Stress distribution in a thick cylinder engineering essay

This laboratory report is based on the experiment carried out to determine the distribution of stress in a thick cylinder (similar to pipes used for transmission and distribution of gas) when subjected to internal pressure. Effort is made to critically analyse the ability of a cylinder to withstand internal pressures and the actual behaviour of stresses developed within a thick cylinder where the diameter and thickness are key to an understanding of the behaviour of the stress distribution in the thick cylinder when pressure is applied from a known source.

During the course of this experiment, the plug is connected to a data logging machine through strain gauges; pressure is exerted by a ram and different values of hoop and longitudinal stresses are recorded by the data machine.

From experimental data obtained, graphs of the strain gauge readings are plotted against penetration depth to ascertain values of constants A and B from Lame’s equations. Various conclusions are drawn from the graphs as well as from the calculated theoretical and practical values of A and B.

Thick cylinders have various applications which include gas, crude oil as well as water transmission systems whereby pipes act as cylinders and stresses are therefore evident when these pipes are subjected to internal pressure.

Section 2: Theory

Stress is the measure of the average amount of force per unit area of a surface within a deformable body on which internal forces act. Internal forces usually build up in pipes during transportation of materials such as gas. Stress is an important property in the criteria for selecting pipes used for gas transmission and distribution.

For thick cylinders such as pipes, hydraulic presses, high pressure hydraulic pipes, wall thickness is relatively large and the stress variation across the thickness is also significant. The problem may be solved by considering an axisymmetry about z-axis and solving the differential equations of stress equilibrium in polar co-ordinates.

Consider a thick walled cylinder with open ends. It is loaded by internal pressure Pi and external pressure Po as seen below. It has inner radius ri and outer radius ro.

Figure 1: A thick cylinder with both external and internal pressure.

Circumferential stress in a cylinder as a result of internal or external pressure is called hoop stress. Hoop stress is mechanical stress acting on rotationally-symmetric objects as a result of forces acting circumferentially (perpendicular both to the axis and to the radius of the object). It is a component of the stress tensor in cylindrical co-ordinates, other components including radial and axial stresses.

Any force applied to an object with rotational symmetry is usually broken down into components parallel to the cylindrical co-ordinates r, z, and θ. These components of force induce corresponding stresses: radial stress, axial stress and hoop stress, respectively.

A classic example of hoop stress is the tension applied to the iron bands, or hoops, of a wooden barrel. In a straight, closed pipe, any force applied to the cylindrical pipe wall by a pressure differential ultimately leads to hoop stresses. Similarly, if this pipe has flat end caps, any force applied to them by static pressure will induce a perpendicular axial stress on the same pipe wall. Thin sections often have negligibly small radial stress, but accurate models of thicker-walled cylindrical shells take such stresses into account.

A thick cylinder has stress in the radial direction as well as circumferential and longitudinal stresses. By rule of thumb, radial stress becomes important when the wall thickness is greater than 1/20th of the diameter. The difference between a thick cylinder and a thin cylinder is that a thin cylinder has its wall thickness less than 1/20th of the diameter. Also, unlike thin cylinders, the radial stress in thick cylinders are not small but instead, varies from inner surface where it is equal to the magnitude of the fluid pressure to the outer surface where its most times equal to zero if exposed to the atmosphere. Hoop stress in thick cylinders also varies with thickness.

The classic equation for hoop stress created by an internal pressure on a thin wall cylindrical pressure vessel is:

(for a cylinder)………………………………………………………………………… (1)

(for a sphere)…………………………………………………………………………… (2)

where

P is the internal pressure

t is the wall thickness

r is the inside radius of the cylinder.

is the hoop stress.

When the vessel has closed ends the internal pressure acts on them to develop a force along the axis of the cylinder. This is known as the axial stress and is usually less than the hoop stress.

