

Integration



<http://sahatmozac.blogspot.com> ADDITIONAL MATHEMATICS FORM 5

MODULE 4 INTEGRATION <http://mathsmozac.blogspot.com>

<http://sahatmozac.blogspot.com> CHAPTER 3 : INTEGRATION Content

Concept Map page 2 3-4 5 6 7 8-9 10 - 11 12 4. 1 Integration of Algebraic

Functions Exercise A 4. 2 The Equation of a Curve from Functions of

Gradients. Exercise B SPM Question Assessment Answer <http://mathsmozac.blogspot.com>

1 <http://sahatmozac.blogspot.com> Indefinite Integral a) $\int a x^n dx = \frac{a x^{n+1}}{n+1} + c$ b) $\int x^n dx = \frac{x^{n+1}}{n+1} + c$ c) $\int \frac{1}{x} dx = \ln|x| + c$ d) $\int a x^n dx = \frac{a x^{n+1}}{n+1} + c$. Integration of Algebraic Functions)) The $\int [f(x) \pm g(x)] dx = \int f(x) dx \pm \int g(x) dx$ y = y

$\int f(x) dx \pm \int g(x) dx$ o Equation of a Curve from Functions of Gradients o $y = y$

$= \int f'(x) dx + c$, $f(x) + \int f'(x) dx = f(x) + c$ <http://mathsmozac.blogspot.com> 2

<http://sahatmozac.blogspot.com> INTEGRATION 1. Integration is the reverse

process of differentiation. dy 2. If y is a function of x and $y = f(x)$ then $\frac{dy}{dx} = f'(x)$ then $\int f'(x) dx = y + c$, c = constant. dx If $\frac{dy}{dx} = f(x)$, then $\int f(x) dx = y + c$ 4. 1.

Integration of Algebraic Functions Indefinite Integral a) b) $\int a dx = ax + c$.

n a and c are constants $\int x^n dx = \frac{x^{n+1}}{n+1} + c$ n c is constant, n is an

integer and $n \neq -1$ c) $\int a x^n dx = \frac{a x^{n+1}}{n+1} + c$ n and c are

constants n is an d) $\int [f(x) \pm g(x)] dx = \int f(x) dx \pm \int g(x) dx$

<http://mathsmozac.blogspot.com> 3 <http://sahatmozac.blogspot.com> Find

the indefinite integral for each of the following. a) $\int 5 dx$ b) $\int x^3 dx$ c) $\int 2 x$

dx 5 d) $\int (x^2 + 3x^2) dx$ Always remember to include '+c' in your answers of

indefinite integrals. Solution : a) $\int 5 dx = 5x + c$ b) $\int x^3 dx = \frac{x^4}{4} + c$ c) $\int 2 x dx = x^2 + c$ d) $\int (x^2 + 3x^2) dx = \frac{x^3}{3} + x^3 + c = \frac{4x^3}{3} + c$

$\int x^2 dx = \frac{x^3}{3} + c$ $\int x^3 dx = \frac{x^4}{4} + c$ $\int 2x dx = x^2 + c$ $\int (x^2 + 3x^2) dx = \frac{x^3}{3} + x^3 + c = \frac{4x^3}{3} + c$ Find the

indefinite integral for each of the following. a) $\int x^2 + 3x^2 dx$ b) $\int x^2 + x^2 dx$

$\int x^2 + x^2 dx = \frac{x^3}{3} + \frac{x^3}{3} + c = \frac{2x^3}{3} + c$ a) Solution : $\int x^2 + 3x^2 dx = \frac{x^3}{3} + x^3 + c = \frac{4x^3}{3} + c$

$\int x^2 + x^2 dx = \frac{x^3}{3} + \frac{x^3}{3} + c = \frac{2x^3}{3} + c$ b) $\int x^2 + x^2 dx = \frac{x^3}{3} + \frac{x^3}{3} + c = \frac{2x^3}{3} + c$

$\int x^2 + x^2 dx = \frac{x^3}{3} + \frac{x^3}{3} + c = \frac{2x^3}{3} + c$ Find the

indefinite integral for each of the following. a) $\int x^2 + 3x^2 dx$ b) $\int x^2 + x^2 dx$

