) Solving Quadratic Equations by Graphing Estimate Solutions Example 2 2(1) x f (x) 1 1 0 2 Often, you may not be able to find exact solutions to quadratic equations by graphing. But you can use the graph to estimate solutions. Solve x 2 2x 2 0 by graphing. If exact roots cannot be found, state the consecutive integers between which the roots are located. The equation of the axis of symmetry of the related function is x 1, so the vertex has x-coordinate 1. Make a table of values. 1 3 2 2 3 1 O f (x ) x The x-intercepts of the graph are between 2 and 3 and between 0 and 1. So one solution is between 2 and 3, and the other solution is between 0 and 1. Exercises Solve the equations by graphing. If exact roots cannot be found, state the consecutive integers between which the roots are located. 1. x2 4x 2 0 2. x2 6x 6 0 3. x2 4x 2 0 between 0 and 1; between 3 and 4 f (x ) between between 2 and 5 and f (x ) 1; 4 between between 1 and 0; 4 and 3 f (x ) O O x x O x 4. x2 2x 4 0 5. 2x2 12x 17 0 6. 1 2 x 2 x 5 2 0 between 3 and 4; between 2 and 1 f (x ) between 2 and 3; between 3 and 4 f (x ) between 2 and between 3 and 4 f (x ) 1; O O x x O x © Glencoe/McGraw-Hill 320 Glencoe Algebra 2 NAME _____________________________________________ DATE ____________ PERIOD _____ 6-2 Skills Practice Solving Quadratic Equations By Graphing Use the related graph of each equation to determine its solutions. 1. x2 2x 3 f (x ) 0 2. x2 f (x ) 6x x2 6x 9 9 O 0 f (x ) x 3. 3x2 4x 3 f (x ) 0 O x f (x ) f (x ) x2 2x 3 3x 2 4x 3 O x 3, 1 3 no real solutions 4. x2 6x 5 0 5. x2 2x 4 0 6. x2 6x 4 0 1, 5 f (x ) no real solutions f (x ) O between 0 and 1; between 5 and 6 f (x ) x O x O x Use a quadratic equation to find two real numbers that satisfy each situation, or show that no such numbers exist. 7. Their sum is 4, and their product is 0. 8. Their sum is 0, and their product is 36. x2 4x 0; 0, f (x ) 4 x2 36 36 24 0; 6, 6 f (x ) O x —12 —6 12 O 6 12 x © Glencoe/McGraw-Hill 321 Glencoe Algebra 2 Lesson 6-2 Solve each equation by graphing. If exact roots cannot be found, state the consecutive integers between which the roots are located. NAME ______________________________________________ DATE ____________ PERIOD _____ 6-2 Practice (Average) Solving Quadratic Equations By Graphing Use the related graph of each equation to determine its solutions. 1. 3x2 f (x ) 3 3x 2 3 0 f (x ) 2. 3x2 x f (x ) 3 0 3. x2 3x 2 f (x ) 0 O x f (x ) O 3x 2 x 3 f (x ) x2 3x 2 O x x 1, 1 no real solutions 1, 2 Solve each equation by graphing. If exact roots cannot be found, state the consecutive integers between which the roots are located. 4. 2x2 6x 5 0 5. x2 10x 24 0 6. 2x2 x 6 0 between 0 and 1; between 4 and 3 12 8 4 —6 —4 —2 O 6, 4 f (x ) between 2 2 and 1, f (x ) x O x Use a quadratic equation to find two real numbers that satisfy each situation, or show that no such numbers exist. 7. Their sum is 1, and their product is f (x ) 6. 8. Their sum is 5, and their product is 8. x2 x 3, 2 6 0; x 2 5x 8 0; no such real numbers exist O x For Exercises 9 and 10, use the formula h(t) v0t 16t 2, where h(t) is the height of an object in feet, v0 is the object’s initial velocity in feet per second, and t is the time in seconds. 9. BASEBALL Marta throws a baseball with an initial upward velocity of 60 feet per second. Ignoring Marta’s height, how long after she releases the ball will it hit the ground? 3. 75 s 10. VOLCANOES A volcanic eruption blasts a boulder upward with an initial velocity of 240 feet per second. How long will it take the boulder to hit the ground if it lands at the same elevation from which it was ejected? 15 s © Glencoe/McGraw-Hill 322 Glencoe Algebra 2 NAME ______________________________________________

DATE ____________ PERIOD _____ 6-2 Reading to Learn Mathematics Solving Quadratic Equations by Graphing How does a quadratic function model a free-fall ride? Read the introduction to Lesson 6-2 at the top of page 294 in your textbook. Write a quadratic function that describes the height of a ball t seconds after 16t 2 125 it is dropped from a height of 125 feet. h(t) Pre-Activity Reading the Lesson 1. The graph of the quadratic function f(x) x2 x 6 is shown at the right. Use the graph to find the solutions of the quadratic equation x2 x 6 0. 2 and 3 y O x 2. Sketch a graph to illustrate each situation. a. A parabola that opens b. A parabola that opens downward and represents a upward and represents a quadratic function with two quadratic function with real zeros, both of which are exactly one real zero. The negative numbers. zero is a positive number. y y c. A parabola that opens downward and represents a quadratic function with no real zeros. y O x O x O x Helping You Remember 3. Think of a memory aid that can help you recall what is meant by the zeros of a quadratic function. Sample answer: The basic facts about a subject are sometimes called the ABCs. In the case of zeros, the ABCs are the XYZs, because the zeros are the x-values that make the y-values equal to zero. © Glencoe/McGraw-Hill 323 Glencoe Algebra 2 Lesson 6-2 NAME ______________________________________________ DATE ____________ PERIOD _____ 6-2 Enrichment Graphing Absolute Value Equations You can solve absolute value equations in much the same way you solved quadratic equations. Graph the related absolute value function for each equation using a graphing calculator. Then use the ZERO feature in the CALC menu to find its real solutions, if any. Recall that solutions are points where the graph intersects the x-axis. For each equation, make a sketch of the related graph and find the solutions rounded to the nearest hundredth. 1. | x

