Time dilation and length contraction

INTRODUCTION:

Time dilationis a phenomenon (or two phenomena, as mentioned below) described by the theory of relativity. It can be illustrated by supposing that two observers are in motion relative to each other, and/or differently situated with regard to nearby gravitational masses.

Length contraction, according to Hendrik Lorentz, is the physical phenomenon of a decrease in length detected by an observer in objects that travel at any non-zero velocity relative to that observer. This contraction (more formally called Lorentz contraction or Lorentz-Fitzgerald contraction) is usually only noticeable, however, at a substantial fraction of the speed of light; and the contraction is only in the direction parallel to the direction in which the observed body is travelling.

SPECIAL RELATIVITY :

When such quantities as length, time interval and mass are considered in elementary physics, no special point is made about how they are measured This theory has a wide range of consequences which have been experimentally verified, including counter-intuitive ones such as length contraction, time dilation and relativity of simultaneity, contradicting the classical notion that the duration of the time interval between two events is equal for all observers. (On the other hand, it introduces the space-time interval, which is invariant.) Combined with other laws of physics, the two postulates of special relativity predict the equivalence of matter and energy, as expressed in the mass-energy equivalence formula E= mc2, where c is the speed of light in a vacuum. The predictions of special relativity agree well with Newtonian mechanics in their common realm of applicability, specifically in experiments in which all velocities are small compared with the speed of light. Special relativity reveals that c is not just the velocity of a certain phenomenon-namely the propagation of electromagnetic radiation (light)-but rather a fundamental feature of the way space and time are unified as space time. One of the consequences of the theory is that it is impossible for any particle that has rest mass to be accelerated to the speed of light.

POSTULATES OF SPECIAL RELATIVITY:

TWO postulates are as follows :

1. The law of physics are the same in all inertial frames of reference.
2. The speed of light in free space has the same value in all inertial frame of reference.

OVERVIEW OF TIME DILATION :

Time dilation can arise from (1) relative velocity of motion between the observers, and (2) difference in their distance from gravitational mass.

In the case that the observers are in relative uniform motion, and far away from any gravitational mass, the point of view of each will be that the other's (moving) clock is ticking at a slower rate than the local clock. The faster the relative velocity, the more is the rate of time dilation. This case is sometimes called special relativistic time dilation. It is often interpreted as time " slowing down" for the other (moving) clock. But that is only true from the physical point of view of the local observer, and of others at relative rest (i.

e. in the local observer's frame of reference). The point of view of the other observer will be that again the local clock (this time the other clock) is correct, and it is the distant moving one that is slow. From a local perspective, time registered by clocks that are at rest with respect to the local frame of reference (and far from any gravitational mass) always appears to pass at the same rate.

There is another case of time dilation, where both observers are differently situated in their distance from a significant gravitational mass, such as (for terrestrial observers) the Earth or the Sun. One may suppose for simplicity that the observers are at relative rest (which is not the case of two observers both rotating with the Earth — an extra factor described below). In the simplified case, the general theory of relativity describes how, for both observers, the clock that is closer to the gravitational mass, i. e. deeper in its " gravity well", appears to go slower than the clock that is more distant from the mass (or higher in altitude away from the center of the gravitational mass). That does not mean that the two observers fully agree: each still makes the local clock to be correct; the observer more distant from the mass (higher in altitude) makes the other clock (closer to the mass, lower in altitude) to be slower than the local correct rate, and the observer situated closer to the mass (lower in altitude) makes the other clock (farther from the mass, higher in altitude) to be faster than the local correct rate. They agree at least that the clock nearer the mass is slower in rate, and on the ratio of the difference. This is gravitational time dilation.