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$x^4 dx$? ? b) $2x^4 - 3x^3 + 4x^2 - 5x + 6$? ? $dx = ? ? 3x^2 - 2x + 1$? ? dx ? ? $x^2 - 3x + 2$? ? dx ? ? $x^2 + 1$? ? $3x^3 = ? 4$? ? c) $3x^2 - 1$? ? $4 = x^3$? ? c) $x^2 - x^3 + 3x^2 - 2$? ? dx ? ? $x^2 - 1$? ? $x^2 = ? 3$? ? c) $2x^2 - 1$? ? $1 - 3 = ? 2$? ? c) $2x^2 - 1$? ? ? ?

<http://mathsmozac.blogspot.com> 4 <http://sahatmozac.blogspot.com> 1.

Find ? ? $3x^2 - 4x + 10 dx$. ? [3m] 2. Find ? ? $x^2 - 1 - 2x + 3$? ? dx . ? [3m]

1 ? ? 3. Find ? ? $2x^2 - 3x + 4$? ? dx . $x^2 - 2$ [3m] 4. Find ? ? $2x^2 - 3x + 3$? ? $2 dx$. $4x^2 - 3$ [3m] 5. Integrate with respect to x. x^3 [3m] 6. Find ? ? $x^5 - 4x^2 - 2x + 4$? ? dx [3m] 3 ? ? 7. Find ? ? $x^6 - 6x^2 + x - 2$ [3m] 8. Integrate $x^2 - 3x + 2$ with respect to x. $x^2 - 1$ [3m] <http://mathsmozac.blogspot.com> 5

<http://sahatmozac.blogspot.com> The Equation of a Curve from Functions of Gradients $dy = f'(x) dx$ If the gradient function of the curve is $y = f(x) + c$, Find the equation of the curve that has the gradient function $3x^2$ and passes through the point (2, 3). Solution The gradient function is $3x^2$. $dy = 3x^2 dx$ $y = \int (3x^2) dx = 3x^2 \cdot \frac{1}{2} + c = \frac{3}{2}x^2 + c$ The curve passes through the point (2, 3). Thus, $x = 2, y = 3$. $3(2)^2 = 3 \cdot 2^2 + c$ $6 = 6 + c$ $c = 0$ Hence, the equation of curve is $y = \frac{3}{2}x^2$ <http://mathsmozac.blogspot.com> 6 <http://sahatmozac.blogspot.com> 1. Given that $dy = 6x^2 dx$, express y in terms of x if $y = 9$ when $x = 2$. dx 2. Given the gradient function of a curve is $4x - 1$. Find the equation of the curve if it passes through the point (1, 6). 3. The gradient function of a curve is given by $dy = kx^3 dx$, where k is a constant. Given that the tangent to the curve at the point (-2, 14) is parallel to the x-axis, find the equation of the curve. <http://mathsmozac.blogspot.com> 7 <http://sahatmozac.blogspot.com> SPM 2003- Paper 2 : Question 3 (a) Given that $y = 2x^2$ and $y = 6$ when $x = 1$, find y in terms of x. dx [3 marks] SPM 2004- Paper 2 : Question 5(a) The <https://assignbuster.com/integration/>

<http://sahatmozac.blogspot.com> The Equation of a Curve from Functions of Gradients $dy = f'(x) dx$ If the gradient function of the curve is $y = f(x) + c$, Find the equation of the curve that has the gradient function $3x^2$ and passes through the point (2, 3). Solution The gradient function is $3x^2$. $dy = 3x^2 dx$ $y = \int (3x^2) dx = 3x^2 \cdot \frac{1}{2} + c = \frac{3}{2}x^2 + c$ The curve passes through the point (2, 3). Thus, $x = 2, y = 3$. $3(2)^2 = 3 \cdot 2^2 + c$ $6 = 6 + c$ $c = 0$ Hence, the equation of curve is $y = \frac{3}{2}x^2$ <http://mathsmozac.blogspot.com> 6 <http://sahatmozac.blogspot.com> 1. Given that $dy = 6x^2 dx$, express y in terms of x if $y = 9$ when $x = 2$. dx 2. Given the gradient function of a curve is $4x - 1$. Find the equation of the curve if it passes through the point (1, 6). 3. The gradient function of a curve is given by $dy = kx^3 dx$, where k is a constant. Given that the tangent to the curve at the point (-2, 14) is parallel to the x-axis, find the equation of the curve. <http://mathsmozac.blogspot.com> 7 <http://sahatmozac.blogspot.com> SPM 2003- Paper 2 : Question 3 (a) Given that $y = 2x^2$ and $y = 6$ when $x = 1$, find y in terms of x. dx [3 marks] SPM 2004- Paper 2 : Question 5(a) The <https://assignbuster.com/integration/>