5| 0 2. | 4x 3| 5 0 3. | x 7| 0 5 No solutions 7 4. | x 3| 8 0 5. | x 9, 3 3| 6 0 6. | x 2| 3 0 11, 5 1, 5 7. | 3x 4| 2 2, 2 3 8. | x 12 | 10 9. | x | 3 0 22, 2 3, 3 10. Explain how solving absolute value equations algebraically and finding zeros of absolute value functions graphically are related. Sample answer: values of x when solving algebraically are the x-intercepts (or zeros) of the function when graphed. Glencoe/McGraw-Hill © 324 Glencoe Algebra 2 NAME ______________________________________________ DATE ____________ PERIOD _____ 6-3 Study Guide and Intervention Solving Quadratic Equations by Factoring When you use factoring to solve a quadratic equation, 0, then either a 0 or b 0, or both a and b 0. Solve Equations by Factoring you use the following property. Zero Product Property For any real numbers a and b, if ab Solve each equation by factoring. a. 15x b. 4x2 5x 3x2 15x Original equation 4x2 3x2 15x 0 Subtract 15x from both sides. 4x2 5x 3x(x 5) 0 Factor the binomial. (4x 7)(x 3x 0 or x 5 0 Zero Product Property 4x 7 0 x 0 or x 5 Solve each equation. x 3x2 The solution set is {0, 5}. Example 21 5x 21 3) or 7 or 4 21 0 0 x Original equation Subtract 21 from both sides. Factor the trinomial. 3 x 0 3 7 , 3 . 4 Zero Product Property Solve each equation. The solution set is Exercises Solve each equation by factoring. 1. 6x2 2x 0 2. x2 7x 3. 20x2 25x 0, 4. 6x2 1 7x {0, 7} 5. 6x2 27x 0 0, 6. 12x2 5 8x 0 0, 7. x2 7 x 30 0 0, 8. 2x2 9 x 3 0 0, 9. x2 2 14x 33 0 {5, 10. 4x2 6} 27x 7 0 3 , 1 29x 10 0 { 11, 12. 6x2 5x 3} 4 0 11. 3x2 1 , , 7 8x 1 0 10, 14. 5x2 1 12 0 15. 2x2 1 4 , 13. 12x2 28x 250x 5000 0 1 1 16. 2x2 11x 40 0 2 , 11, 6 21x 11 0 {100, 25} 18. 3x2 2x 21 0 17. 2x2 8, 19. 8x2 5 14x 3 0 1 2 0 7 , , , 5 3 17x 12 0 20. 6x2 11x 21. 5x2 3 1 , 2, 25x 12 0 23. 12x2 1 18x 6 0 3 4 36x 5 0 22. 12x2 24. 7x2 4 , 3 1 , 1 325 1 © Glencoe/McGraw-Hill Glencoe Algebra 2 Lesson 6-3 NAME ______________________________________________ DATE ____________

PERIOD _____ 6-3 Study Guide and Intervention q) (continued) Solving Quadratic Equations by Factoring Write Quadratic Equations (x p)(x To write a quadratic equation with roots p and q, let 0. Then multiply using FOIL. Example in the form a. 3, 5 (x p)(x q) (x 3)[x ( 5)] (x 3)(x 5) x2 2x 15 The equation x2 3 and 5. ax2 Write a quadratic equation with the given roots. Write the equation bx c 0. 0 0 0 0 2x b. Write the pattern. Replace p with 3, q with Simplify. Use FOIL. 5. 7 1 , 8 3 (x x 7 8 p)(x x x (3x 3 7)(3x 24 q) 1 3 1 3 1) 1) 0 0 0 0 24 0 0 13x 7 0 has 15 0 has roots (8x x 7) 8 24 (8x 7 8 24x2 13x 7 The equation 24x2 roots 7 1 and . 8 3 Exercises Write a quadratic equation with the given roots. Write the equation in the form ax2 bx c 0. 1. 3, 4 2. 8, 2 3. 1, 9 x2 4. 5 x 10x 12 25 0 0 x2 5. 10, 7 10x 17x 16 70 0 6. x2 x2 9. 7, 3 4 10x 13x 9 30 0 0 2, 15 x2 7. 1 , 5 3 3x 2 2 5 5x 2 2 , 3 2 3 x2 8. 2, 2 3 3x 2 4 , 9 9x 2 5 , 4 1 2 0 14x 5 0 11. 8x 1 4 0 4x 2 12. 9, 1 6 6x 2 3 1 , 7 5 25x 21 0 10. 3, 17x 6 0 14. 13x 4 0 15. 55x 9 0 13. 9x 2 16. 7 7 , 8 2 16x 2 4 0 17. 8x 2 1 3 , 2 4 8x 2 6x 5 0 18. 35x 2 1 1 , 8 6 22x 3 0 42x 49 10x 326 3 0 48x 2 14x 1 0 © Glencoe/McGraw-Hill Glencoe Algebra 2 NAME ______________________________________________ DATE ____________ PERIOD _____ 6-3 Skills Practice Solving Quadratic Equations by Factoring Solve each equation by factoring. 1. x2 64 { 8, 8} 2. x2 100 0 {10, 10} 3. x2 3x 2 0 {1, 2} 4. x2 4x 3 0 {1, 3} 5. x2 2x 3 0 {1, 3} 6. x2 3x 10 0 {5, 2} 7. x2 6x 5 0 {1, 5} 8. x2 9x 0 {0, 9} 9. x2 6x 0 {0, 6} 10. x2 6x 8 0 { 2, 4} 11. x2 5x {0, 5} 12. x2 14x 49 0 {7} 13. x2 6 5x {2, 3} 14. x2 18x 81 { 9} 17. 4x2 5x 6 0 3 , 2 18. 3x2 13x 10 0 2 , 5 Write a quadratic equation with the given roots. Write the equation in the form ax2 bx c 0, where a, b, and c are integers. 19. 1, 4 x 2 5x 7x 4 0 10 3 0 0 20. 6, 9 x2 3x 7x 0 2x 54 0 21. 2, 1 , 3 5 x2 22. 0, 7 x 2 1 3 , 2 4 23. 3 3x 2 10x 24. 8x 2 3 0 25. Find two consecutive integers whose product is 272. 16, 17 © Glencoe/McGraw-Hill 327 Glencoe Algebra 2 Lesson 6-3 15. x2 4x 21 { 3, 7} 16. 2x2 5x 3 0 1 , 3 NAME