FORMULAE OF TIME DILATION AND LENGTH CONTRACTION:

TIME DILATION:

- t0 is the proper time between events A and B for a slow-ticking observer within the gravitational field,
- tf is the coordinate time between events A and B for a fast-ticking observer at an arbitrarily large distance from the massive object (this assumes the fast-ticking observer is using Schwarzschild coordinates, a coordinate system where a clock at infinite distance from the massive sphere would tick at one second per second of coordinate time, while closer clocks would tick at less than that rate),
- G is the gravitational constant,
- M is the mass of the object creating the gravitational field,
- r is the radial coordinate of the observer (which is analogous to the classical distance from the center of the object, but is actually a Schwarzschild coordinate),
- c is the speed of light, and r0 = 2GM / c2 is the called the Schwarzschild Radius of M. If a mass collapses so that its surface lies at less than this radial coordinate (or in other words covers an area of less than 4pG2M2 / c4), then the object exists within a black hole.

LENGTH CONTRACTION:

This effect is negligible at everyday speeds, and can be ignored for all regular purposes. It is only when an object approaches greater speeds, that it becomes important. At a speed of 13, 400, 000 m/s, the length is 99. 9% of the length at rest and at a speed of 42, 300, 000 m/s still 99%. As the magnitude of the velocity approaches the speed of light, the effect becomes dominant, as can be seen from the formula:

Note that in this equation it is assumed that the object is parallel with its line of movement. Also note that for the observer in relative movement, the length of the object is measured by subtracting the simultaneously measured distances of both ends of the object. For more general conversions, see the Lorentz transformations.

AN EXAMPLE OF TIME DILATION:

A spaceship is flying a distance of 5lighthours, for example from Earth to the dwarf planet which Earth and Pluto are motionless. Formula used :

- t.. time indicated by the spaceship clock
- t.. time indicated by the clocks of the Earth-Pluto-system
- v.. speed of the spacecraft relatively to the system of Earth and Pluto
- c.. speed of light

REMARKS:

1. In a simplifying way there was assumed an inertial system in which Earth and Pluto are motionless; especially the motion around the Sun was neglected.
2. According to an important result of the theory of relativity, an observer in the Earth-Pluto-system would see the spacecraft shortened in the direction of motion. This so-called Lorentz contraction was not taken into consideration in order to make it possible to read off the spaceship’s clock.

BASIS IN RELATIVITY:

The origin of length contraction in the special theory of relativity can be traced to the operational definitions of simultaneity and length. According to Milne and Bondi the following operational definitions are assigned to simultaneity and length: an observer moving uniformly along a straight line sends out a light signal at time t0 to a distant point (stationary according to the observer), where it arrives and is immediately reflected at time tr, arriving back at the observer at time ta. What time does the observer ascribe to the time of reflection tr, or, what event is simultaneous with the reflection? Let l be the distance to the point of reflection. An observer, with his or her definition of c, says it takes time l / c for light to reach the reflector. Because light travels at the same speed c in both directions, it takes the same time both ways, so it returns to the observer at time ta = t0 + 2 l / c, or in other words, the distance to the point of reflection is l = c (ta – t0) / 2, and the time at which reflection occurred is simultaneous with the clock registering (t0 + ta) / 2. With these operational definitions for determining length and simultaneous events, two observers in constant relative motion at velocity v are considered, and their time and length scales compared. The result of the above definitions is that time and length are connected by the Lorentz factor ?:

PHYSICAL ORIGIN OF LENGTH CONTRACTION:

Length contraction as a physical effect on bodies composed of atoms held together by electromagnetic forces was proposed independently by George FitzGeraldand by Hendrik Lorentz . The following quote from Joseph Larmor is indicative of the pre-relativity view of the effect as a consequence of James Clerk Maxwell's electromagnetic theory:

"… if the internal forces of a material system arise wholly from electromagnetic actions between the system of electrons which constitute the atoms, then the effect of imparting to a steady material system a uniform velocity of translation is to produce a uniform contraction of the system in the direction of motion, of amount (1-v2/c2)1/2

The extension of this specific result to a general result was (and is) considered " ad hoc" by many who prefer Einstein's deduction of it from the Principle of Relativity without reference to any physics. In other words, length contraction is an inevitable consequence of the postulates of special relativity. To gain a little physical insight on why length contractions occur, consider what those postulates involve: by requiring the speed of light (a quantity dependent on the fundamental properties of space and time) to be invariant in all frames of reference (including ones in motion) one can appreciate that it would require the " distortion" of the measures of length and time. Apparently Lorentz did not agree to the criticism that his proposal was " ad hoc".