<http://mathsmozac.blogspot.com> 6 <http://sahatmozac.blogspot.com> 1. Given that $dy = 6x^2 dx$, express y in terms of x if $y = 9$ when $x = 2$. dx 2. Given the gradient function of a curve is $4x - 1$. Find the equation of the curve if it passes through the point (1, 6). 3. The gradient function of a curve is given by $dy = kx^3 dx$, where k is a constant. Given that the tangent to the curve at the point (-2, 14) is parallel to the x-axis, find the equation of the curve. <http://mathsmozac.blogspot.com> 7 <http://sahatmozac.blogspot.com> SPM 2003- Paper 2 : Question 3 (a) Given that $y = 2x^2$ and $y = 6$ when $x = 1$, find y in terms of x. dx [3 marks] SPM 2004- Paper 2 : Question 5(a) The <https://assignbuster.com/integration/>

<http://mathsmozac.blogspot.com> 7 <http://sahatmozac.blogspot.com> SPM 2003- Paper 2 : Question 3 (a) Given that $y = 2x^2$ and $y = 6$ when $x = 1$, find y in terms of x. dx [3 marks] SPM 2004- Paper 2 : Question 5(a) The <https://assignbuster.com/integration/>

gradient function of a curve which passes through A(1, 12) is $3x^2 + 6x$.

Find the equation of the curve. [3 marks] <http://mathsmozac.blogspot.com>

8 <http://sahatmozac.blogspot.com> SPM 2005- Paper 2 : Question 2 A curve has a gradient function $px^2 + 4x$, where p is a constant. The tangent to the curve at the point (1, 3) is parallel to the straight line $y + x + 5 = 0$. Find (a) the value of p, [3 marks] (b) the equation of the curve. [3 marks]

<http://mathsmozac.blogspot.com> 9 <http://sahatmozac.blogspot.com> 1.

Find the indefinite integral for each of the following. (a) $\int (4x^3 + 3x^2) dx$

(b) $\int (3x^2 + 2x + 6) dx$ (c) $\int (x^5 + 5x^3 + 3x) dx$

(d) $\int (x^2 + 2x + 2) dx$. If $dy = 4x^3 + 4x$, and $y = 0$ when $x = 2$, find y in terms of x. <http://mathsmozac.blogspot.com> 10 <http://sahatmozac.blogspot.com>

3. If $dp = v^3 + 2v$, and $p = 0$ when $v = 0$, find the value of p when $v = 1$. $dv = 2$ 4. Find the equation of the curve with gradient $2x^2 + 3x + 1$, which passes through the origin. 5. $\frac{d^2y}{dy^2}$ Given that $\frac{dy}{dx} = 4x$, and that $y = 2$ when $x = 0$. Find and y in terms of x. <http://mathsmozac.blogspot.com> 11 <http://sahatmozac.blogspot.com>

EXERCISE A 1) 2) 3) 4) 5) 6) 7) 8) $x^2 + 10x + c$ 3 2 SPM QUESTIONS 1) $y = x^2 + 2x + 7$ 2) $y = x^3 + 3x^2 + 10$ 3) $p = 3$, $y = x^3 + 2x^2 + 4x + 4$ $x^3 + 3x^2 + c$ 2 4 3 1 $x = 4x + c$ 3 $x^4 + 2x^3 + c$ 2 ASSESSMENT 1) (a) $x^4 + 3x^2 + 2x + c$ 2 2 3 (b) $3x^2 + 2x + c$ $x^6 + x^4 + c$ 9 24 $x^4 + x^3 + 9$ (d) $6x^2 + c$ 3 $x^4 + 2x^2 + 8$ $p = 7$ 8 2 3 3 2 $x^2 + x^3 + 2x^2 + 3x + 2$ 3 EXERCISE B 1) $y = 3x^2 + 2x + 1$ 3 $x^2 + 24x + 2$ 2 2 $x^2 + 2x + 3$ 3) $y = 3$ 4) $y = 5$ <http://mathsmozac.blogspot.com> 12