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_____ 6-3 Practice (Average) Solving Quadratic Equations by Factoring Solve each equation by factoring. 1. x2 4. x2 7. x2 10. 10x2 12. x2 14. 36x2 16. 3x2 18. 3x2 20. 3x2 4x 6x 4x 12 8 0 {6, 0 {2, 4} 2} 2. x2 5. x2 8. 7x2 16x 3x 64 2 0 {8} 0 { 2, 3. x2 20x 9x 25 100 14 0 {10} 0 {2, 7} 1} 6. x2 9. x2 0 {0, 4} 9x 0, 4x 0, 4 2x 35x 8x 10x {5} 9 11. x2 13. 5x2 15. 2x2 99 { 9, 11} 60 90 0 {3, 4} 0 {9, 12x 25 2x 24x 8x 36 { 6} 5 1 , 0 5 1 5} , 1 3} 17. 6x2 19. 15x2 21. 6x2 9x 0, 19x 5x 6 3 6 0 45 0 { 5, 3 2 , 2 4 2, 2 3 , Write a quadratic equation with the given roots. Write the equation in the form ax2 bx c 0, where a, b, and c are integers. 22. 7, 2 23. 0, 3 24. 5, 8 x2 25. 7, 9x 8 14 56 0 26. x2 6, 3x 3 0 18 0 x2 27. 3, 4 3x x 7 2 40 12 0 0 x2 28. 1, 1 2 2x 2 1 , 3 15x 0 29. x2 1 , 2 3 3x 2 1 3 3x 2 9x x2 30. 0, 3x 3 1 0 7x 2 0 33. 2x 2 2 , 3 7x 4 5 0 31. 32. 4, 3x 2 24, 26 8x 3 0 13x 4 0 15x 2 22x 8 0 34. NUMBER THEORY Find two consecutive even positive integers whose product is 624. 35. NUMBER THEORY Find two consecutive odd positive integers whose product is 323. 17, 19 36. GEOMETRY The length of a rectangle is 2 feet more than its width. Find the dimensions of the rectangle if its area is 63 square feet. 7 ft by 9 ft 37. PHOTOGRAPHY The length and width of a 6-inch by 8-inch photograph are reduced by the same amount to make a new photograph whose area is half that of the original. By how many inches will the dimensions of the photograph have to be reduced? 2 in. © Glencoe/McGraw-Hill 328 Glencoe Algebra 2 NAME

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_____ 6-3 Reading to Learn Mathematics Solving Quadratic Equations by

