# The key to understanding fractions 

Among the many difficult mathematical concepts for children to understand, fractions ranks as one of the most difficult. The complex concept poses a lot of problems not only for children, but for adults as well. The only way to build a conceptual understanding of fractions is to build on children's informal knowledge of partitioning and fair share. Throughout the paper I will discuss how we can use these simpler topics as a starting point in the uphill battle to understanding this extremely difficult concept.

I will identify common partitioning strategies used by children, and shed light on the way these strategies can be used to tackle the remaining four subconstructs, as well as guide in the understanding of multiplication and division of fractions. Five subconstructs make up a whole, which is used to explain all definitions of the fraction concept. " The understanding of fractions depends on gaining and understanding of each of these different meanings, as well as of their confluence" (Charalambous \& Pitta-Pantazi, 2007, p. 295).

The ratio construct of fractions focuses on the comparison of two quantities of the same type. An important realization students need to make is that there is a direct relationship between two quantities. When one quantity in the relationship changes so does the other, therefore the relationship of the two quantities remains the same. If both numbers in the ratio are multiplied by any number other than zero the ratio remains the same. The operator construct can be explained as taking a set or region and mapping it into another set or region.

The quotient construct allows fractions to be seen as division problems. These problems are usually situations where students must partition objects into an equal number of parts. They need to understand that the dividend is the number of parts in each share, while the divisor refers to the fraction name of the share. The measure construct is usually used with a number line where students must identify the length of a certain segment (Charalambous \& Pitta-Pantazi, 2007). Finally, the fifth, and newly added is the part-whole construct.

Before this was added as the fifth part to the whole, it was thought to be so embedded in the other constructs that it did not need to be specified on its own. The part-whole subconstruct of rational numbers, along with the process of partitioning, is considered a fundament for developing understanding of the four subordinate constructs of fractions" (Charalambous \& Pitta-Pantazi, 2007, p. 295). The part-whole subconstruct of fractions is a situation where a continuous area or a set of discrete objects is partitioned into equal parts. In this case, " the fraction represents a comparison between the number of parts of the partitioned unit to the total number of parts in which the unit is partitioned" (Charalambous \& PittaPantazi, 2007, p. 95).

Students need to develop an understanding of the part-whole subconstruct before they can begin to understand those that follow. The key to developing an understanding of this concept is to build on students' informal knowledge of partitioning. Knowing that sets and quantities can be partitioned into equal-sized parts, and understanding the importance of equal-sized partitions is crucial to recognising the part-whole relationship between the numerator and denominator in fractions. According to Kieren (1983), partitioning experiences may be as important to the development of rational number concepts as counting experiences are to the development of whole number concepts. That being said, it is important to understand the strategies our students use to partition objects, so we can help them mature in their understanding of the fraction concept.

Charles and Nason (2000) do a great job explaining and categorizing the strategies most young children use when partitioning. In their study they chose to focus on learning fractions in relation to the partitive quotient construct. The partitive quotient fraction construct can be operationally defined as the process in which one starts with two quantities $x$ and $y$, treats $x$ as the dividend and $y$ as the divisor and by the operation of partitive division obtains a single quantity $\mathrm{x} / \mathrm{y}$ " (Charles \& Nason, 2000, p. 193). Another important part of this study was identifying how well these strategies were able to be transferred and used in a different context.

The study resulted in a list of twelve different partitioning strategies, classified into three categories. The categories were partitive quotient construct strategies, multiplicative strategies and iterative sharing strategies. All of the strategies in the partitive quotient construct category revolved around using the number of people sharing an object to define the denominator of the fraction. For example, if 6 people shared 7 candy bars, the value of each share would be expressed in sixths. The multiplicative strategies on the other hand, used a multiplication algorithm to determine the number of parts there would be in each whole.

According to Charles and Nason (2000) the students who used these strategies had a much easier time creating an odd number of partitions. An example of these strategies would be one where a student multiplies the number of people sharing the object, by the number of objects being shared. If a student had 3 cakes to split between 4 people they would multiply 3(4) and partition each cake into 12 pieces. The student would then distribute $3 / 12$ or $1 / 4$ of every cake making a total of $3 / 4$ for each student. The final category contains iterative sharing strategies. An example of an iterative sharing strategy is one where students continually halve the object.

For example, if a student was trying to share 3 pancakes between 2 people they would halve each pancake until they had eighths. Once the pancake was partitioned into eighths they would distribute $4 / 8$ of each three pancakes to the two students. Ultimately each student would end up with $12 / 8$ or one and one half of a pancake (Charles \& Nason, 2000). The different strategies in each category vary slightly according to the situation and child employing the strategy. Children also used a variety of strategies to solve problems rather than just one. Young children's selection of partitioning strategies depends not only on their prior knowledge and experiences but also on the context of the task, the type of analog objects being shared, the number of analog objects being shared and number of shares" (Charles \& Nason, p.
216). While there is nothing wrong with using multiple strategies, students must understand three concepts in order for these strategies to work all of the time. Students need to understand that their partitioning strategy must yield equal parts that are able to be quantified accurately. In addition, the
strategy they choose must clearly show a relationship between the number of people and the fraction name, as well as a relationship between the number of objects being shared and the number of parts in each share.

Charles and Nason (2000) suggest that teachers use these three things to asses their students' understanding of partitioning and fractions. Students who are using partitioning strategies which employ all three of these concepts have a deeper understanding of the content than those using only one or two of these concepts when partitioning objects and sets. This information can be used to plan and implement activities at the level the learner is functioning at. According to the NCTM Standards by the third grade students should be " developing an understanding of fractions as parts of unit wholes, as parts of a collection, as locations on number lines, and as divisions of whole numbers" (2000).

The Ohio Academic Content Standards state that by the end of second grade students should be able to " Identify and illustrate parts of a whole and parts of sets of objects" (123). Again, these skills need to be mastered before students move on to more difficult fraction concepts. As I stated previously we should be building on our students' informal knowledge of partitioing and fair share in order to develop these skills. Problems should be written in different contexts so students may get the most varied experience possible. Along with this the best way to develop understanding of these difficult concepts is to practice them.

There are numerous real life opportunities we can take advantage of to implement these concepts into the curriculum. Students deal with fair share
situations on a daily basis. We should strive to make these opportunities teachable moments, and build on their real life experiences. According to Saxe, Gearhart and Nasir (2001), it is very possible that many children struggle with fractions because their teacher has difficulty understanding fractions. As I said at the beginning, fractions are not just a rough spot for children, but for many adults as well.

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