

# Examining matrices of relation



History of matrix had to be going back to the ancient times, because it is not applied until 1850.

Matrix is the Latin word for womb, and is same in English. It can also mean something is formed or produced.

Matrix was introduced by James Joseph Sylvester, who have brief career at the University of Virginia, which came to an abrupt end after an enraged Sylvester, hit a newspaper-reading student with a sword stick and fled the country, believing he had killed the student!

An important Chinese text from between 300 BC and AD 200, Nine Chapters of the Mathematical Art (Chiu Chang Suan Shu), gives the use in matrix method to solve simultaneous equations. And this is origins of matrix.

“ Too much and not enough,” is the concept of a determinant first appears in the treatise’s seventh chapter. These concepts is invented nearly two millennia before Japanese mathematician Seki Kowa in 1683 or his German contemporary Gottfried Leibnitz (who is also credited with the invention of differential calculus, separately from but simultaneously with Isaac Newton) found it and use it widely.

In chapter eight “ Methods of rectangular arrays,” using a counting board that is mathematically identical to the modern matrix method of solution to solve the simultaneous equation is more widely use. This is also called Gaussian elimination outlined by Carl Friedrich Gauss (1777-1855). Matrices has its important in ancient China and today it is not only solve simultaneous equation, but also for designing the computer games graphics, describing

the quantum mechanics of atomic structure, analysing relationships, and even plotting complicated dance steps!

## Background of Matrices

More and larger with amount of numerical data, measurements of one form or another gathered from their lab is confronting the scientists. However the mere collecting and recording data have been collected, data must analyze and interpreted. And here, matrix algebra is useful in both simplifying and promoting much development of many analysis methods but also in organizing computer techniques to execute those methods and present its results.

## Definition

An  $M \times N$  matrix is a rectangular array of members having  $m$  rows and  $n$  columns. The number comprising the array are called element of the matrix. The numbers  $m$  and  $n$  are called dimensions of the matrix. The set of all  $m \times n$  matrices is denoted by  $R_{m \times n}$ .

We shall ordinarily denote a matrix by an upper case Latin or Greek letter, whenever possible, an element of a matrix will be denoted by the corresponding lower case Greek letter with two subscripts, the first specifying the row that contains the element and the second the column.

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Thus the  $3 \times 3$  matrix has the form:

$A_{3 \times 3}$

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The matrix is read as A with r rows and c columns has order r x c (read as “r” by “c”) or  $A_{r \times c}$

And 4 x 3 matrix has the form:

( )

In some applications, notably those involving partitioned matrices, considerable notational simplification can be achieved by permitting matrices with one or both its dimensions zero. Such matrices will be said to be void.

### **Row and column matrix**

The  $n \times 1$  matrix A has the form

Such matrix is called a column vector which has a single column only, which looks exactly like a member of  $R^n$ . We shall not distinguish between  $n \times 1$  matrices and n-vectors; they will be denoted by upper or lower case Latin letters as convenience dictates.

Example: the  $1 \times n$  matrix R has the form

$$R' = (i_{11}, i_{12}, \dots, i_{1n}).$$

$$R' = (5, 6, 7, \dots, n)$$

Such a matrix will be called a row vector.

A well-organized notation is that of denoting matrices by uppercase letters and their elements by the lowercase counterparts with appropriate subscripts. Vectors are denoted by lowercase letters, often from the end of

the alphabet, using the prime superscript to distinguish a row vector from a column vector. Thus  $A$  is a column vector and  $R'$  is a row vector,  $\hat{I}$  is use for scalar whereby scalar represent a single number such as 2,-4

## Equal matrices

For two matrices to be equal, every single element in the first matrix must be equal to the corresponding element in the other matrix.

So these two matrices are equal:

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But these two are not:

Of course this means that if two matrices are equal, then they must have the same numbers of rows and columns as each other. So a  $3 \times 3$  matrix could never be equal to a  $2 \times 4$  matrix, for instance.

Also remember that each element must be equal to that element in the other matrix, so it's no good if all the values are there but in different places:

Combining the ideas of subtraction and equality leads to the definition of zero matrix algebra. For when  $A = B$ , then  $a_{ij} = b_{ij}$

And so

$$A - B = \{ a_{ij} - b_{ij} \} = \{ 0 \} = 0$$

Which mean in matrix are

## Square Matrix

A square matrix is a matrix which has the same number of rows and columns. An  $m \times n$  matrix  $A$  is said to be a square matrix if  $m = n$

Example: number of rows = number of columns.

\*provided no ambiguity

In the sequel the dimensions and properties of a matrix will often be determined by context. As an example of this, the statement that  $A$  is of order  $n$  carries the implication that  $A$  is square.

An  $n$ -by- $n$  matrix is known as a square matrix of order  $n$ . Any two square matrices of the same order can be added and multiplied. A square matrix  $A$  is called invertible or non-singular if there exists a matrix  $B$  such that

$$\mathbf{AB} = \mathbf{I}$$

This is equivalent to  $\mathbf{BA} = \mathbf{I}$ . Moreover, if  $B$  exists, it is unique and is called the inverse matrix of  $A$ , denoted  $A^{-1}$ .