"… the interpretation given by me and FitzGerald was not artificial. It was more so that it was the only possible one, and I added the comment that one arrives at the hypothesis if one extends to other forces what one could already say about the influence of a translation on electrostatic forces. Had I emphasized this more, the hypothesis would have created less of an impression of being invented ad hoc." (emphasis added)

The Trouton-Rankine experiment in 1908 showed that length contraction of an object according to one frame, did not cause changes in the resistance of the object in its rest frame. This is in agreement with some current theories at the time (Special Relativity and Lorentz ether theory) but in disagreement with FitzGerald's ideas on length contraction.

EXPERIMENTAL CONFIRMATION:

Time dilation has been tested a number of times. The routine work carried on in particle accelerators since the 1950s, such as those at CERN, is a continuously running test of the time dilation of special relativity. The specific experiments include:

Velocity time dilation tests

- Ives and Stilwell (1938, 1941), " An experimental study of the rate of a moving clock", in two parts. The stated purpose of these experiments was to verify the time dilation effect, predicted by Lamor-Lorentz ether theory, due to motion through the ether using Einstein's suggestion that Doppler effect in canal rays would provide a suitable experiment. These experiments measured the Doppler shift of the radiation emitted from cathode rays, when viewed from directly in front and from directly behind. The high and low frequencies detected were not the classical values predicted.
- Rossi and Hall (1941) compared the population of cosmic-ray-produced muons at the top of a mountain to that observed at sea level. Although the travel time for the muons from the top of the mountain to the base is several muon half-lives, the muon sample at the base was only

moderately reduced. This is explained by the time dilation attributed to their high speed relative to the experimenters. That is to say, the muons were decaying about 10 times slower than if they were at rest with respect to the experimenters.

- Hasselkamp, Mondry, and Scharmann(1979) measured the Doppler shift from a source moving at right angles to the line of sight (the transverse Doppler shift). The most general relationship between frequencies of the radiation from the moving sources is given by:

as deduced by Einstein (1905). For phi = 90^circ(cosphi = 0,) this reduces to fdetected = frest?. Thus there is no transverse Doppler shift, and the lower frequency of the moving source can be attributed to the time dilation effect alone.

Gravitational time dilation tests

- Pound, Rebka in 1959 measured the very slight gravitational red shift in the frequency of light emitted at a lower height, where Earth’s gravitational field is relatively more intense. The results were within 10% of the predictions of general relativity. Later Pound and Snider (in 1964) derived an even closer result of 1%. This effect is as predicted by gravitational time dilation.

Velocity and gravitational time dilation combined-effect tests

- Hafele and Keating, in 1971, flew caesium atomic clocks east and west around the Earth in commercial airliners, to compare the elapsed time against that of a clock that remained at the US Naval Observatory. Two opposite effects came into play. The clocks were expected to age more quickly (show a larger elapsed time) than the reference clock, since they were in a higher (weaker) gravitational potential for most of the trip (c. f. Pound, Rebka). But also, contrastingly, the moving clocks were expected to age more slowly because of the speed of their travel. The gravitational effect was the larger, and the clocks suffered a net gain in elapsed time. To within experimental error, the net gain was consistent with the difference between the predicted gravitational gain and the predicted velocity time loss. In 2005, the National Physical Laboratory in the United Kingdom reported their limited replication of this experiment. The NPL experiment differed from the original in that the caesium clocks were sent on a shorter trip (London-Washington D. C. return), but the clocks were more accurate. The reported results are within 4% of the predictions of relativity.

- The Global Positioning System can be considered a continuously operating experiment in both special and general relativity. The in-orbit clocks are corrected for both special and general relativistic time dilation effects as described above, so that (as observed from the Earth’s surface) they run at the same rate as clocks on the surface of the Earth. In addition, but not directly time dilation related, general relativistic correction terms are built into the model of motion that the satellites broadcast to receivers – uncorrected, these effects would result in an approximately 7-metre (23ft) oscillation in the pseudo-ranges measured by a receiver over a cycle of 12 hours.