Factoring How is the Zero Product Property used in geometry? Read the introduction to Lesson 6-3 at the top of page 301 in your textbook. What does the expression x(x 5) mean in this situation? Pre-Activity It represents the area of the rectangle, since the area is the product of the width and length. Reading the Lesson 1. The solution of a quadratic equation by factoring is shown below. Give the reason for each step of the solution. x2 x2 (x x 10x 3)(x 3 x 10x 21 7) 0 0 7 x 0 7 21 Original equation Add 21 to each side. Factor the trinomial. Zero Product Property Solve each equation. . 0 or x 3 The solution set is {3, 7} Marla (x 7)(x x2 2x 5) 35 0 0 Rosa (x 7)(x x2 2x 5) 35 0 0 Larry (x 7)(x x2 2x 5) 35 0 0 Who is correct? Rosa Explain the errors in the other two students' work. Sample answer: Marla used the wrong factors. Larry used the correct factors but multiplied them incorrectly. Helping You Remember 3. A good way to remember a concept is to represent it in more than one way. Describe an algebraic way and a graphical way to recognize a quadratic equation that has a double root. Sample answer: Algebraic: Write the equation in the standard form ax 2 bx c 0 and examine the trinomial. If it is a perfect square trinomial, the quadratic function has a double root. Graphical: Graph the related quadratic function. If the parabola has exactly one x-intercept, then the equation has a double root. © Glencoe/McGraw-Hill 329 Glencoe Algebra 2 Lesson 6-3 2. On an algebra quiz, students were asked to write a quadratic equation with 7 and 5 as its roots. The work that three students in the class wrote on their papers is shown below. NAME ______________________________________________ DATE ____________ PERIOD _____ 6-3 Enrichment Euler's Formula for Prime Numbers Many mathematicians have searched for a formula that would generate prime numbers. One such formula was proposed by Euler and uses a quadratic polynomial, x2 x 41. Find the values of x2 x 41 for the given values of x. State whether each value of the polynomial is or is not a prime number. 1. x 0 2. x 1 3. x 2 4. x 3 5. x 4 6. x 5 7. x 6 8. x 17 9. x 28 10. x 29 11. x 30 12. x 35 13. Does the formula produce all prime numbers greater than 40? Give examples in your answer. 14. Euler's formula produces primes for many values of x, but it does not work for all of them. Find the first value of x for which the formula fails. (Hint: Try multiples of ten.) © Glencoe/McGraw-Hill 330 Glencoe Algebra 2 NAME ______________________________________________ DATE ____________ PERIOD _____ 6-4 Study Guide and Intervention Completing the Square Square Root Property Use the following property to solve a quadratic equation that is in the form " perfect square trinomial constant. " Square Root Property For any real number x if x 2 n, then x n. Example a. x2 x2 x Solve each equation by using the Square Root Property. b. 4x2 20x 25 32 8x 16 25 4x2 20x 25 8x 16 25 32 2 2 (x 4) (2x 5) 25 32 4 2x 5 25 or x 4 25 32 or 2x 5 x 5 4 9 or x 5 4 1 2x 5 4 2 or 2x 5 1}. x 5 4 2 2 5 32 4 2 The solution set is {9, The solution set is 4 2 . 2 Exercises Solve each equation by using the Square Root Property. 1. x2 18x 81 49 2. x2 20x 100 64 3. 4x2 4x 1 16 {2, 16} { 2, 18} 3 , 5 4. 36x2 12x 1 18 5. 9x2 12x 4 4 6. 25x2 40x 16 28 1 3 2 0, 4 4 2 7 7. 4x2 28x 49 64 8. 16x2 24x 9 81 9. 100x2 60x 9 121 15 , 1 3 , 3 { 0. 8, 1. 4} 10. 25x2 20x 4 75 11. 36x2 48x 16 12 12. 25x2 30x 9 96 2 5 3 2 3 3 4 6 © Glencoe/McGraw-Hill 331 Glencoe Algebra 2 Lesson 6-4 NAME ______________________________________________ DATE ____________ PERIOD _____ 6-4 Study Guide and Intervention Completing the Square (continued) Complete the Square x2 1. Find b . 2 bx, follow these steps. To complete the square for a quadratic expression of the form b . 2 ! 2. Square ! 3. Add b 2 to x2 2 bx. Find the value of 2 c that makes x 22x c a perfect square trinomial. Then write the trinomial as the square of a binomial. Step 1 b 22; b 2 Example 1 Solve 2x2 completing the square. 2x2 2x2 Example 2 8x 8x 2 8x 24 0 by 24 24 0 0 2 Original equation Divide each side by 2. x2 4x 12 is not a perfect square. 4 2 2 11 x2 x2 121, Step 2 112 121 Step 3 c 121 The trinomial is x2 22x which can be written as (x 11)2. 4x 12 x2 4x 4x 4 (x 2)2 0 12 12 Add 12 to each side. 4 Since 4, add 4 to each side. 16 x 2 4 x 6 or x 2 The solution set is {6, Factor the square. Square Root Property Solve each equation. 2}. Exercises Find the value of c that makes each trinomial a perfect square. Then write the trinomial as a perfect square. 1. x2 10x c 2. x2 60x c 3. x2 3x c 25; (x 4. x2 3. 2x 5)2 c 900; (x 5. x2 1 x 2 30)2 c 9 6. x2 ; x 2. 5x 3 2 c 2. 56; (x 1. 6)2 1 ; x 1 2 1. 5625; (x 1. 25)2 Solve each equation by completing the square. 7. y2 4y 5 0 8. x2 8x 65 0 9. s2 10s 21 0 1, 5 10. 2x2 3x 1 0 5, 13 11. 2x2 13x 7 0 3, 7 12. 25x2 40x 9 0 1, 13. x2 1 4x 1 0 14. y2 1 , 7 12y 4 0 1 15. t2 , 9 3t 8 0 2 © 3 6 4 2 332 3 2 41 Glencoe/McGraw-Hill Glencoe Algebra 2 NAME ________________________________________________ DATE ____________ PERIOD _____ 6-4 Skills Practice Completing the Square Solve each equation by using the Square Root Property. 1. x2 8x 16 1 3, 5 2. x2 4x 4 1 1, 3 3. x2 12x 36 25 1, 11 2 4. 4x2 4x 1 9 1, 2 5 8 15 5. x2 4x 4 2 2 7 6. x2 2x 1 5 1 7. x2 6x 9 7 3 8. x2 16x 64 15 Find the value of c that makes each trinomial a perfect square. Then write the trinomial as a perfect square. 9. x2 10x c 25; (x 5)2 12)2 9 2 10. x2 14x c 49; (x 7)2 5 2 11. x2 24x c 144; (x 12. x2 5x c 25 ; x 13. x2 9x c 81 ; x 14. x2 x c 1 ; x 1 2 Solve each equation by completing the square. 15. x2 13x 36 0 4, 9 16. x2 3x 0 0, 3 0 2 17. x2 x 6 0 2, 3 4, 1 3 2 33 18. x2 4x 13 17 , 1 13 2 19. 2x2 7x 4 0 20. 3x2 2x 1 0 1 21. x2 3x 6 0 22. x2 x 3 0 1 23. x2 11 i 11 24. x2 2x 4 0 1 i 3 Glencoe Algebra 2 ©

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_____ 6-4 Practice (Average) Completing the Square Solve each equation by using the Square Root Property. 1. x2 8x 16 1 2. x2 6x 9 1 3. x2 10x 25 16 5, 4. x2 3 14x 49 9 4, 5. 4x2 2 12x 9 4 6. x2 9, 4 1 2 9. 9x2 1 8x 16 8 4, 10 7. x2 6x 9 5 8. x2 1 , 2x 5 2 2 6x 1 2 3 5 1 2 1 3 2 Find the value of c that makes each trinomial a perfect square. Then write the trinomial as a perfect square. 10. x2 12x c 11. x2 20x c 12. x2 11x c 36; (x 13. x2 0. 8x 6)2 c 100; (x 14. x2 2. 2x 10)2 c 121 15. x2 ; x 0. 36x 11 2 c 0. 16; (x 16. x2 5 x 6 0. 4)2 c 1. 21; (x 17. x2 1 x 4 1. 1)2 c 0. 0324; (x 18. x2 5 x 3 0. 18)2 c 25 ; x 5 2 1 ; x 1 2 25 ; x 5 2 Solve each equation by completing the square. 19. x2 22. x2 6x 18 8x 8 9x 3 0 0 4, 2 20. 3x2 23. x2 x 14x 2 19 5 0 2 0 , 1 21. 3x2 24. x2 5x 16x 2 7 0 1, 0 0 2 6, 3 25. 2x2 7 26. x2 x 30 0 8 27. 2x2 10x 71 5 4 28. x2 22 2 3x 6 0 1 29. 2x2 21 2 5x 6 0 5 30. 7x2 15 2 6x 2 0 3 i 2 15 5 i 4 23 3 7 i 5 31. GEOMETRY When the dimensions of a cube are reduced by 4 inches on each side, the surface area of the new cube is 864 square inches. What were the dimensions of the original cube? 16 in. by 16 in. by 16 in. 32. INVESTMENTS The amount of money A in an account in which P dollars is invested for 2 years is given by the formula A P(1 r)2, where r is the interest rate compounded annually. If an investment of $800 in the account grows to $882 in two years, at what interest rate was it invested? 5% © Glencoe/McGraw-Hill 334 Glencoe Algebra 2 NAME

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_____ 6-4 Reading to Learn Mathematics Completing the Square How can you find the time it takes an accelerating race car to reach the finish line? Read the introduction to Lesson 6-4 at the top of page 306 in your textbook.