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<http://sahatmozac.blogspot.com> CONTENT CONCEPT MAP INTEGRATION BY
 SUBSTITUTION DEFINITE INTEGRALS EXERCISE A EXERCISE B ASSESSMENT
 SPM QUESTIOS ANSWERS 2 3 5 6 7 8 9 10 <http://mathsmozac.blogspot.com>
 14 <http://sahatmozac.blogspot.com> CONCEPT MAP INTEGRATION BY
 SUBSTITUTION $\int (ax + b)^n dx = \frac{(ax + b)^{n+1}}{a(n+1)}$ DEFINITE INTEGRALS If $\int_a^b g(x) f(x) dx = \int_a^b g(x) dx \int_a^b f(x) dx$ then $\int_a^b g(x) f(x) dx = \int_a^b g(x) dx \int_a^b f(x) dx$ where $u = ax + b$, a and b are constants, n is an integer and $n \neq -1$ OR (a) $\int_a^b f(x) dx = \int_a^b g(x) dx \int_a^b g(x) dx = \int_a^b g(x) dx \int_a^b f(x) dx$ (b) $\int_a^b f(x) dx = \int_a^b f(x) dx \int_a^b f(x) dx = \int_a^b f(x) dx \int_a^b f(x) dx$ (c) $\int_a^b f(x) dx = \int_a^b f(x) dx \int_a^b f(x) dx = \int_a^b f(x) dx \int_a^b f(x) dx$ $\int_a^b (ax + b)^n dx = \frac{(ax + b)^{n+1}}{a(n+1)}$ where a , b , and c are constants, n is integer and $n \neq -1$
<http://mathsmozac.blogspot.com> 15 <http://sahatmozac.blogspot.com>
 INTEGRATION BY SUBSTITUTION $\int (ax + b)^n dx = \frac{(ax + b)^{n+1}}{a(n+1)}$ where $u = ax + b$, a and b are constants, n is an integer and $n \neq -1$ OR $\int (ax + b)^n dx = \frac{(ax + b)^{n+1}}{a(n+1)}$ where a , b , and c are constants, n is integer and $n \neq -1$
 Find the indefinite integral for each of the following. (a) $\int (2x + 1)^3 dx$ (b) $\int 4(3x + 5)^7 dx$ (c) $\int dx (5x + 3)^3$ SOLUTION (a) $\int (2x + 1)^3 dx$ Let $u = 2x + 1$ $du = 2 dx$ $dx = \frac{du}{2}$ $\int u^3 \frac{du}{2} = \frac{1}{2} \int u^3 du = \frac{1}{2} \cdot \frac{u^4}{4} + c = \frac{1}{8} (2x + 1)^4 + c$ OR $\int (2x + 1)^4 dx = \frac{(2x + 1)^5}{5} + c$ (b) $\int 4(3x + 5)^7 dx$ (c) Let $u = 3x + 5$ $du = 3 dx$ $dx = \frac{du}{3}$ $\int 4u^7 \frac{du}{3} = \frac{4}{3} \int u^7 du = \frac{4}{3} \cdot \frac{u^8}{8} + c = \frac{1}{3} (3x + 5)^8 + c$ (3) $\int dx (5x + 3)^3$ Let $u = 5x + 3$ $du = 5 dx$ $dx = \frac{du}{5}$ $\int \frac{u^3}{5} du = \frac{1}{5} \int u^3 du = \frac{1}{5} \cdot \frac{u^4}{4} + c = \frac{1}{20} (5x + 3)^4 + c$ OR $\int 4(3x + 5)^8 dx = \frac{4}{3} \int (3x + 5)^8 dx = \frac{4}{3} \cdot \frac{(3x + 5)^9}{9} + c = \frac{4}{27} (3x + 5)^9 + c$ DEFINITE INTEGRALS If $\int_a^b g(x) f(x) dx = \int_a^b g(x) dx \int_a^b f(x) dx$ then (a) $\int_a^b g(x) f(x) dx = \int_a^b g(x) dx \int_a^b f(x) dx$ (b) $\int_a^b g(x) f(x) dx = \int_a^b g(x) dx \int_a^b f(x) dx$ (c) $\int_a^b g(x) f(x) dx = \int_a^b g(x) dx \int_a^b f(x) dx$