Muon lifetime

A comparison of muon lifetimes at different speeds is possible. In the laboratory, slow muons are produced, and in the atmosphere very fast moving muons are introduced by cosmic rays. Taking the muon lifetime at rest as the laboratory value of 2. 22 μs, the lifetime of a cosmic ray produced muon traveling at 98% of the speed of light is about five times longer, in agreement with observations. In this experiment the “ clock” is the time taken by processes leading to muon decay, and these processes take place in the moving muon at its own “ clock rate”, which is much slower than the laboratory clock.

TIME DILATION AND SPACE FLIGHT:

Time dilation would make it possible for passengers in a fast-moving vehicle to travel further into the future while aging very little, in that their great speed slows down the rate of passage of on-board time. That is, the ship’s clock (and according to relativity, any human travelling with it) shows less elapsed time than the clocks of observers on Earth. For sufficiently high speeds the effect is dramatic. For example, one year of travel might correspond to ten years at home. Indeed, a constant 1g acceleration would permit humans to travel as far as light has been able to travel since the big bang (some 13. 7 billion light years) in one human lifetime. The space travellers could return to Earth billions of years in the future. A scenario based on this idea was presented in the novel Planet of the Apes by Pierre Boulle.

A more likely use of this effect would be to enable humans to travel to nearby stars without spending their entire lives aboard the ship. However, any such application of time dilation during Interstellar travel would require the use of some new, advanced method of propulsion.

Current space flight technology has fundamental theoretical limits based on the practical problem that an increasing amount of energy is required for propulsion as a craft approaches the speed of light. The likelihood of collision with small space debris and other particulate material is another practical limitation. At the velocities presently attained, however, time dilation is not a factor in space travel. Travel to regions of space-time where gravitational time dilation is taking place, such as within the gravitational field of a black hole but outside the event horizon (perhaps on a hyperbolic trajectory exiting the field), could also yield results consistent with present theory.

LORENTZ TRANSFORMATION:

In physics, the Lorentz transformation, named after the Dutch physicist Hendrik Lorentz, describes how, according to the theory of special relativity, two observers' varying measurements of space and time can be converted into each other's frames of reference. It reflects the surprising fact that observers moving at different velocities may measure different distances, elapsed times, and even different orderings of events.

The Lorentz transformation was originally the result of attempts by Lorentz and others to explain observed properties of light propagating in what was presumed to be the luminiferous aether; Albert Einstein later reinterpreted the transformation to be a statement about the nature of both space and time, and he independently re-derived the transformation from his postulates of special relativity. The Lorentz transformation supersedes the Galilean transformation of Newtonian physics, which assumes an absolute space and time (see Galilean relativity). According to special relativity, this is only a good approximation at relative speeds much smaller than the speed of light.

LORENTZ TRANSFORMATION

RELATIVISTIC LENGTH CONTRACTION:

One of the peculiar aspects of Einstein's theory of special relativity is that the length of objects moving at relativistic speeds undergoes a contraction along the dimension of motion. An observer at rest (relative to the moving object) would observe the moving object to be shorter in length. That is to say, that an object at rest might be measured to be 200 feet long; yet the same object when moving at relativistic speeds relative to the observer/measurer would have a measured length which is less than 200 ft. This phenomenon is not due to actual errors in measurement or faulty observations. The object is actually contracted in length as seen from the stationary reference frame. The amount of contraction of the object is dependent upon the object's speed relative to the observer.

Temporal coordinate systems and clock synchronization

In Relativity, temporal coordinate systems are set up using a procedure for synchronizing clocks, discussed by Poincaré (1900) in relation to Lorentz's local time (see relativity of simultaneity). It is now usually called the Einstein synchronization procedure, since it appeared in his 1905 paper.

An observer with a clock sends a light signal out at time t1 according to his clock. At a distant event, that light signal is reflected back to, and arrives back to the observer at time t2 according to his clock. Since the light travels the same path at the same rate going both out and back for the observer in this scenario, the coordinate time of the event of the light signal being reflected for the observer tE is tE = (t1 + t2) / 2. In this way, a single observer's clock can be used to define temporal coordinates which are good anywhere in the universe.