Explain what it means to say that the driver accelerates at a constant rate of 8 feet per second square. Pre-Activity If the driver is traveling at a certain speed at a particular moment, then one second later, the driver is traveling 8 feet per second faster. Reading the Lesson 1. Give the reason for each step in the following solution of an equation by using the Square Root Property. x2 12x (x x x x 6 x 36 6)2 6 6 6 x 9 9 3 81 81 81 Original equation Factor the perfect square trinomial. Square Root Property 81 9 Rewrite as two equations. Solve each equation. 9 or x 15 2. Explain how to find the constant that must be added to make a binomial into a perfect square trinomial. Sample answer: Find half of the coefficient of the linear term and square it. 3. a. What is the first step in solving the equation 3x2 6x 5x 5 by completing the square? 12 0 by completing the Divide the equation by 3. b. What is the first step in solving the equation x2 square? Add 12 to each side. Helping You Remember 4. How can you use the rules for squaring a binomial to help you remember the procedure for changing a binomial into a perfect square trinomial? One of the rules for squaring a binomial is (x y) 2 x 2 2xy y 2. In completing the square, you are starting with x 2 bx and need to find y 2. This shows you that b 2y, so y b . That is why you must take half of the coefficient and square it to get the constant that must be added to complete the square. © Glencoe/McGraw-Hill 335 Glencoe Algebra 2 Lesson 6-4 NAME ________________________________________________ DATE ____________ PERIOD _____ 6-4 Enrichment The Golden Quadratic Equations A golden rectangle has the property that its length can be written as a b, where a is the width of the rectangle and a a b divided into a square and a smaller golden rectangle, as shown. a b a . Any golden rectangle can be b a a The proportion used to define golden rectangles can be used to derive two quadratic equations.

These are sometimes called golden quadratic equations. Solve each problem. a b 1. In the proportion for the golden rectangle, let a equal 1. Write the resulting quadratic equation and solve for b. 2. In the proportion, let b equal 1. Write the resulting quadratic equation and solve for a. 3. Describe the difference between the two golden quadratic equations you found in exercises 1 and 2. 4. Show that the positive solutions of the two equations in exercises 1 and 2 are reciprocals. 5. Use the Pythagorean Theorem to find a radical expression for the diagonal of a golden rectangle when a 1. 6. Find a radical expression for the diagonal of a golden rectangle when b 1. © Glencoe/McGraw-Hill 336 Glencoe Algebra 2 NAME ______________________________________________ DATE ____________ PERIOD _____ 6-5 Study Guide and Intervention The Quadratic Formula and the Discriminant Quadratic Formula The Quadratic Formula can be used to solve any quadratic equation once it is written in the form ax2 bx c 0. Quadratic Formula The solutions of ax 2 bx c 0, with a 0, are given by x b b2 2a 4ac . Example x b ( 5) 5 5 2 2 9 81 b2 2a Solve x2 x2 4ac ( 5)2 2(1) 5x 5x 14 by using the Quadratic Formula. 14 0. Rewrite the equation as Quadratic Formula 4(1)( 14) Replace a with 1, b with 5, and c with 14. Simplify. 7 or 2 2 and 7. The solutions are Exercises Solve each equation by using the Quadratic Formula. 1. x2 2x 35 0 2. x2 10x 24 0 3. x2 11x 24 0 5, 4. 4x2 7 19x 5 0 4, 5. 14x2 6 9x 1 0 3, 8 6. 2x2 x 15 0 1 , 5 5x 2 1 8. 2y2 , y 1 3, 15 0 9. 3x2 5 7. 3x2 16x 16 0 2, 1 10. 8x2 6x 9 0 5 11. r2 , 3 3r 5 2 25 4, 4 0 12. x2 10x 50 0 13. x2 6x 23 0 14. 4x2 12x 63 0 15. x2 6x 21 0 3 © 4 2 3 6 2 3 2i 3 Glencoe Algebra 2 Glencoe/McGraw-Hill 337 Lesson 6-5 3 3 , 2 1 , 5 5 3 NAME ______________________________________________ DATE ____________ PERIOD _____ 6-5 Study Guide and Intervention (continued) The Quadratic

Formula and the Discriminant Roots and the Discriminant Discriminant The expression under the radical sign, b2 the discriminant. 4ac, in the Quadratic Formula is called Roots of a Quadratic Equation Discriminant b2 b2 b2 b2 4ac 4ac 4ac 4ac 0 and a perfect square 0, but not a perfect square 0 0 Type and Number of Roots 2 rational roots 2 irrational roots 1 rational root 2 complex roots Find the value of the discriminant for each equation. Then describe the number and types of roots for the equation. b. 3x2 2x 5 a. 2x2 5x 3 The discriminant is The discriminant is b2 4ac 52 4(2)(3) or 1. b2 4ac ( 2)2 4(3)(5) or 56. The discriminant is a perfect square, so The discriminant is negative, so the the equation has 2 rational roots. equation has 2 complex roots. Example Exercises For Exercises 1 12, complete parts a c for each quadratic equation. a. Find the value of the discriminant. b. Describe the number and type of roots. c. Find the exact solutions by using the Quadratic Formula. 1. p2 12p 4 128; 2. 9x2 6x 1 0 0; 3. 2x2 7x 4 0 81; two irrational roots; 4 6 4 2 4. x2 4x 4 0 32; one rational root; 1 2 rational roots; 1 , 5. 5x2 36x 7 0 1156; 6. 4x2 4x 11 0 2 irrational roots; roots; 2 2 2 7. x2 7x 6 0 25; 2 rational roots; 1 160; 2 complex 1 i 10 40x 16 0; , 7 8m 14 8; 8. m2 9. 25x2 2 rational roots; 1, 6 10. 4x2 20x 29 0 2 irrational roots; 4 2 1 rational root; 4 12. 4x2 4x 11 0 192; 64; 11. 6x2 26x 8 0 484; 2 complex roots; 2 rational roots; 338 2 irrational roots; Glencoe Algebra 2 © Glencoe/McGraw-Hill NAME ______________________________________________ DATE ____________ PERIOD _____ 6-5 Skills Practice The Quadratic Formula and the Discriminant Complete parts a c for each quadratic equation. a. Find the value of the discriminant. b. Describe the number and type of roots. c. Find the exact solutions by using the Quadratic Formula. 1. x2 8x 16 0 2. x2 11x 26 0 0; 1 rational root; 4 3. 3x2 2x 0 225; 2 rational roots; 4. 20x2 7x 3 0 2, 13 3 1 4; 2