$\int \frac{dx}{x^2}$? ? $\int f(x) dx$? ? $\int f(x) dx$ $b a c c$ <http://mathsmozac.blogspot.com> 17
<http://sahatmozac.blogspot.com> Evaluate each of the following $\int (x^2 + 3)(x^2 + 3)$
 3) (a) $\int_{-1}^1 x^4 dx$ (b) $\int_0^2 (2x + 1)^2 dx$ SOLUTION (a) $\int_{-1}^1 x^4 dx = \left[\frac{x^5}{5} \right]_{-1}^1 = \frac{1}{5} - \frac{(-1)^5}{5} = \frac{1}{5} + \frac{1}{5} = \frac{2}{5}$ (b) $\int_0^2 (2x + 1)^2 dx = \int_0^2 (4x^2 + 4x + 1) dx = \left[\frac{4x^3}{3} + 2x^2 + x \right]_0^2 = \left(\frac{32}{3} + 8 + 2 \right) - 0 = \frac{42}{3} = 14$
 $\int \frac{dx}{x^2}$? ? $\int \frac{dx}{x^4}$? ? $\int \frac{dx}{x^2}$? ? $\int \frac{dx}{x^4}$? ? $\int \frac{dx}{x^2}$? ? $\int \frac{dx}{x^4}$? ? $\int \frac{dx}{x^2}$? ? $\int \frac{dx}{x^4}$? ? $\int \frac{dx}{x^2}$? ? $\int \frac{dx}{x^4}$? ?
 $\int \frac{dx}{x^2}$? ? $\int \frac{dx}{x^4}$? ? $\int \frac{dx}{x^2}$? ? $\int \frac{dx}{x^4}$? ? $\int \frac{dx}{x^2}$? ? $\int \frac{dx}{x^4}$? ? $\int \frac{dx}{x^2}$? ? $\int \frac{dx}{x^4}$? ?
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<http://mathsmozac.blogspot.com> 18 Distributed: 18. 1. 09 Return: 20. 1. 09
 INTEGRATE THE FOLLOWING USING SUBSTITUTION METHOD. (1) $\int (x^2 + 1)^3 dx$
 (2) $\int \frac{dx}{4 + 3x^2 + 5}$? ? $\int \frac{dx}{4 + 3x^2 + 5}$? ? (3) $\int \frac{dx}{1 + 5x^2 + 3}$? ? $\int \frac{dx}{1 + 5x^2 + 3}$? ? (4) $\int \frac{dx}{5 + x^2}$? ? ?
 $\int \frac{dx}{5 + x^2}$? ? (5) $\int \frac{dy}{5 + 4y^2}$? ? $\int \frac{dy}{5 + 4y^2}$? ? (6) $\int \frac{du}{5 + u^2}$? ? $\int \frac{du}{5 + u^2}$? ? 19
<http://sahatmozac.blogspot.com> EXERCISE B 8 1. Evaluate $\int_3^4 (x^3 + 4) dx$
 Answer : 1023. 75 2. Evaluate Answer: $\int_3^4 x(x^2 + 5) dx = \int_3^4 (x^3 + 5x) dx = \left[\frac{x^4}{4} + \frac{5x^2}{2} \right]_3^4 = \left(\frac{256}{4} + \frac{100}{2} \right) - \left(\frac{81}{4} + \frac{45}{2} \right) = \left(64 + 50 \right) - \left(20.25 + 22.5 \right) = 114 - 42.75 = 71.25$
 2 ? 3. Integrate $\int x^5$? ? with respect to x ? ? 4 4. Evaluate $\int \frac{1}{3 + 1/x}$? ? ? ?
 $\int \frac{dx}{3 + 1/x}$? ? $\int \frac{dx}{3 + 1/x}$? ? $\int \frac{dx}{3 + 1/x}$? ? $\int \frac{dx}{3 + 1/x}$? ? $\int \frac{dx}{3 + 1/x}$? ? $\int \frac{dx}{3 + 1/x}$? ? $\int \frac{dx}{3 + 1/x}$? ?
 Answer: $\int \frac{dx}{3 + 1/x} = \int \frac{x dx}{3x + 1} = \frac{1}{3} \int \frac{3x dx}{3x + 1} = \frac{1}{3} \left(\int \frac{3x + 1 - 1}{3x + 1} dx \right) = \frac{1}{3} \left(\int \left(1 - \frac{1}{3x + 1} \right) dx \right) = \frac{1}{3} \left(x - \frac{1}{3} \ln |3x + 1| \right) + c$
 Answer : $\frac{x}{3} - \frac{1}{9} \ln |3x + 1| + c$
 Evaluate $\int \frac{1}{3 + 2x^2}$? ? $\int \frac{1}{3 + 2x^2}$? ? $\int \frac{1}{3 + 2x^2}$? ? $\int \frac{1}{3 + 2x^2}$? ? $\int \frac{1}{3 + 2x^2}$? ? $\int \frac{1}{3 + 2x^2}$? ? $\int \frac{1}{3 + 2x^2}$? ?
 10 , find the value 5 Answer: $\int \frac{1}{3 + 2x^2} dx = \frac{1}{\sqrt{6}} \int \frac{1}{\sqrt{3} + \sqrt{2}x} dx = \frac{1}{\sqrt{6}} \ln \left| \frac{\sqrt{3} + \sqrt{2}x}{\sqrt{3} - \sqrt{2}x} \right| + c$
 Answer : $\frac{1}{\sqrt{6}} \ln \left| \frac{\sqrt{3} + \sqrt{2}x}{\sqrt{3} - \sqrt{2}x} \right| + c$
<http://mathsmozac.blogspot.com> 20 <http://sahatmozac.blogspot.com>
 ASSESSMENT ? 6 and 2. (a) $\int \frac{dv}{5(2 + 3v)^4}$ (b) $\int \frac{dx}{5 + 3x^2 + 1}$? ? $\int \frac{dx}{5 + 3x^2 + 1}$? ?
 that ? ? $\int f(x) dx = 3 + 2 + 3 + f(x) dx = 7$. Find (a) the value of k if (b) ? ?
 $\int kx^2 f(x) dx = 8 + \frac{1}{5} \int f(x) dx$? ? Answer : (a) k = (b) $\frac{48}{22} + \frac{3}{5}$.
 Show that $\frac{d}{dx} (x^2 + 2x^2 + 6x + 4) = 4x + 2$? ? $\frac{d}{dx} (x^2 + 2x^2 + 6x + 4) = 4x + 2$? ? $\frac{d}{dx} (x^2 + 2x^2 + 6x + 4) = 4x + 2$? ?
 $\int \frac{40}{f(x)} dx = 3$ and Hence, find the value of Answer : $\int \frac{40}{f(x)} dx = 3$ and Hence, find the value of Answer : $\frac{1}{10} \ln \left| \frac{3 + 2x}{0 + 1} \right| + c$

