Experiment 1: penny pinching

Experiment 1 PENNY PINCHING: STATISTICAL TREATMENT OF DATA Objective Interpretation is one of the important steps for a chemical analysis. Upon receiving raw data, anyone whether scientists or non-scientists can give some thoughts about the results, such as the similarity or difference between the values or the connection between measurements. Scientists are believed to give a better interpretation as they are able to recognize a significant difference between raw data and final results.

These results, which are mainly based on the mean values, average values, and standard deviations, however, can still be biased and misinterpreted without using appropriate statistical tools, such as the Q test and the Student’s t test. The Q test allows us to determine if a value can be discarded or retained, while the Student’s t test is used to determine the uncertainty and confidence associated with the assignment of a value. These analysis tools are proved to be very helpful as the data and results can be interpreted in a less biased manner. In this experiment each group of students obtained a sample of 20 pennies.

Each penny was weighted and the mass was recorded along with the year of penny. The Q test was performed to determine if any values would be rejected. The whole class data set was used to construct a frequency histogram and two distinct distributions in the penny masses were noticed. The Student’s t test was then used to evaluate whether the two distributions of pennies really represent two distinct sets of pennies or simply one set of penny weights. By using these two statistical tests, it was concluded that there is a correlation between the year of the pennies and the apparent bimodal distribution of penny weights.

Data Table 1. Individual Data for the masses and the years of pennies (n= 16 measurements). | Pennies | Year | Weight (g) | | 1 | 1987 | 2. 4533 | | 2 | 2004 | 2. 4691 | | 3 | 1988 | 2. 4723 | | 4 | 1993 | 2. 4760 | | 5 | 1999 | 2. 4763 | 6 | 2002 | 2. 4832 | | 7 | 1997 | 2. 4912 | | 8 | 1998 | 2. 5033 | | 9 | 1993 | 2. 5055 | | 10 | 2002 | 2. 5155 | | 11 | 1985 | 2. 5161 | | 12 | 1984 | 2. 568 | | 13 | 1975 | 3. 0969 | | 14 | 1979 | 3. 1063 | | 15 | 1973 | 3. 1316 | | 16 | 1982 | 3. 1873 | Table 2. Frequency Table for Class Data of the penny masses over the mass range of 2. 300g and 3. 300g (n= 176 measurements). | Range (g) |# of pennies | | Range (g) |# of pennies | | 2. 00 to 2. 325 | 0 | | 2. 800 to 2. 825 | 0 | | 2. 325 to 2. 350 | 0 | | 2. 825 to 2. 850 | 0 | | 2. 350 to 2. 375 | 1 | | 2. 850 to 2. 875 | 0 | | 2. 375 to 2. 400 | 0 | | 2. 875 to 2. 900 | 0 | | 2. 400 to 2. 425 | 0 | | 2. 900 to 2. 925 | 0 | | 2. 425 to 2. 450 | 3 | | 2. 25 to 2. 950 | 0 | | 2. 450 to 2. 475 | 11 | | 2. 950 to 2. 975 | 0 | | 2. 475 to 2. 500 | 18 | | 2. 975 to 3. 000 | 0 | | 2. 500 to 2. 525 | 27 | | 3. 000 to 3. 025 | 0 | | 2. 525 to 2. 550 | 8 | | 3. 025 to 3. 050 | 1 | | 2. 550 to 2. 575 | 2 | | 3. 050 to 3. 075 | 5 | | 2. 75 to 2. 600 | 0 | | 3. 075 to 3. 100 | 9 | | 2. 600 to 2. 625 | 0 | | 3. 100 to 3. 125 | 8 | | 2. 625 to 2. 650 | 0 | | 3. 125 to 3. 150 | 4 | | 2. 650 to 2. 675 | 0 | | 3. 150 to 3. 175 | 0 | | 2. 675 to 2. 700 | 0 | | 3. 175 to 3. 200 | 2 | | 2. 700 to 2. 725 | 0 | | 3. 200 to 3. 25 | 0 | | 2. 725 to 2. 750 | 0 | | 3. 225 to 3. 250 | 0 | | 2. 750 to 2. 775 | 0 | | 3. 250 to 3. 275 | 0 | | 2. 775 to 2. 800 | 0 | | 3. 275 to 3. 300 | 0 | Calculation and Graphs Q-test Calculations for Individual Data: Qcalc = Gap (1)Gap (low) = 2. 4691 – 2. 4533 = 0. 0158 g RangeGap (high) = 3. 1873– 3. 1316 = 0. 0557 g Range (high-low) = 3. 1577 – 2. 3922 = 0. 7340 g Qcalc(low) = 0. 0158 = 0. 0215 . 7340 Qcalc(high) = 0. 0557 = 0. 0759 0. 7340 Qcrit (n= 20, 90% confidence level) = 0. 3000 Qcalc(low) < QcritQcalc(high) < Qcrit Q calculated in both cases (high and low) is less than Q critical (table), so we retain both values. Mean and Standard Deviation for Individual Data: Mean = ? xi? xi = sum of measured values(2) n n = number of measurements Standard Deviation (s) = ((? (xi – mean)2) / (n-1))1/2(3) ? xi = sum of measured values n = number of measurements Mean = 42. 4407 = 2. 6525 g 16 s = ((1. 5990)/(19))1/2 = 0. 3668 g Mean and Standard Deviation for Low Distribution:

