Good thesis proposal about modelling the dynamics of impact system using the math...

[Government](#)

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INTRODUCTION

Various impact dynamic systems have been modelled postulating the expected drilling speed and displacement at given speeds. One of the most basic model [Fig 1(a)] that has been used involves a mass loaded with a force composed of static and harmonic components. Though the model is simple, the model also produces highly complex dynamics where the progression occurs at periodic system response.

However, the stick slip phenomenon observed in this motion of the rock and the drill bit is not analyzed and therefore a more realistic model is required. The other model involves modeling the motion between the rock and the drill as a frictional slider and the impacting machine as shown [Fig 1(b)].

A new model is desired to predict the motion more accurately and therefore it will be described in this paper.

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PREVIOUS MATHEMATICAL MODEL

If we consider the model in [Fig 1(b)] then it is considered to be a three degrees of motion system with a force

Pd being the harmonic component of the force that is producing the motion

M mass driven by the external force

Ps Static component of the force

Ψ being the shift in phase

Ω the frequency of the motion

k is the linear spring stiffness

Pf the dry friction force

Xm Xt and Xb represents the absolute displacements in the mass the slider top and the slider bottom

G is the gap between the drill bit and the rock at the initial stage which also is the gap between the mass and the slider top for the model. Since the model forces are directed vertically, directed towards the ground, then a static force from gravity is included.

Once the system is in motion then various phases will observed. Where if G> 0 then this is a transitional phase and the mass moves freely. However if G= 0 then the mass is touching the slider top without compressing the slider spring is not compressed. The other phase occurs when G <0 then there is a pre-compression of the spring in the slider.

In the case where G> 0 i. e. there is no contact between the mass and the slider then dynamic equations of motions are represented as shown below

Where G= 0 the equations of motion are described by the following set of equations

For the equations [2] and [3] the motion of the top of the slider is in phase with the mass and therefore the following equation is valid in describing the gap.

If we take the following set of undefined variable

Allowing the Pmax to be the normalization contact force.

Then the following three regimes of solutions can be defined:

- No contact between the mass and the slider

- Contact between mass and slider without pre-compression

- Progression of the slider

Regime A: No contact

In this case the displacement of the mass is larger than the sum of displacement of slider top and the gap,

Taking the following set of relations,

Where the ‘ is taken to denote d/dτ. The equations of motion become

z'= -12ς(z-ς) (6)

v'= 0(7)

Regime B: C with contact without progression

It occurs when

x≥z+g(8)

In this case the force acting on the mass from slider is greater than zero but smaller than threshold of dry friction force

0> 2ςz'+z-v <1(9)

Therefore in this scenario the mass and the slider top move together without progression. The solution to this motion is given by

x'= y

y'=-2ςz'-z-v+acosωτ+ψ+b(10)

In this case the velocity of the slider top is equal to velocity of the slider top and is equal to the velocity of the mass.

x'= y'(11)

x= z+g(12)

There is no progression in the bottom of the slider and hence the velocity is zero.

v'= 0(13)

Regime C: Contact with progression

In this case the displacement of the mass is equal to or greater than the displacement of the slider top and it is there described as shown

2ςz'+(z-v)≥1(14)

If equation [14] is satisfied then the mass and the bottom of the slider are moving together therefore its concluded that there is progression. The equations of motion are

x'= y

y'= acosωτ+ψ+b-1(15)

The motion of slider top in velocity and displacement is given in equations [11] and [12] . However the velocity of slider bottom is given by

v'= z'+12ς(z-v-1)

Since all the regimes of motion have been described then Heaviside function H(·) is used by introducing the parameters P1 P2 P3 P4. And defining the parameters as follows

The equations of motions then may be written as

Through numerical integration the solution to equation [18] is calculated.

Non-linear dynamic analysis

The motion of such a system usually presents a periodic response or sometimes a very complex chaotic response in the intermediary time. However at the steady state response the motion can be described as shown in Figure [2]. The solid line represents absolute displacements of impacting mass and the dashed line represents motion of the bottom of the slider.

The motion in location A and B is zoomed up to show details in figure [3(a)]

Other factors such as the static force and spring stiffness to get various results.

NEW MATHEMATICAL MODEL

The frame of the drill bit can be assumed to be suspended in air and is held onto the drill bit and then the motion analyzed. Therefore the drill is considered to be a two mass system and a static force is added due to gravity affecting the motion.

The diagram below represents the new model.

Figure 1001: New Mathematical Model

The frame mass at m2 is considered to be the exciter and has more weight than that of the drill bit. The drill bit is modelled by a mass m1 which is initially located at a distance G from the surface of the rock. For small contact forces the spring in the slider mechanism then behaves in an elastic manner however if the contact forces exceed the frictional forces then there is a displacement of the drill bit. In this motion the three regimes of motion described earlier also present themselves. The motion of the mass m2 can be as follows

$m2\ddot{x}2+cx2-x1+k2x2-x1= Fs(20)$

The drill bit mass m1 can be modeled in the three regimes as done to the previous model.