$\int_0^5 (x^3 + 3x^2 + 2x + 1) dx = 5$. Find $\int_0^4 (x^2 + 3x + 2) dx$. (a) $\int_0^4 (x^2 + 3x + 2) dx$ (b) $\int_0^4 (x^2 + 3x + 2) dx$ Answer: (a) -15 (b) 4 <http://mathsmozac.blogspot.com> 21 <http://sahatmozac.blogspot.com> SPM QUESTIONS SPM 2003 - PAPER 1, QUESTION 17 1. Given that $\int_0^1 (kx^2 + 1) dx = 2$, find the value of k. [4 marks] Answer: k = 5 5 Answer: k = -3 3 5 4 SPM 2005 - PAPER 1, QUESTION 21 6 6 3. Given that $\int_0^2 (2x^2 + kx) dx = 10$, find the value of k. Answer: k = 1 4 <http://mathsmozac.blogspot.com> 22 <http://sahatmozac.blogspot.com> ANSWERS EXERCISE A 1. $3(x+1)^4 + c$ 2. $60(3x+5)^{-4} + c$ 3. 20 EXERCISE B 1. 1023.75 2. $5x^3 + 3x^2 + c$ 3. 8396 4. $3x^5 + 1$ 5. 17 6. 10 7. 4 8. 2 9. 6 10. 4 11. 2 12. 6 13. 4 14. 3 15. 2 16. 5 17. 5 18. u 19. c 20. 3 21. (a) k = 3 (b) 48 2. (a) $90(2 - 3v) + c$ 100 (b) $(1 + 5x)^4 + c$ 3. 1 10 -5 SPM QUESTIONS 1. k = 2. k = 5 3. = 1 4 5 3 n = -3 4. (a) -15 (b) 4 <http://mathsmozac.blogspot.com> 23 <http://sahatmozac.blogspot.com> ADDITIONAL MATHEMATICS MODULE 6 INTEGRATION <http://mathsmozac.blogspot.com> 24 <http://sahatmozac.blogspot.com> CHAPTER 3 : INTEGRATION Content Concept Map 9. 1 Integration as Summation of Areas page 2 3 4-6 7-8 9 - 11 12 - 14 15 Exercise A 9. 2 Integration as Summation of Volumes Exercise B SPM Question Answer <http://mathsmozac.blogspot.com> 25 <http://sahatmozac.blogspot.com> a) The area under a curve which enclosed by x-axis, x = a and x = b is a) The volume generated when a curve is rotated through 360° about the x-axis is $\int_a^b \pi y^2 dx$ b) The area under a curve which enclosed by y-axis, y = a and y = b is $\int_a^b x dy$ b) The volume generated when a curve is rotated through 360° <https://assignbuster.com/integration/>