The entries  $A_{ii}$  form the main diagonal of a matrix. The trace,  $\text{TR}(A)$  of a square matrix  $A$  is the sum of its diagonal entries. While, as mentioned above, matrix multiplication is not commutative, the trace of the product of two matrices is independent of the order of the factors:

$$\text{TR}(AB) = \text{TR}(BA).$$

Also, the trace of a matrix is equal to that of its transpose, i. e.  $\text{TR}(A) = \text{TR}(A^T)$ .

If all entries outside the main diagonal are zero,  $A$  is called a diagonal matrix.

If only all entries above (below) the main diagonal are zero,  $A$  is called a lower triangular matrix (upper triangular matrix, respectively). For example,

if  $n = 3$ , they look like

(Diagonal), (lower) and (upper triangular matrix).

## Properties of Square Matrix

- Any two square matrices of the same order can be added.
- Any two square matrices of the same order can be multiplied.
- A square matrix  $A$  is called invertible or non-singular if there exists a matrix  $B$  such that

$$AB = In.$$

## Examples for Square Matrix

For example:  $A =$  is a square matrix of order 3  $\tilde{A}$ - 3.

## Relations of matrices

If  $R$  is a relation from  $X$  to  $Y$  and  $x_1, \dots, x_m$  is an ordering of the elements of  $X$  and  $y_1, \dots, y_n$  is an ordering of the elements of  $Y$ , the matrix  $A$  of  $R$  is obtained by defining  $A_{ij} = 1$  if  $x_i R y_j$  and 0 otherwise. Note that the matrix of  $R$  depends on the orderings of  $X$  and  $Y$ .

Example: The matrix of the relation

$$\mathbf{R} = \{(1, a), (3, c), (5, d), (1, b)\}$$

From  $X = \{1, 2, 3, 4, 5\}$  to  $Y = \{a, b, c, d, e\}$  relative to the orderings 1, 2, 3, 4, 5 and a, b, c, d, e is

Example: We see from the matrix in the first example that the elements (1, a), (3, c), (5, d), (1, b) are in the relation because those entries in the matrix are 1. We also see that the domain is {1, 3, 5} because those rows contain at least one 1, and the range is {a, b, c, d} because those columns contain at least one.

## Symmetric and anti-symmetric

Let  $R$  be a relation on a set  $X$ , let  $x_1, \dots, x_n$  be an ordering of  $X$ , and let  $A$  be the matrix of  $R$  where the ordering  $x_1, \dots, x_n$  is used for both the rows and columns. Then  $R$  is reflexive if and only if the main diagonal of  $A$  consists of all 1's (i. e.,  $A_{ii} = 1$  for all  $i$ ).  $R$  is symmetric if and only if  $A$  is symmetric (i. e.,  $A_{ij} = A_{ji}$  for all  $i$  and  $j$ ).  $R$  is anti-symmetric if and only if for all  $i = j$ ,  $A_{ij}$  and  $A_{ji}$  are not both equal to 1.  $R$  is transitive if and only if whenever  $A_{ij}$  is nonzero,  $A_{jk}$  is also nonzero.

Example:

The matrix of the relation  $R = \{(1, 1), (1, 2), (1, 3), (2, 2), (2, 3), (3, 3), (4, 3)\}$  on  $\{1, 2, 3, 4\}$  relative to the ordering 1, 2, 3, 4 is  $A =$

We see that  $R$  is not reflexive because  $A$ 's main diagonal contains a 0.  $R$  is not symmetric because  $A$  is not symmetric; for example,  $A_{12} = 1$ , but  $A_{21} = 0$ .  $R$  is anti-symmetric because for all  $i = j$ ,  $A_{ij}$  and  $A_{ji}$  are not both equal to 1.

## Reflexive Matrices

In functional analysis, reflexive operator is an operator that has enough invariant subspaces to characterize it. The matrices that obey the reflexive



rules also called reflexive matrices. A relation is reflexive if and only if it contains  $(x, x)$  for all  $x$  in the base set. Nest algebras are examples of reflexive matrices. In dimensions or spaces of matrices, finite dimensions are the matrices of a given size whose nonzero entries lie in an upper-triangular pattern.

This 2 by 2 matrices is NOT a reflexive matrices

The matrix of the relation which is reflexive is

$R = \{(a, a), (b, b), (c, c), (d, d), (b, c), (c, b)\}$  on  $\{a, b, c, d\}$ , relative to the ordering  $a, b, c, d$  is

Or

In generally reflexive matrices are in the case if and only if it contains  $(x, x)$  for all  $x$  in the base set.

## **Transitive Matrices**

When we talk about transitive matrices, we have to compare the  $A$ (matrix) to the  $A^2$ (matrix). Whenever the element in the  $A$  is nonzero then the element in the  $A^2$  have to be nonzero or vice versa to show that the matrices is transitive.

For examples of transitive matrices:

Then the  $A^2$  is

Now we can have a look where all the element  $a_{ij}$  in  $A$  and  $A^2$  is either both nonzero or both are zero.

Another example:

## **Conclusion**

In conclusion, the matrix we are discussed previous is useful and powerful in the mathematical analysis and collecting data. Besides the simultaneous equations, the characteristic of the matrices are useful in the programming where we putting in array that is a matrix also to store the data. Lastly, the matrices are playing very important role in the computer science and applied mathematics. So we can manage well of matrix, then we can play easy in computer science but the matrix is not easy to understand whereby these few pages of discussion and characteristic just a minor part of matrix. With this mini project, we know more about matrix and if we need to know all about how it uses in the computer science subject, I personally think that it will be difficult as it can be very complicated.