Symmetric time dilation occurs with respect to temporal coordinate systems set up in this manner. It is an effect where another clock is being viewed as running slowly by an observer. Observers do not consider their own clock time to be time-dilated, but may find that it is observed to be time-dilated in another coordinate system.

SIMPLE INFERENCE OF TIME DILATION :

Time dilation can be inferred from the observed fact of the constancy of the speed of light in all reference frames.

This constancy of the speed of light means, counter to intuition, that speeds of material objects and light are not additive. It is not possible to make the speed of light appear faster by approaching at speed towards the material source that is emitting light. It is not possible to make the speed of light appear slower by receding from the source at speed. From one point of view, it is the implications of this unexpected constancy that take away from constancies expected elsewhere.

Consider a simple clock consisting of two mirrors A and B, between which a light pulse is bouncing. The separation of the mirrors is L, and the clock ticks once each time it hits a given mirror.

In the frame where the clock is at rest (diagram at right), the light pulse traces out a path of length 2L and the period of the clock is 2L divided by the speed of light:

From the frame of reference of a moving observer traveling at the speed v (diagram at lower right), the light pulse traces out a longer, angled path. The second postulate of special relativity states that the speed of light is constant in all frames, which implies a lengthening of the period of this clock from the moving observer's perspective. That is to say, in a frame moving relative to the clock, the clock appears to be running more slowly. Straightforward application of the Pythagorean theorem leads to the well-known prediction of special relativity:

The spacetime geometry of velocity time dilation

Time dilation in transverse motion.

The green dots and red dots in the animation represent spaceships. The ships of the green fleet have no velocity relative to each other, so for the clocks onboard the individual ships the same amount of time elapses relative to each other, and they can set up a procedure to maintain a synchronized standard fleet time. The ships of the " red fleet" are moving with a velocity of 0. 866 of the speed of light with respect to the green fleet.

The blue dots represent pulses of light. One cycle of light-pulses between two green ships takes two seconds of “ green time”, one second for each leg.

As seen from the perspective of the reds, the transit time of the light pulses they exchange among each other is one second of “ red time” for each leg. As seen from the perspective of the greens, the red ships’ cycle of exchanging light pulses travels a diagonal path that is two light-seconds long. (As seen from the green perspective the reds travel 1. 73 (sqrt{3}) light-seconds of distance for every two seconds of green time.)

One of the red ships emits a light pulse towards the greens every second of red time. These pulses are received by ships of the green fleet with two-second intervals as measured in green time. Not shown in the animation is that all aspects of physics are proportionally involved. The light pulses that are emitted by the reds at a particular frequency as measured in red time are received at a lower frequency as measured by the detectors of the green fleet that measure against green time, and vice versa.

The animation cycles between the green perspective and the red perspective, to emphasize the symmetry. As there is no such thing as absolute motion in relativity (as is also the case for Newtonian mechanics), both the green and the red fleet are entitled to consider themselves motionless in their own frame of reference.

Again, it is vital to understand that the results of these interactions and calculations reflect the real state of the ships as it emerges from their situation of relative motion. It is not a mere quirk of the method of measurement or communication.

The four dimensions of space time

In Relativity the world has four dimensions: three space dimensions and one dimension that is not exactly time but related to time. In fact, it is time multiplied by the square root of -1. Say, you move through one space dimension from point A to point B. When you move to another space coordinate, you automatically cause your position on the time coordinate to change, even if you don’t notice. This causes time to elapse. Of course, you are always travelling through time, but when you travel through space you travel through time by less than you expect. Consider the following example:

Time dilation; the twin paradox

There are two twin brothers. On their thirtieth birthday, one of the brothers goes on a space journey in a superfast rocket that travels at 99% of the speed of light. The space traveller stays on his journey for precisely one year, whereupon he returns to Earth on his 31st birthday. On Earth, however, seven years have elapsed, so his twin brother is 37 years old at the time of his arrival. This is due to the fact that time is stretched by factor 7 at approx. 99% of the speed of light, which means that in the space traveller’s reference frame, one year is equivalent to seven years on earth. Yet, time appears to have passed normally to both brothers, i. e. both still need five minutes to shave each morning in their respective reference frame.