………………………………………………………………………… (3)

Though this may be approximated to

……………………………………………………………………………………………………… (4)

Also in this situation a radial stress is developed and may be estimated in thin walled cylinders as:

……………………………………………………………………………………………… (5)

When the cylinder has a d/t ratio of less than 10 the thin-walled cylinder equations no longer hold since stresses vary significantly between inside and outside surfaces and shear stress through the cross section can no longer be neglected.

In order to calculate the stresses and strains, a set of equations known as the Lame’s equations must be used.

……………………………………………………………………………………… (6)

……………………………………………………………………………………………… (7)

where

A and B are constants of integration, and may be discovered from the boundary conditions

r is the radius at the point of interest (at the inside or outside walls)

It can be shown from boundary conditions;

=-p at and = 0 at that

A= and B= where are inside and outside radii of cylinder and P is internal pressure.

Lame's equations give the relationship of a radial distance, r, with radial and circumferential stresses and are based on the following assumptions;

• The material in focus being homogeneous and isotropic.

• The plane sections perpendicular to the longitudinal axis of the cylinder remaining plane even after the application of internal pressure.

• That the material is stressed within the elastic limit

• All the fibres of the material expand or contract individually without being constrained by the adjacent fibres.

Given the stresses occurring in a cylinder in two dimensions as radial and circumferential stress, corresponding strains will be radial and circumferential strains. Thus,

=………………………………………………………………………… (8)

………………………………………………………………………….. (9)

Where E = Young's modulus of elasticity

= Poisson's ratio

And

………………………………………………………………………………………………

(10)

……………………………………………………………………………………… (11)

These stress-strain equations are constitutive because they depend on what the part is made of and are derived from the axisymetric equation of equilibrium and the strain displacement equations shown below;

……………………………………………………………………… (12)

Where

And the strain displacement equations;

…………………………………………………………………… (13), and

……………………………………………………………………… (14)

Section 3: Description of Apparatus

The apparatus consists of a thick cylinder which has an outer radius of 152mm and inner radius as 77. 5mm fixed to ten strain gauges. A tapered plug was forced into the bore with the use of hydraulic ram to produce the internal pressure (pi) which is now lubricated with oil supplied from the second hydraulic cylinder. The results obtained are fed into the data logger which interprets the readings in micro strain.

Figure 2: Cut-away view of thick cylinder and plug

Figure 3:

Section 4: Procedure

The procedures to carry out this experiment are outlined below:-

A reliable datum was first achieved by setting all gauges to zero levels with the plug already inserted in the ring at about 0. 3mm.

The plug was then fit in the cylinder up to 4mm with o. 75mm increments.

The hydraulic ram was been pumped and the oil was used to lubricate the cylinder and the next plug penetration depth recorded at an increment of 0. 75mm.

The lubrication was still done at every new penetration depth, the load removed from the plug and the strain values recorded from the data logger.

Finally the hydraulic ram was carefully removed whilst holding the spacer bar firmly to avoid accidental discharge by the ram.

Section 5: Results

Experimental Data

Radial strain (negative)

Hoop strain (positive)

Penetration depth(mm)

1

3

5

7

9

11

13

15

17

19

0. 78

-10

-14

-17

-25

-37

44

50

57

63

74

1. 42

-22

-31

-42

-58

-110

79

95

109

131

159

2. 32

-42

-57

-76

-111

-200

131

159

181

221

274

3. 00

-53

-72

-105

-152

-204

176

207

242

292

362

4. 02

-73

-101

-144

-213

-320

238

278

326

395

493

Table A: strain gauge readings for hoop and radial stress

Where 1 to 9 = radial strain (negative values)

11 to 19 = hoop strain (positive values)

R (mm)

90. 5

103

115. 5

128

140. 5

1/r2(mm-2 10-5)

12. 21

9. 43

7. 50

6. 10

5. 07

â„“ Ñ³(10-6)

274

221

181

159

131

â„“ Ð³(10-6)

-200

-111

-76

-57

-42

Table B: Results for penetration depth of 2. 32mm for hoop and radial stresses.

The table above consists of â„“ r and â„“ Ñ³ at different depth

(penetrations).