Mean = 397. 7725 = 2. 5017 g 159 s = ((0. 1212)/(158))1/2 = 0. 0277 g Mean and Standard Deviation for High Distribution: Mean = 185. 9168 = 3. 0986 g 60 s = ((0. 0562)/(59))1/2 = 0. 0073 g Student's T-test: Spooled = (((? Set 1 (xi-mean1)2) + (? Set 2 (xi-mean2)2))) ? (4) (n1+n2 – 2) Spooled = (((0. 1092 + 0. 0562))) ? = 0. 0276g (159 + 60 – 2) tcalc = ¦Mean1 – Mean2¦ ((n1n2)) ? (5) Spooled ((n1+n2)) tcalc = ¦2. 5017 – 3. 0986¦ ((9660)) ? = 143. 6358 0. 0276 ((219)) ttable (99% confidence level, DOF = ?) = 2. 576 tcalc > ttable [pic] Figure 1.

The frequency histogram of the penny masses using the class data set over the mass range between 2. 300g and 3. 300g (n= 219 measurements). [pic] Figure 2. The graph shows the year of the pennies and the correspondent masses (n= 219 pennies). Summarized Results For the individual data in Table 1, the Q test is used to determine whether to retain or discard the lowest value 2. 3922g and the highest value 3. 1879g. By Equation (1), the values Qcalc for the lowest and highest measurements are calculated to be 0. 0874 and 0. 0362 respectively. As the value Qcrit for 20 measurements at 90% confidence level is 0. 00, both of Qcalc values are smaller than Qcrit. The mean of the individual data is 2. 6753g and the standard deviation is 0. 3668g. For each distribution of pennies' masses in Figure 1, the mean and standard deviation are calculated by Equation (2) and (3). The low distribution ranges from 2. 350g to 2. 600g, its mean is 2. 5017g and its standard deviation is 0. 0277g. The high distribution ranges from 3. 025g to 3. 250g with the mean of 3. 0986g and the standard deviation of 0. 0073g. To perform the Student's t test, Spooled value is calculated using Equation (4) and the obtained value is 0. 0276g.

The tcalc value is then found by Equation (5) to be 143. 6358. As ttable for infinity degree of freedom (DOF=?) at 99% confidence level is 2. 576, the tcalc is much greater than the ttable value. Conclusions The Q test for the individual data set in Table 1 shows that the values Qcalc of both lowest and highest measurements are smaller than the value Qcrit for 20 measurements at 90% confidence level. This result indicates that there are no outliers for the data, and the measurements of 2. 3922g and 3. 1879g are both retained. It also means that the experiment was carefully performed and less error was made.

By using the class data set, a frequency table and a frequency histogram of the penny masses, in which the mass range is divided between 2. 300g and 3. 300g into 40 equal intervals, are shown in Table 2 and Figure 2. It is noticeable that there are two distributions of pennies on the graph. The low distribution ranges from 2. 350g to 2. 600g, and the high distribution ranges from 3. 025g to 3. 200g. To answer the question whether the two distributions of pennies represent two distinct sets of pennies or simply the random error associated with one set of penny weights, the Student's T test was performed.

The result shows that the value tcalc is much greater than the ttable for infinity degree of freedom at 99% confidence level, which means that the difference between the two distributions is statistically significant. In other word the two distributions of pennies truly represent two distinct sets of pennies at 99% confidence level. One explanation for this could be that the pennies have been worn out over time. Figure 2, however, shows that most of the pennies before the year 1982 are heavier than the pennies after 1982, so the previous explanation seems invalid.

There must be a correlation between the identity of the pennies and the apparent bimodal distribution of penny weights. The materials for penny throughout years might have been different. Without proper control, the differences between measurements could be considered insignificant. In this experiment the collected data for the masses and years of pennies have been treated statistically. With proper statistical analysis tools, such as the Q test and the Student's t test, the results of the experiment are interpreted in a less biased manner.