rational roots; 0, 5. 5x2 6 0 2 289; 2 rational roots; 6. x2 6 0 , 120; 2 irrational roots; 7. x2 8x 13 0 30 5 24; 2 irrational roots; 8. 5x2 x 1 0 6 21 10 12; 2 irrational roots; 9. x2 2x 17 0 4 3 21; 2 irrational roots; 1 10. x2 49 0 72; 2 irrational roots; 1 11. x2 x 1 0 3 2 i 2 3 196; 2 complex roots; 12. 2x2 3x 2 7i i 4 7 3; 2 complex roots; 1 7; 2 complex roots; 3 Solve each equation by using the method of your choice. Find exact solutions. 13. x2 15. x2 17. x2 19. x2 21. 2x2 23. 8x2 64 x 4x 25 10x 1 4x 8 30 11 0 14. x2 30 0 24x 27 30 0 5, 6 0 2 16. 16x2 9 , 3 15 18. x2 20. 3x2 8x 36 7x 2x 17 0 4 3 0 4 33 3 5i 11 0 2i 0 0 5 2 3 22. 2x2 24. 2x2 7 4 2 1 17 4 25. PARACHUTING Ignoring wind resistance, the distance d(t) in feet that a parachutist falls in t seconds can be estimated using the formula d(t) 16t2. If a parachutist jumps from an airplane and falls for 1100 feet before opening her parachute, how many seconds pass before she opens the parachute? about 8. 3 s © Glencoe/McGraw-Hill 339 Glencoe Algebra 2 Lesson 6-5 1 i i 5 NAME ________________________________________________ DATE _____________ PERIOD _____ 6-5 Practice (Average) The Quadratic Formula and the Discriminant Complete parts a c for each quadratic equation. a. Find the value of the discriminant. b. Describe the number and type of roots. c. Find the exact solutions by using the Quadratic Formula. 1. x2 16x 64 0 2. x2 3x 3. 9x2 24x 16 0 0; 1 rational; 8 4. x2 3x 40 9; 2 rational; 0, 3 5. 3x2 9x 2 0 105; 0; 1 rational; 6. 2x2 7x 0 4 169; 2 rational; 7. 5x2 2x 4 0 5, 8 76; i 19 5 2 irrational; 8. 12x2 x 6 9 6 0 289; 105 49; 2 rational; 0, 9. 7x2 6x 2 0 7 20; i 7 5 2 complex; 1 10. 12x2 2x 4 2 rational; 3 , 11. 6x2 2x 1 2 0 28; 2 complex; 12. x2 3x 6 0 3 0 196; 15; 3 i 15 2 rational; 13. 4x2 3x2 1 6 , 2 0 105; 2 irrational; 1 14. 16x2 8x 1 0 7 6 1 2 complex; 15. 2x2 5x 6 0 73; 2 irrational; 3 105 8 0; 1 rational; 2 irrational; 5 73 4 Solve each equation by using the method of your choice. Find exact solutions. 16. 7x2 18. 3x2 20. 3x2 22. x2 24. 3x2 26. 4x2 28. x2 5x 8x 13x 6x 3 54 4x 4x 0 0, 3 4 5 17. 4x2 9 21 0 4x 3 1 , 0 3 1 19. x2 3, 7 8 , 4 6 21. 15x2 23. x2 25. 25x2 22x 14x 20x 1 53 6 2 0 7 , 4 0 3 2i 10 5 3i 17 0 2 1 4i 0 2 27. 8x 29. 4x2 4x2 2 7 3 2 0 3 15 2 i 11 12x 2 2 30. GRAVITATION The height h(t) in feet of an object t seconds after it is propelled straight up from the ground with an initial velocity of 60 feet per second is modeled by the equation h(t) 16t2 60t. At what times will the object be at a height of 56 feet? 1. 75 s, 2 s 31. STOPPING DISTANCE The formula d 0. 05s2 1. 1s estimates the minimum stopping distance d in feet for a car traveling s miles per hour. If a car stops in 200 feet, what is the fastest it could have been traveling when the driver applied the brakes? about 53. 2 mi/h © Glencoe/McGraw-Hill 340 Glencoe Algebra 2 NAME ______________________________________________ DATE ____________ PERIOD _____ 6-5 Reading to Learn Mathematics The Quadratic Formula and the Discriminant How is blood pressure related to age? Read the introduction to Lesson 6-5 at the top of page 313 in your textbook. Describe how you would calculate your normal blood pressure using one of the formulas in your textbook. Pre-Activity Sample answer: Substitute your age for A in the appropriate formula (for females or males) and evaluate the expression. Reading the Lesson 1. a. Write the Quadratic Formula. x b b2 2a 4ac 5x 7, but do b. Identify the values of a, b, and c that you would use to solve 2x2 not actually solve the equation. a 2 b 5 c 7 2. Suppose that you are solving four quadratic equations with rational coefficients and have found the value of the discriminant for each equation. In each case, give the number of roots and describe the type of roots that the equation will have. Value of Discriminant 64 8 21 0 Number of Roots Type of Roots 2 2 2 1 real, rational complex real, irrational real, rational Helping You Remember 3. How can looking at the Quadratic Formula help you remember the relationships between the value of the discriminant and the number of roots of a quadratic equation and whether the roots are real or complex? Sample answer: The discriminant is the expression under the radical in the Quadratic Formula. Look at the Quadratic Formula and consider what happens when you take the principal square root of b2 4ac and apply in front of the result. If b2 4ac is positive, its principal square root will be a positive number and applying will give two different real solutions, which may be rational or irrational. If b2 4ac 0, its principal square root is 0, so applying in the Quadratic Formula will only lead to one solution, which will be rational (assuming a, b, and c are integers). If b 2 4ac is negative, since the square roots of negative numbers are not real numbers, you will get two complex roots, corresponding to the and in the symbol. © Glencoe/McGraw-Hill 341 Glencoe Algebra 2 Lesson 6-5 NAME _______________________________________________ DATE ____________ PERIOD _____ 6-5 Enrichment Sum and Product of Roots Sometimes you may know the roots of a quadratic equation without knowing the equation itself. Using your knowledge of factoring to solve an equation, you can work backward to find the quadratic equation. The rule for finding the sum and product of roots is as follows: Sum and Product of Roots If the roots of ax 2 then s1 s2 bx c s2 b and s1 a 0, with a ! 0, are s1 and s2, c . a A road with an initial gradient, or slope, of 3% can be represented by the formula y ax2 0. 03x c, where y is the elevation and x is the distance along the curve. Suppose the elevation of the road is 1105 feet at points 200 feet and 1000 feet along the curve. You can find the equation of the transition curve. Equations of transition curves are used by civil engineers to design smooth and safe roads. The roots are x 3 ( 8) 5 3( 8) 24 Equation: x2 5x 3 and x Add the roots. Multiply the roots. Example 8. 10 —8 —6 —4 —2 O —10 —20 5 —(— —, —301) 2 4 y 24 0 2 4 x —30 Write a quadratic equation that has the given roots. 1. 6, 9 2. 5, 1 3. 6, 6 x2 3x 54 0 x2 2 2 , 5 7 35x 2 4x 5 0 x2 2 12x 3 5 36 0 4. 4 3 6. 6. 7 x2 8x 13 0 4x 4 0 49x 2 42x 205 0 Find k such that the number given is a root of the equation. 7. 7; 2x2 kx 21 0 8. 2; x2 13x k 0 11 30 © Glencoe/McGraw-Hill 342 Glencoe Algebra 2 NAME ______________________________________________ DATE ____________ PERIOD _____ 6-6 Study Guide and Intervention Analyzing Graphs of Quadratic Functions The graph of y a(x h)2 k has the following characteristics: - Vertex: (h, k ) - Axis of symmetry: x h - Opens up if a 0 - Opens down if a 0 - Narrower than the graph of y x 2 if " a" 1 - Wider than the graph of y x 2 if " a" 1 Vertex Form of a Quadratic Function Example each graph. Identify the vertex, axis of symmetry, and direction of opening of a. y 2(x 4)2 11 The vertex is at (h, k) or ( 4, 11), and the axis of symmetry is x up, and is narrower than the graph of y x2. a. y 1 (x 4 4. The graph opens 2)2 10 2. The graph opens The vertex is at (h, k) or (2, 10), and the axis of symmetry is x down, and is wider than the graph of y x2. Exercises Each quadratic function is given in vertex form. Identify the vertex, axis of symmetry, and direction of opening of the graph. 1. y (x 2)2 16 2. y 4(x 3)2 7 3. y 1 (x 2 5)2 3 (2, 16); x 4. y 7(x 2; up 1)2 9 ( 3, 5. y 1 (x 5 7); x 4)2 3; up 12 (5, 3); x 6. y 6(x 5; up 6)2 6 ( 1, 7. y 2 (x 5 9); x 9)2 1; down 12 (4, 8. y 12); x 8(x 3)2 4; up 2 ( 6, 6); x 9. y 3(x 1)2 6; up 2 (9, 12); x 10. y 5 (x 2 9; up 5)2 12 (3, 11. y 2); x 4 (x 3 3; up 22 (1, 12. y 2); x 16(x 4)2 1; down 1 7)2 ( 5, 12); x 13. y 3(x 1. 2)2 5; down 2. 7 (7, 22); x 14. y 0. 4(x 7; up 0. 6)2 0. 2 (4, 1); x 15. y 1. 2(x 4; up 0. 8)2 6. 5 (1. 2, 2. 7); x 1. 2; up (0. 6, 0. 2); x down 343 0. 6; ( 0. 8, 6. 5); x up