about the y-axis is $\int_a^b y^2 dx$

c) The area enclosed by a curve and a straight line $y = f(x)$ and $y = g(x)$ between $x = a$ and $x = b$ is $\int_a^b (f(x) - g(x)) dx$

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The area under a curve which is enclosed by $x = a$ and $x = b$ is $\int_a^b y dx$

The area under a curve which is enclosed by $y = a$ and $y = b$ is $\int_a^b x dy$

Note : The area is preceded by a negative sign if the region lies below the x-axis.

Note : The area is preceded by a negative sign if the region is to the left of the y-axis.

The area enclosed by a curve and a straight line $y = f(x)$ and $y = g(x)$ between $x = a$ and $x = b$ is $\int_a^b (f(x) - g(x)) dx$

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<http://sahatmozac.blogspot.com> 1. Find the area of the shaded region in the diagram.

$y = x^2 - 2x$

$y = -x^2 + 3x + 4$

$x = -1$ $x = 4$

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$y = 2$

$y = x^2 + 4x + 4$

$x = -2$ $x = 2$

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<http://sahatmozac.blogspot.com> 5. Find the area of the shaded region in the diagram

$y = 1$

$y = x^3 - yx$

$y = (x - 1)^2$

$x = k - 1$

Given that the area of the shaded region in the diagram above is 3 units². Find the value of k .

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<http://sahatmozac.blogspot.com> 3. 2 Integration as Summation of Volumes

The volume generated when a curve is rotated through 360° about the x-axis is $\int_a^b \pi y^2 dx$

The volume generated when a curve is rotated through 360° about the y-axis is $\int_a^b \pi x^2 dy$

The volume generated when a curve is rotated through 360° about the y-axis is $\int_a^b \pi x^2 dy$

<http://mathsmozac.blogspot.com> 31 <http://sahatmozac.blogspot.com> $y = x(x+1)$ Find the volume generated when the shaded region is rotated through 360° about the x-axis. $x=0$ Answer : $x=2$ $y^2 dx$ Volume generated $\int_0^2 x^2(x+1)^2 dx = \int_0^2 (x^4 + 2x^3 + x^2) dx = \left[\frac{x^5}{5} + \frac{2x^4}{4} + \frac{x^3}{3} \right]_0^2 = \frac{32}{5} + 8 + \frac{8}{3} = \frac{128}{15}$ units³.