As it can be seen from the above function, the effect of time dilation is negligible for common speeds, such as that of a car or even a jet plane, but it increases dramatically when one gets close to the speed of light. Very close to c, time virtually stands still for the outside observer.

Time expands, space contracts

Interestingly, while time expands from the perspective of the stationary observer, space contracts from the perspective of the moving observer. This phenomenon is known as Lorentz contraction, which is exactly the reciprocal of the above time dilation formula: $l' = l * sqr(1-v^2/c^2)$. Thus the space traveller passing by Earth at a speed of 0. 99c would see it's shape as an ellipsis with the axis parallel to his flight direction contracted to a seventh of its original diameter. That is of course, if he sees it at all, given the enormous speed. Therefore, space travel is shortened with the velocity of the traveller. A journey to the 4. 3 light-years distant Alpha Centauri C, the closest star to our Sun, would take only 7. 4 months in a space ship moving at 0. 99c.

The effect of time dilation has been experimentally confirmed thanks to very precise caesium clocks that can measure extremely small periods of time. Unfortunately, time dilation is completely outside of human experience, because we have not yet devised a way of travelling at speeds where relativistic effects become noticeable. Even if you spent your whole life in a jet plane that moves at supersonic speed, you would barely win a second over your contemporaries on the ground. And, not even today's astronauts can perceive the Lorentz contraction. Imagine you are a cosmonaut on board of space station Mir, moving at 7700 meters per second relative to Earth. Looking down upon Europe from space, you would see the entire 270 kilometre east to west extent of Switzerland contracted by a mere 0. 08 millimetres.

Can we travel at the speed of light?

The hope that one day mankind will be able to travel at near-to-speed-of-light velocities seems farfetched, because of the incredible amounts of energy needed to accelerate a spacecraft to these speeds. The forces are likely to destroy any vehicle before it comes even close to the required speed. In addition, the navigational problems of near-to-speed-of-light travel pose another tremendous difficulty. Therefore, when people say they have to hurry in order to “ win time”, they probably don’t mean it in a relativistic way.

Kant: Space and time are properties of thought

The German philosopher, Immanuel Kant (1724-1804), maintained that time and space are a priori particulars, which is to say they are properties of perception and thought imposed on the human mind by nature. This subtle position allowed Kant to straddle the well-known differences about the reality of space and time that existed between Newton and Leibniz. Newton held that space and time have an absolute reality, in the sense of being quantifiable objects. Leibniz held against this that space and time weren’t really “ things”, such as cup and a table, and that space and time have a different quality of being. Kant’s position agrees with Newton in the sense that space and time are absolute and real objects of perception, hence, science can make valid propositions about them. At the same time, he agrees with Leibniz by saying that time and space are not “ things in themselves,” which means they are fundamentally different from cups and tables. Of course, this view of space and time also introduces new problems. It divides the world into a phenomenal (inner) reality sphere and an noumenal (outer) reality sphere. From this academic separation arise many contradictions in epistemology. We will, however, not deal with this particular problem at this point.

Life in a spacetime cubicle

From Relativity we learn that time and space is seemingly independent of human experience, as the example of time dilation suggests. Since our own perception of time and space is bound to a single reference frame, time appears to be constant and absolute to us. Physics teaches us that this is an illusion and that our perception deceived us within living memory. Thanks to Einstein, we are now able to draw relativistic spacetime diagrams, compute gravitational fields, and predict trajectories through the four-dimensional spacetime continuum. Still, we are hardly able to visualise this spacetime continuum, or deal with it in practical terms, because human consciousness is bound to the human body, which is in turn bound to a single reference frame. We live within the confinements of our own spacetime cubicle.

Considering that in Relativity, spacetime is independent of human perception, the Kantian understanding of space and time as a priori particulars seems to be obsolete. They are no longer properties of perception, but properties of nature itself. But, there is more trouble looming for Kant. Relativity stretches the distincti