0. 8; © Glencoe/McGraw-Hill Glencoe Algebra 2 Lesson 6-6 Analyze Quadratic Functions NAME ______________________________________________ DATE ____________ PERIOD _____ 6-6 Study Guide and Intervention (continued) Analyzing Graphs of Quadratic Functions Write Quadratic Functions in Vertex Form A quadratic function is easier to graph when it is in vertex form. You can write a quadratic function of the form y ax2 bx c in vertex from by completing the square. Example y y y y 2x2 2(x2 2(x2 2(x Write y 2x2 12x 25 in vertex form. Then graph the function. y 12x 25 6x) 25 6x 9) 25 3)2 7 18 2(x 3)2 7. The vertex form of the equation is y O x Exercises Write each quadratic function in vertex form. Then graph the function. 1. y x2 10x 32 2. y x2 6x 3. y x2 8x 6 y (x y 5)2 7 y (x 3)2 y O 9 x y (x 8 4 —4 O —4 —8 4)2 y 10 4 8 x O x —12 4. y 4x2 16x 11 5. y 3x2 12x 5 6. y 5x2 10x 9 y 4(x y 2)2 5 y 3(x y 2)2 7 y 5(x y 1)2 4 O x O x O x © Glencoe/McGraw-Hill 344 Glencoe Algebra 2 NAME ______________________________________________ DATE ____________ PERIOD _____ 6-6 Skills Practice Analyzing Graphs of Quadratic Functions Lesson 6-6 Write each quadratic function in vertex form, if not already in that form. Then identify the vertex, axis of symmetry, and direction of opening. 1. y (x 2)2 2. y x2 4 3. y x2 6 y (x (2, 0); x 4. y 3(x 2)2 0; 2; up 5)2 5 y (x (0, 4); x 5. y 5x2 4; 0; down 9 0)2 y (x 0)2 6; (0, 6); x 0; up 6. y (x 2)2 18 5)2 y 3(x ( 5, 0); x 7. y x2 2x 0; 5; down y 5(x 9; (0, 9); x 0; down 8. y x2 6x 2 0)2 y (x (2, 18); x 9. y 3x2 2)2 24x 18; 2; up y (x (1, 6); x 1)2 6; 1; up y (x ( 3, 7); x 3)2 7; 3; up y 3(x (4, 48); x 4)2 48; 4; down Graph each function. 10. y (x y 3)2 1 11. y (x 1)2 y 2 12. y y O (x 4)2 4 x O x O x 13. y 1 (x 2 2)2 y O 14. y 3x2 y 4 15. y x2 6x 4 y x O O x x Write an equation for the parabola with the given vertex that passes through the given point. 16. vertex: (4, 36) point: (0, 20)