Using values of â„“ r and â„“ Ñ³ at depth 1. 82mm the hoop stress and radial

stress is calculated below.

Using,

σÑ³ = ——————————————————————————(15)

σr = ——————————————————————————(16)

Calculations for hoop and radial stresses

At radius r = 90. 5 from Table B, using

=

Where, 1-μ2 = 1- (0. 3)2 = 0. 91

E= 208 KN/mm2 = 208-103 N/mm2

μ = 0. 3

@ r = 90. 5mm

= 208 x103 [274+ (0. 3x-200)]/ 0. 91 x 10-6

= 48. 91N/mm2

@ r = 103mm

σÑ³ = 208 x103 [221 + (0. 3x–111)]/ 0. 91 x -6

= 42. 90N/mm2

@ r = 115. 5mm

σÑ³ = 208 x103 [181 + (0. 3x-76)]/ 0. 91 x 10-6

= 36. 16N/mm2

@ r = 128mm

σÑ³ = 208 x103 [159+ (0. 3x–57)]/ 0. 91 x 10-6

= 32. 43N/mm2

@ r = 140. 5mm

σÑ³ = 208 x103 [131 + (0. 3x-42)]/ 0. 91 x 10-6

= 27. 06N/mm2

The values of for radius 90. 5mm, 103mm, 115. 5mm, 128mm, and 140. 5mm are (48. 91, 42. 91, 36. 16, 32. 43 and 27. 06)N/mm2 respectively.

For radial stress:

At radius r = 90. 5 from Table B.

Using

E = 208KN/mm2 = 208-103 N/mm2

μ = 0. 3

@ r = 90. 5mm

σð‘ Ÿ = 208 x103 [-200 + (0. 3×274)]/ 0. 91 x 10-6

= -26. 93N/mm2

@ r = 103mm

σð‘ Ÿ = 208 x103 [-111 + (0. 3×221)]/ 0. 91 x 10-6

= -10. 22N/mm2

@ r = 115. 5mm

σð‘ Ÿ = 208 x103 [-76 + (0. 3×181)]/ 0. 91 x 10-6

= -4. 96N/mm2

@ r = 128mm

σð‘ Ÿ = 208 x103 [-57 + (0. 3×159)]/ 0. 91 x 10-6

= -2. 13 N/mm2

@ r = 140. 5mm

σð‘ Ÿ = 208 x103 [-42+ (0. 3×131)]/ 0. 91 x 10-6

= -0. 62 N/mm2

The values of for radius 90. 5mm, 103mm, 115. 5mm, 128mm, 140. 5mm are (-26. 93,-10. 22 -4. 96, -2. 13 and -0. 62N/mm2) respectively.

Hence at depth 2. 32mm,

R (mm)

90. 5

103

115. 5

128

140. 5

1/r2(mm-2×10-5)

12. 21

9. 43

7. 50

6. 10

5. 07

â„“ Ñ³(10-6)

274

221

181

159

131

â„“ Ð³ (10-6)

-200

-111

-76

-57

-42

Ñ³(N/mm²)

48. 91

42. 90

36. 16

32. 43

27. 06

Ð³(N/mm²)

-26. 93

-10. 22

-4. 96

-2. 13

-0. 62

Table C: Table showing values of Hoop and Radial stress at depth of 2. 32mm

Also at depth 4. 02mm,

1/r²(mm-²10-5)

12. 21

9. 43

7. 50

6. 10

5. 07

σÑ³ (N/mm²)

90. 74

75. 68

64. 64

56. 62

49. 39

σr (N/mm²)

-39. 34

-21. 60

-10. 56

-4. 02

-0. 37

Table D: Table showing values of Hoop and Radial stress at depth of 4. 02mm

Hence considering values obtained at depth 2. 32mm and 4. 02mm, using equations 15 and 16, two graphs of σÑ³ , σr and 1/r2 were plotted and the following deductions made.