The figure shows the shaded region that is enclosed by the curve $y = 6 - x^2$, the x-axis and the y-axis. Calculate the volume generated when the shaded region is revolved through 360° about y-axis. Answer : Given $y = 6 - x^2$ substitute $x = \sqrt{6 - y}$ into $y = 6 - x^2$ Then, $y = 6 - 0$ $y = 6 - 2^2$ $x = 2$ Volume generated $\int_0^6 x^2 dy = \int_0^6 \sqrt{6 - y} dy = \left[-\frac{2}{3}(6 - y)^{3/2} \right]_0^6 = \frac{16}{3}$ units³.

<http://mathsmozac.blogspot.com> 32 <http://sahatmozac.blogspot.com> 1. $y = x(2 - x)$ The above figure shows the shaded region that is enclosed by the curve $y = x(2 - x)$ and x-axis. Calculate the volume generated when the shaded region is revolved through 360° about the y-axis. [4 marks]

<http://mathsmozac.blogspot.com> 33 <http://sahatmozac.blogspot.com> 2. y R (0, 4) Q (3, 4) P (0, 2) $y = 4(x + 1)$ $x = 3$ The figure shows the curve $y = (x + 2)^2$. Calculate the volume generated when the shaded region is revolved through 360° about the x-axis. <http://mathsmozac.blogspot.com>

34 <http://sahatmozac.blogspot.com> 3. y R (0, 4) $x = k$ The above figure shows part of the curve $y = 3 - x$ and the straight line $x = k$. If the volume generated when the shaded region is revolved through 360° about the x-axis is 12π units³, find the value of k. <http://mathsmozac.blogspot.com>

35 <http://sahatmozac.blogspot.com> SPM 2003- Paper 2 : Question 9 (b) Diagram 3 shows a curve $x^2 + y^2 = 1$ which intersects the straight line $3y = 2x$ at point A. $3y = 2x$ $3y = 2x$ $x^2 + y^2 = 1$ $x = 0$ Diagram

3 Calculate the volume generated when the shaded region is involved 360° about the y-axis. [6 marks] <http://mathsmozac.blogspot.com> 36

<http://sahatmozac.blogspot.com> SPM 2004- Paper 2 : Question 10 Diagram 5 shows part of the curve $y = y^3 - 2x^2 + 1$ which passes through A(1, 3). A(1, 3) $y > 0$ a) b) Diagram 5 $y = y^3 - 2x^2 + 1$ Find the equation of the tangent to the curve at the point A. [4 marks] A region is bounded by the curve, the x-axis and the straight lines $x = 2$ and $x = 3$. i) Find the area of the region. ii) The region is revolved through 360° about the x-axis. Find the volume generated, in terms of π . [6 marks] <http://mathsmozac.blogspot.com> 37

<http://sahatmozac.blogspot.com> SPM 2005- Paper 2 : Question 10 In Diagram 4, the straight line PQ is normal to the curve $y = y^3 - 2x^2 + 1$ at A(2, 3). The straight line AR is parallel to the y-axis. $y = y^3 - 2x^2 + 1$ A(2, 3) R

Diagram 4 Find (a) (b) (c) Q(k, 0) x the value of k, [3 marks] the area of the shaded region, [4 marks] the volume generated, in terms of π when the region bounded by the curve, the y-axis and the straight line $y = 3$ is revolved through 360° about y-axis. [3 marks] <http://mathsmozac.blogspot.com> 38

<http://sahatmozac.blogspot.com> EXERCISE A EXERCISE B 1. 1 1 ? unit 2 15 1. 1 1 units 2 3 5 units 2 6 2. 2. 20 3 6 ? unit 3 5 k ? ? 2 3. 3. 2 2 units 2 3 2 units 2 3 SPM QUESTIONS SPM 2003 Volume Generated ? 52 ? units 3 15 4. 24 SPM 2004 i) Area ? 1 units 2 5 49 ? units 3 1125 5. 1 units 2 2 k? 4 ii) Volume Generated ? 6. SPM 2005 a) k ? 8 1 b) Area ? 12 units 2 3 c) Volume Generated ? 4? units? <http://mathsmozac.blogspot.com> 39