Definition of game theory

Psychology

Introduction

Game theory is a branch of mathematics concerned with studying the strategic decision making of multiple intelligent parties. In determining the set of equilibrium strategies for each player, a game theorist factors in the possible outcomes, actions, and knowledge present to the parties. These equilibrium strategies sets an equilibrium to the game, in which outcomes occur with known probabilities. The decisions of a party, furthermore, affect the outcomes of the other parties. Each party, therefore, is prone to considering the possible decisions of the other parties. This tension between parties compel a game theorist to focus on the conflict and cooperation between these parties, in which each party desires the most beneficial outcome for themselves (Wikipedia, “ Game Theory”).

The above table shows the possible decisions and outcomes for Michael and Lucifer. If Michael and Lucifer both choose Loyalty, Lucifer will be 80% happy while Michael will be 100% happy. If Lucifer chooses Loyalty and Michael chooses Betrayal, Lucifer and Michael will both be 80% happy. The table also shows that the best strategy for Michael is Loyalty, since choosing Loyalty will make him 100% or 90% happy, while choosing Betrayal will make him 80% happy. Contrarily, the best strategy for Lucifer is Betrayal, since that decision will bring him the most happiness.

Motivation

Describing the optimal strategy for each player, a nash equilibrium exists when no player in the game takes a different action, provided that the actions of the other players remain the same. The player does not have an incentive to change his or her strategy after taking into factor the strategy of the opponent(s); there is no benefit from changing strategy, assuming the other players do not change their strategies (Investopedia, “ Nash Equilibrium”).

Game theorists utilize Nash equilibrium to analyze the possible outcomes of multiple parties, predicting the most optimal decisions of multiple people or institutions dealing with one another. The underlying idea behind Nash equilibrium is that one cannot predict the outcome of multiple parties by only analyzing one decision at a time. Instead, one must understand what each party would do, while each takes into account the decision of the other parties (“ Nash Equilibrium and Dominant Strategies”).

The new table representing the decisions and outcomes for Lucifer and Michael shows that there is a Nash equilibrium. Since Michael now knows that the best strategy for Lucifer is Loyalty, since Lucifer will be either 50% or 70% happy. Lucifer, on the other hand, knows that the best strategy for Michael is Loyalty as well, since Michael will also be either 50% or 70% happy. The Nash equilibrium, therefore, is Loyalty for both players.

Background/Related Work

The Prisoner’s’ Dilemma is an example of game theory, in which two individuals choose not to cooperate, even if they were to be better off through cooperation. Each player has the choice of either confessing on being silent. No matter the choice of Player 1, Player 2 can have a better outcome by confessing. If Player 1 confesses, then Player 2 should confess as well to avoid a harsher punishment. If Player 1 chooses to be silent, Player

2 can obtain a very favorable outcome by confessing. When both choose to keep silent, however, the outcome is better than had they both chosen to confess. These individuals, therefore, choose the most dominant strategy for their own interest (Dixit). Using the Prisoner's' Dilemma, one can better understand how the outcomes will be a Nash equilibrium.

Dean and Sam Winchester are brothers who know the location of the Demon Tablets, which are capable of closing the gates of Hell. Currently, they are captured by Crowley, the King of Hell. Each brother knows that spending the least amount of years in Hell is the best outcome. If both brothers talk, they each get 333 years in Hell. If they both refuse to talk, they each spend 100 years in Hell. If one brother talks, while the other does not, the one who talks gets 0 year while the other gets 666 years.

Both brothers know that talking is the dominant strategy. Since the brothers will both choose to talk, 333 years is the outcome for both and is also the outcome in the Nash equilibrium. One can make sure that this strategy is the Nash equilibrium by understanding that neither brother would want to refuse, since if one brother refuses to talk, the other would choose to talk and be free from Hell.

Analysis/Illustration

Nash equilibrium can be defined as a set of strategies, in which no player can do better by changing his or her strategy, assuming that each player knows the strategies of the others. Each player considers the best outcome knowing that the strategies of the other players would not change. If any player, however, would benefit by changing his or her strategy, then the set of strategies is not a Nash equilibrium. The set of strategies must be the best, optimal decisions from each player, based on all the other strategies in the set (“ Nash Equilibrium and Dominant Strategies”).

Definition

Pure Strategy is the set of strategies for a particular player. The pure strategy will completely define how a player will play the game, determining the move a player will make for a situation that he or she faces.

Mixed Strategy is created when a player assigns a probability to each pure strategy, allowing the player to randomly select a pure strategy. There are an infinite amount of mixed strategies for each player (Mobius).

Conditions

1. The players aim to maximize their expected payoffs.
2. The players are capable of carrying out their decisions.
3. The players are intelligent enough to understand the outcomes of their strategies.
4. The players know the equilibrium strategies of all the other players.
5. The players understand that a change in their own strategies will not cause changes in the strategies of the other players.
6. The players know that the other players are following the same rules (Wikipedia, “ Nash Equilibrium”).

Theorem – Nash equilibrium Existence

Every finite game has a Nash equilibrium.

Nash equilibrium Existence in a 2 by 2 Game

Consider the above 2 by 2 game about Dean and Sam Winchester. Keep in mind that the sets of strategies and outcomes for Dean and Sam are the same.

Let SDean = Talk + Refuse be the set of strategies for Dean.

Let ODean = 0, 100, 333, + 666 years be the set of outcomes for Dean.

Let XDean to be the strategy for Dean and O(XDean, XSam) to be the outcome factoring in the strategies of both brothers.

XDean and XSam are either Talk or Refuse.

From the perspective of Dean, he knows that the best strategy for Sam is Talk, since Sam is guaranteed that he would not receive 666 years in Hell and that he could even be set free if Dean chooses Refuse. Dean, therefore, has to choose Talk as the dominant strategy.

X*Dean and X*Sam, therefore, are Talk.

O(X*Dean, X*Sam) ≥ O(X*Sam, X*Dean); each brother serves the next 333 years in Hell.

Nash equilibrium Existence in a General Finite Case

Assume that n players are playing a game.

Let set S be the set of all strategies, with Si being the strategy for player i and i ∈{1, 2, …, n}. Set S cannot be empty, since each player has at least one strategy. Therefore, S = S1 x S2 x … x Sn.

Let set O to be the set of all outcomes. Therefore, O = O1(X), O2(X), … , On(X) is the outcome function for X ∈ S. Set O also cannot be empty, since there is at least one possible outcome from a strategy.

Let Xi be the strategy of player i and X-i be the strategy of all players except player i. Therefore, X = (X1, X2, …, Xn), with each player i obtaining Oi(X).

Let X*∈S represents a Nash equilibrium in which there is no deviation by any player from the strategy that is most optimal for each player, while factoring in the unchanging strategy of the other players.

Then ∀i, Xi ∈ Si : Oi(Xi*, X-i*) ≥ Oi(Xi, Xi*) represents an equilibrium, in which when the inequality holds true for all players, that is classified as a Nash equilibrium (Wikipedia, " Game Theory", and Mobius).