At depth 2. 32mm,

Pressure = -27. 3N/mm2

Radial stress gradient (σr. g) = = (10. 22-0. 62) / (9. 43-5. 07) X 10-â µ = 2. 20 X 10â » â µ

Hoop stress gradient (σÑ³. g) = = (48. 91-27. 06) / (12. 21-5. 07) x 10-â µ = 3. 06 X 10â » â µ

Hence Average gradient (B) = (σr. g+ σÑ³. g) / 2

= (2. 20+3. 06) /2 = 2. 63 X 10â » â µ

The intercept A = 11. 5

And the theoretical values can be calculated using lames theory, where pressure, Râ‚€ is 152mm and Râ‚ IS 77. 5mm.

Thus A = = =

B = = =

Then σÑ³ = = 36. 45N/mm^2

σð' Ÿ = A – = -17. 27N/mm^2

EXPERIMENTAL VALUES

THEORETICAL VALUES

A(N/mm2)

11. 5

9. 59

B(N)

2. 63X 10â ∎ » â ∎ µ

2. 216 x 10â ∎ µ

Table E: Table showing values of Experimental and Theoretical at depth of 2. 32mm

% Error =

=

= 19. 92%

Also, at a depth 4. 02mm a graph of σÑ³ , σr and 1/r2 was plotted and the following deductions made.

Pressure = -52. 5N/mm2

Radial stress gradient (σr. g) = = (21. 60-0. 37) / (9. 43-5. 07) = 4. 869

Hoop stress gradient (σÑ³. g) = = (90. 74-56. 62) / (12. 21-9. 43) = 12. 273

Hence Average gradient (B) = (σr. g+ σÑ³. g) / 2

= (4. 869+12. 273) / 2 = 8. 571

The intercept A = 22

Also the theoretical values can be calculated using Lame’s equations, where pressure, Râ,€ is 152mm and Râ, is 77. 5mm.

Thus A = = =

B = = =

Then σÑ³ = = 70. 45 N/mm^2

σ r = A – = -33. 57N/mm^2

EXPERIMENTAL VALUES

THEORETICAL VALUES

A

22

18. 442

B

8. 571

4. 26x 10â µ

Table F: Table showing values of Experimental and Theoretical at depth of 4. 02mm

% Error =

= = 19. 29%

Section 6: Discussion

From the results of the experiment and graphs, it can be can be deduced that there is a resultant increase in force being exerted on the cylinder as a result of increasing plug penetration values. Thus, the internal pressure of the thick cylinder also increases and leads to an increase in strain gauge readings.

The graph of hoop and radial stresses versus 1/r2 show a similar progression of both the hoop stress and radial stress and also a parallel movement to the radial position axis.

The values of A and B deduced experimentally from the graph are greater than the theoretically calculated values of A and B. This might be as a result of some sources of error when carrying out the experiment.

PENETRATION DEPTH(mm)

EXPERIMENTAL VALUE OF A

THEORETHICAL VALUE OF A

EXPERIMENTAL VALUE OF B

THEORETICAL VALUE OF B

2. 32

11. 5

9. 59

2. 63

2. 216

4. 02

22

18. 442

8. 571

4. 261

Table G: Table showing values of experimental and theoretical values of A and B at penetration depths of 2. 32mm and 4. 02mm.

Sources of Error

Some of the sources of errors, which may have affected the results, are listed below:

Inadequate lubrication of the cylinder from the oil pump.

Not zeroing the data logger before taking readings.

Excess pressure exerted by hydraulic ram.

Error due to approximation of values.

Section 7: Conclusion

The importance of Lame's theory in stress-strain analysis over various penetration depths for a thick cylinder under internal pressure is shown in this experiment which also sheds light on the importance of stress distribution determination, an important property for selection consideration for appropriate materials.

Stress in pipes is always caused by an evident build up of internal pressure during gas transmission. Hoop stress is largest when r is smallest (this is the same for radial stress), and therefore cracks in pipes should theoretically start from inside the pipe as a result of internal pressure which leads to subsequent development of stresses within pipes similar to thick cylinders.

Section 8: References

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