Regime A: No contact

The equation below governs this motion

x1

During the happenings of this phase the contact force is given by

Pc= k1xt-xb+c1xt-xb= 0

The equations of motion are given by

$m1\ddot{x}1+c2x1-x2+k2x1-x2= F0\cos(\Omega t+\psi)$

xt=-k1c1(xt-xb)

xb= 0 (21)

Regime B: C with contact without progression

During this stage the drill bit reaches the slider. The following conditions are satisfied

x1≥xt+G , x1> 0 , 0

The contact force is computed as

Pc= k1xt-xb+c1xt-xb (23)

As the contact force increases then the phase ends when the contact force exceeds the static friction force. The massless rock surface move together with the drill bit with the equations

xt= x1-G, xt= x1, xb= 0

The equations of motion of this section are given by

xt= x1-G

m1ẍ1+c2x1-x2+c1x1+k2x1-x2-k1(G-xb)= F0cos(Ωt+ψ)

xb= 0 (24)

Regime C: Contact with progression

At this instant the following conditions are satisfied

x1≥xt+G, x1> 0, k1xt-xb+c1xt-xb≥Pr(25)

The slider moves along counteracting the frictional force. Therefore we have the equation

k1xt-xb+c1xt-xb-Pr= 0 (26)

The contact force and friction resistance forces are equal. Also, the top of the slider moves at the same speed as the drill bit therefore xt= x1. The equations of motion of the slider are given by

xb= xt+k1c1xt-xb-Prc1 (27)

The resulting equations of motion of this phase are given by

m1ẍ1+c2x1-x2-c1x1-xb+k2x1-x2-k1(xt-xb)= F0cos(Ωt+ψ)

k1xt-xb+c1x1-G-xb= Pr

xt= x1-G(28)

(29)

Where

P1= H(z7-z8)

P2= H(2çz6-z7)

P3= H(2çz6+z7-1)

P4= H(z6)

H(·) is the Heaviside step function.

Solving this using numerical integration we have the following plots

APPENDIX

Matlab code

```
% 5g = 0. 1;

% zeta = 0. 01;

%alpha2 = 0. 1;

%alpha3 = 0. 01;

%beta2 = 0. 1;

%beta3 = 0. 1;

%gamma2 = 1. 0;

%gamma3 = 50;

%phi = 0;

%a = 0. 5;

%b = 0. 32;

%omega = 0. 4;

%initializing of variables

t = 0;

x1 = 0;

x2 = 0;

x3 = 0;

xt = 0;
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```
xb = 0;

%constants

m3 = 1000;

k3 = 100000;

c3 = 650;

fs = 1000;

m2 = 500;

k2 = 2030;

c2 = 784;

m1 = 50;

f0 = 1000;

ohm = 50;

phi = 0;

G = 0. 1;

k1 = 2000;

c1 = 654;

pr = 30;

ohm0 = sqrt( k1/m1 )

omega = ohm / ohm0;

a = f0 / pr;

b = fs / pr;

zeta = c1 / (2 * m1 * ohm0);

g = k1 * G / pr;

alpha3 = m1 / m3;

alpha2 = m1 / m2;
```

```
beta3 = k3 / k1;

beta2 = k2 / k1;

gamma3 = c3 / c1;

gamma2 = c2 / c1;

tao = ohm0 * t;

for i = 2500 : 2700 % start-up phase for iteration

t = i;

zed1 = x3 * k1 / pr;

syms x3 tao

zed2 = diff(x3, tao);

zed2 = double(zed2);

zed3 = x2 * k1 / pr;

syms x2 tao

zed4 = diff(x2, tao);

zed4 = double(zed4);

zed5 = x1 * k1 / pr;

syms x1 tao

zed6 = diff(x1, tao);

zed6 = double(zed6);

zed7 = xt * k1 / pr;

zed8 = xb * k1 / pr;

P1 = heaviside(zed1 - zed8);

P2 = heaviside(2 * zeta * zed6 - zed7);

P3 = heaviside(2 * zeta * zed6 + zed7 - 1);

P4 = heaviside(zed6);
```

```
vzed1 = zed2;
vzed2 = -alpha3 * beta3 * zed1 - 2 * zeta * alpha3 * gamma3 * zed2 +
alpha3 * beta3 * zed3 + 2 * zeta * alpha3 * gamma3 * zed4;
vzed3 = zed4;
vzed4 = alpha2 * beta2 * zed1 + 2 * zeta * alpha2 * gamma2 * zed2 - alpha2
* (beta2 + beta3) * zed3 - 2 * zeta * alpha2 * (gamma2 + gamma3) * zed4 +
alpha2 * beta2 * zed5 + alpha2 * b;
vzed5 = zed6;
vzed6 = a * cos( omega * tao + phi) - beta2 * (zed5 - zed3) - 2 * zeta *
gamma2 * (zed6 - zed4)- P1 * P2 *(1 - P3) * (2 * zeta * zed6 + zed7 -zed8) -
P1 * P3;
vzed7 = P1 * zed6 - (1 -P1) * (zed7 - zed8) / (2 * zeta);
vzed8 = P1 * P2 * P3 * ((zed7 - zed8 - 1) / (2 * zeta) + zed6);
end;
%plotting of various velocities
subplot(2, 2, 1); plot(vzed1, zed1, zed2, zed3)
%plot mode2
subplot(2, 2, 2); plot(vzed2, t)
%plot mode3
subplot(2, 2, 3); plot(vzed3, t)
%plot mode4
subplot(2, 2, 4); plot(vzed4, t)
```

Reference

Pavlovskaia Ekaterina, Marian Wierciegroch, 1st June 2001. Modelling Impact with a System Drift.