17. vertex: (3, 1) point: (2, 0) 18. vertex: ( 2, 2) point: ( 1, 3) y © (x 4)2 36 y (x 3)2 345 1 y (x 2)2 2 Glencoe/McGraw-Hill Glencoe Algebra 2 NAME ______________________________________________ DATE ____________ PERIOD _____ 6-6 Practice (Average) Analyzing Graphs of Quadratic Functions Write each quadratic function in vertex form, if not already in that form. Then identify the vertex, axis of symmetry, and direction of opening. 1. y 6(x 2)2 1 2. y 2x2 2 3. y 4x2 8x y 6(x ( 2, 1); x 4. y x2 10x 2)2 1; 2; down 5; 5; up y 2(x (0, 2); x 5. y 2x2 0)2 0; up 2; 18 y 4(x 1)2 4; (1, 4); x 1; down 3x2 6x y (x ( 5, 5); x 7. y 2x2 5)2 16x 20 12x y x 8. y 2(x 3; up 3x2 3)2; 6. y ( 3, 0); 21 y 3(x (1, 2); x 9. y 2x2 1)2 2; 1; up 29 5 y 2(x ( 4, 0); x 10. y (x 3)2 4)2; 32 18x 4; down 1 y y 3(x 6; (3, 6); x 3; down 11. y x2 y 3)2 y 2(x 4)2 ( 4, 3); x 2x2 2x 16x 3; 4; up 1 Graph each function. 6x 5 12. y O O x x Write an equation for the parabola with the given vertex that passes through the given point. 13. vertex: (1, 3) point: ( 2, 15) 14. vertex: ( 3, 0) point: (3, 18) 15. vertex: (10, point: (5, 6) 4) y y y 1 2(x (x (x 1)2 4)2 3 4 y 1 (x 3)2 y 2 (x 10)2 4 16. Write an equation for a parabola with vertex at (4, 4) and x-intercept 6. 17. Write an equation for a parabola with vertex at ( 3, 1) and y-intercept 2. 3)2 1 18. BASEBALL The height h of a baseball t seconds after being hit is given by h(t) 16t2 80t 3. What is the maximum height that the baseball reaches, and when does this occur? 103 ft; 2. 5 s 19. SCULPTURE A modern sculpture in a park contains a parabolic arc that starts at the ground and reaches a maximum height of 10 feet after a horizontal distance of 4 feet. Write a quadratic function in vertex form that describes the shape of the outside of the arc, where y is the height of a point on the arc and x is its horizontal distance from the left-hand 5 starting point of the arc. 2 10 ft y (x 4) 10 4 ft © Glencoe/McGraw-Hill 346 Glencoe Algebra 2 NAME

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_____ 6-6 Reading to Learn Mathematics Analyzing Graphs of Quadratic Equations Read the introduction to Lesson 6-6 at the top of page 322 in your textbook. - What does adding a positive number to x2 do to the graph of y x2? It moves the graph up. - What does subtracting a positive number to x before squaring do to the graph of y x2? It moves the graph to the right. Reading the Lesson 1. Complete the following information about the graph of y a. What are the coordinates of the vertex? (h, k) b. What is the equation of the axis of symmetry? x c. In which direction does the graph open if a d. What do you know about the graph if " a" a(x h)2 k. h 0? up; down 0? If a 1? It is wider than the graph of y If " a" x 2. x 2. 1? It is narrower than the graph of y 2. Match each graph with the description of the constants in the equation in vertex form. a. a c. a i. O 0, h 0, h y 0, k 0, k 0 iii 0 ii ii. y b. a d. a iii. x O 0, h 0, h y 0, k 0, k 0 iv 0 i iv. y O x O x x Helping You Remember 3. When graphing quadratic functions such as y (x 4)2 and y (x 5)2, many students have trouble remembering which represents a translation of the graph of y x2 to the left and which represents a translation to the right. What is an easy way to remember this? Sample answer: In functions like y (x 4)2, the plus sign puts the graph " ahead" so that the vertex comes " sooner" than the origin and the translation is to the left. In functions like y (x 5)2, the minus puts the graph " behind" so that the vertex comes " later" than the origin and the translation is to the right. © Glencoe/McGraw-Hill 347 Glencoe Algebra 2 Lesson 6-6 Pre-Activity How can the graph of y function? x2 be used to graph any quadratic NAME

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_____ 6-6 Enrichment Patterns with Differences and Sums of Squares Some whole numbers can be written as the difference of two squares and some cannot. Formulas can be developed to describe the sets of numbers algebraically. If possible, write each number as the difference of two squares. Look for patterns. 1. 0 02 5. 4 22 9. 8 32 13. 12 42 02 02 12 22 2. 1 12 6. 5 32 10. 9 32 14. 13 72 02 22 02 62 3. 2 cannot 7. 6 cannot 11. 10 cannot 15. 14 cannot 4. 3 22 8. 7 42 12. 11 62 16. 15 42 12 32 52 12 Even numbers can be written as 2n, where n is one of the numbers 0, 1, 2, 3, and so on. Odd numbers can be written 2n 1. Use these expressions for these problems. 17. Show that any odd number can be written as the difference of two squares. 2n 1 (n 1)2 n2 18. Show that the even numbers can be divided into two sets: those that can be written in the form 4n and those that can be written in the form 2 4n. Find 4n for n 0, 1, 2, and so on. You get {0, 4, 8, 12, …}. For 2 4n, you get {2, 6, 10, 12, …}. Together these sets include all even numbers. 19. Describe the even numbers that cannot be written as the difference of two squares. 2 4n, for n 0, 1, 2, 3, … 20. Show that the other even numbers can be written as the difference of two squares. 4n (n 1)2 (n 1)2 Every whole number can be written as the sum of squares. It is never necessary to use more than four squares. Show that this is true for the whole numbers from 0 through 15 by writing each one as the sum of the least number of squares. 21. 0 02 24. 3 12 27. 6 12 30. 9 32 33. 12 12 36. 15 12 © 22. 1 12 23. 2 12 26. 5 12 12 22 22 12 22 32 32 12